

A note on Horwich's notion of grounding

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Abstract Horwich (Deflationism and paradox, Oxford University Press, Oxford, pp 75–84, 2005) proposes a solution to the liar paradox that relies on a particular notion of grounding—one that, unlike Kripke's (J Philos 72:690–716, 1975) notion of grounding, does not invoke any “Tarski-style compositional principles”. In this short note, we will formalize Horwich's construction and argue that his solution to the liar paradox does not justify certain generalizations about truth that he endorses. We argue that this situation is not resolved even if one appeals to the ω -rule. In the final section, we briefly discuss how Horwich might respond to the situation.

Keywords Truth · Liar paradox · Deflationism · Minimalism · ω -Rule · Kripke's theory of truth

1 Introduction

According to the minimalist position (Horwich 1998b), the basic axioms of truth are instances of the so-called Equivalence Schema

The proposition that p is true if and only if p

An immediate problem for this account is that some instances of the Equivalence Schema may lead to contradictions due to the liar and other semantic paradoxes. This raises the question of what axioms the theory of truth actually comprises—what are the admissible instances of the Equivalence Schema?

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In this short note we shall be concerned with Horwich's response to this problem. The plan is as follows. In the next section we will look at a suggestion made by Horwich in his paper (2005) on how to characterize the admissible instances of the Equivalence Schema, which involves the idea of grounding. While this leads to a definite theory of truth, we will show that no interesting generalizations about truth (such as 'A conjunction is true if and only if both conjuncts are true'), which Horwich deems to be true, are entailed by the theory. This is a problem for Horwich because "an adequate theory of *any* phenomenon (e.g., truth) must explain *all* the facts concerning that phenomenon (e.g., general facts about truth)." (Horwich 2005, p. 83) In the third section, we will therefore look at a second formalization that involves the ω -rule. While the appeal to this rule is somewhat problematic and has been criticized, it is nevertheless a move that Horwich has made at several places. However, we will show that even by appealing to the ω -rule, the problem of deriving general facts about truth is still not resolved. In the final section, we briefly discuss how Horwich might respond to this situation.

2 The semantic paradoxes and grounding

Horwich (1998b, pp. 40–42) considers four possible solutions to the liar paradox:

1. to reject classical logic;
2. to adopt a typed theory of truth;
3. to deny that paradoxical sentences express propositions;
4. to reject certain instances of the Equivalence Scheme.

He claims that (1) and (2) are too radical and that (3) is wrong. The argument goes roughly as follows. For any condition $C(x)$, i.e., predicate with one free variable, it is possible to believe that the proposition satisfying that condition is false. Since any object of belief is a proposition, this implies that paradoxical propositions exist. For instance, let $C(x)$ be the condition 'x is being said by the least intelligent person in the room'. Then it is easy to imagine circumstances in which someone might say, and firmly believe, 'What is being said by the least intelligent person in the room is false'. Hence, since any object of belief is a proposition, 'What is being said by the least intelligent person in the room is false' must express a proposition. However, it may just so happen that the least intelligent person in the room is identical with the person believing that proposition. Hence that proposition would be paradoxical in classical logic.

Horwich therefore opts for the fourth route, to deny that all instances of the Equivalence Schema are correct. In an often cited remark, he proposes three constraints on the restriction on the Equivalence Schema:

- (a) that the minimal theory not engender 'liar-type' contradictions;
- (b) that the set of excluded instances be as small as possible; and—perhaps just as important as (b)—
- (c) that there be a constructive specification of the excluded instances that is as simple as possible. (Horwich 1998b, p. 42)

Many authors have interpreted that passage as saying that the minimalist theory will consist of the largest possible collection of instances of the Equivalence Schema that

does not generate paradoxes—e.g., Gupta (2000). However, McGee (1992) has shown that if the above constraints are interpreted as such a maximality criterion, i.e., that the set of acceptable instances of the Equivalence Schema ought to be a maximal consistent set of T-biconditionals, then that criterion will not give us a unique theory at all. There are uncountably many maximal consistent sets of T-biconditionals which are pairwise inconsistent and, moreover, none of them is recursively enumerable (semi-computable). Thus, the maximality constraint doesn't give us a unique theory and none which has a constructive specification.

The upshot of McGee's result is that one needs another (or additional) criterion for singling out the acceptable instances of the Equivalence Schema. Horwich (2005) considers the popular idea that an instance of the Equivalence Schema is acceptable as long as the proposition is grounded in the non-truth-theoretic facts. However, he rejects Kripke's celebrated approach (Kripke 1975) because it "invokes Tarski-style compositional principles" (p. 82, footnote 11). Horwich suggests, however, that the concept of grounding may be adapted in such a way that it squares with minimalism. The idea is to use a Kripke-like construction but one that merely involves the notion of logical consequence. More precisely, he proposes to regard a sentence as grounded if and only if that sentence or its negation

is entailed either by the non-truth-theoretic facts, or by those facts together with whichever truth-theoretic facts are 'immediately' entailed by them (via the already legitimized instances of the equivalence schema), or ... and so on. (Horwich 2005, p. 81)

We may try to formalize Horwich's notion of groundedness as follows. As our toy language, we will use the language of Peano arithmetic augmented with a primitive unary predicate Tr . We identify a sentence with its Gödel number (relative to some fixed coding). Our language contains a name for each sentence φ , namely the numeral of its code, denoted by $\ulcorner \varphi \urcorner$. We will identify the non-truth-theoretic facts with the set of sentences that are true in the standard model of arithmetic, \mathbb{N} . We denote this set of sentences by $Th(\mathbb{N})$, the theory of \mathbb{N} . For any set of sentences, X , let $X^- := \{\neg\varphi \mid \varphi \in X\}$ and let $T \upharpoonright X$ denote the T-schema restricted to members of $X \cup X^-$. Now, define by transfinite induction

$$\begin{aligned} H_0 &:= Th(\mathbb{N}) \\ H_{\alpha+1} &:= \{\varphi \mid H_\alpha \cup T \upharpoonright H_\alpha \models \varphi\} \\ H_\gamma &:= \bigcup_{\alpha < \gamma} H_\alpha \end{aligned}$$

where \models is the relation of logical consequence in classical first-order logic.

Since logical consequence is monotonic,¹ this sequence of sets reaches a fixed point,² which we denote by H . Now we may let the set of grounded sentences, G , be defined as follows:

¹ That is, $\Sigma \models \varphi$ implies $\Gamma \models \varphi$ for all sets of premisses $\Gamma \supseteq \Sigma$.

² That is, there is an ordinal α such that $H_\alpha = H_\beta$ for all $\beta \geq \alpha$.

$$G := H \cup H^-$$

As in the case of Kripke's notion of grounding, there will be no effective method for enumerating all the grounded sentences, because the non-truth-theoretic facts are not recursively enumerable. Of course, one might replace $Th(\mathbb{N})$ by the set of theorems of Peano arithmetic, thus obtaining a recursively enumerable set of grounded sentences. Perhaps this is preferable for another reason as well. The definition of $Th(\mathbb{N})$ involves the notion of truth in a model, and it is not clear whether this notion is available to the minimalists.

Either way, it is not hard to see that the fixed point, H , of the above hierarchy is reached after the first limit ordinal, that is:

Proposition 1 $H_\omega = H_{\omega+1}$

It suffices to show $H_{\omega+1} \subseteq H_\omega$. The other inclusion follows from the monotonicity of logical consequence. So let $\varphi \in H_{\omega+1}$. Then by definition $H_\omega \cup T \upharpoonright H_\omega \models \varphi$. By compactness of first-order logic, there must be some finite $\Sigma \subseteq H_\omega \cup T \upharpoonright H_\omega$ such that $\Sigma \models \varphi$. Since Σ is finite, we can find some $n \in \omega$ such that $\Sigma \subseteq H_n \cup T \upharpoonright H_n$. Since logical consequence is monotonic, $H_n \cup T \upharpoonright H_n \models \varphi$ and therefore $\varphi \in H_{n+1} \subseteq H_\omega$.

This creates an immediate problem. First, it is not possible to show that infinite iterations of applications of the truth predicate to a non-truth-theoretic sentence are grounded. That is, while e.g., $Tr\ulcorner 1 \urcorner = 1\urcorner$, $Tr\ulcorner Tr\ulcorner 1 \urcorner \urcorner = 1\urcorner\urcorner$, and so on will turn out to be grounded and true, the statement $Tr^{\omega}\ulcorner 1 \urcorner = 1\urcorner$ won't.³ Perhaps this is not much of a problem after all. One might think that such infinite iterations of applications of the truth predicate to a sentence are not important, as they don't add any content to the plain claim that $1 = 1$.

However, a more serious problem is that no non-trivial truth-theoretic generalization will turn out to be grounded. For example, while both $Tr\ulcorner \varphi \urcorner$, $Tr\ulcorner \neg\varphi \urcorner$ will be in G for every arithmetical sentence φ , the (formalization of the) sentence 'For all sentences of arithmetic, x , either x is true or the negation of x is true' won't be in G , because in general universal statements are not implied by the set of their instances.

Hence, while the above definitions lead to a definite theory of truth, it does not seem to lead to a *good* theory of truth, because "a good theory of truth [...] is a body of axioms that can explain all the facts about truth—and such facts include generalizations." (Horwich 2005, p. 84, fn 14)

3 Adding the ω -rule

Perhaps Horwich can help himself by appealing to the ω -rule. The ω -rule is an infinitary rule that allows us to derive a universal statement $\forall x \varphi$ whenever we have a proof of $\varphi(\bar{n})$ for every $n \in \omega$. Horwich actually considers that option (Horwich 2005, p. 84, fn 14):

³ The expression $Tr^{\omega}\ulcorner 1 \urcorner = 1\urcorner$ is shorthand for $\forall x T\bar{f}(x, \ulcorner \varphi \urcorner)$, where \bar{f} is a function symbol for the function f that maps the code of a sentence to the code of the sentence preceded by $x + 1$ -many applications of the truth predicate, Tr . For details, see Halbach (2011, p. 157).

As for the minimalist, he needs to show how general facts about truth could be explained in terms of what he alleges to be the *basic* facts about truth—i.e., facts of the form, ‘ $\langle p \rangle$ is true $\leftrightarrow p$ ’. But he is licensed to cite further explanatory factors (as long as they do not concern truth). And this license yields a solution. For it is possible to suppose that there is a truth-preserving rule of inference that will take us from a set of premises attributing to each proposition of a certain form some property, G , to the conclusion that *all* propositions have property G . And this rule—not *logically* valid, but none the less necessarily truth-preserving given the nature of propositions—enables the general facts about truth to be explained by their instances. [...] The idea comes from Tarski himself that generalizations about truth may be deduced from their instances by means of some such rule (“infinite induction”). (Horwich 2005, p. 84, fn 14)

What Horwich has in mind here seems indeed to be the ω -rule. Horwich’s appeal to the ω -rule has been criticized by Raatikainen (2005). The main problem here is that the ω -rule, because of its infinitary character, cannot be applied by any finite agent, not even an idealized one. Let us, however, set this problem aside and investigate whether the appeal to the ω -rule can actually help Horwich. Let us define

$$\begin{aligned}
 H_0^* &:= Th(\mathbb{N}) \\
 H_{\alpha+1}^* &:= \{\varphi \mid H_\alpha^* \cup T \upharpoonright H_\alpha^* \vdash_\omega \varphi\} \\
 H_\gamma^* &:= \bigcup_{\alpha < \gamma} H_\alpha^*
 \end{aligned}$$

where \vdash_ω refers to deducibility in ω -logic. Again, this construction is monotonic and therefore reaches a fixed point, which we will denote by H^* .

How strong is H^* ? Does it entail any non-trivial truth-theoretic generalizations? We will answer this question by embedding H^* into a well-known supervaluational fixed point theory by Kripke. We assume the reader is familiar with the basics of (Kripke 1975). We say that X is an *admissible expansion* of Y if and only if $X \supseteq Y$ and $X \cap Y^- = \emptyset$. Define

$$\begin{aligned}
 VF_0 &:= Th(\mathbb{N}) \\
 VF_{\alpha+1} &:= \{\varphi \mid \text{for all admissible } X \supseteq VF_\alpha : (\mathbb{N}, X) \models \varphi\} \\
 VF_\gamma &:= \bigcup_{\alpha < \gamma} VF_\alpha
 \end{aligned}$$

Here, $(\mathbb{N}, X) \models \varphi$ means that φ is true in the standard model of arithmetic when the predicate Tr is interpreted by the set of sentences X .

Let VF denote the fixed point of the VF_α -hierarchy. Of course, VF is Kripke’s minimal fixed point theory relative to van Fraassen’s supervaluational scheme.

The following result is straightforward:

Proposition 2 For all α , $H_\alpha^* \subseteq VF_\alpha$

The proof is by induction on α . The case $\alpha = 0$ and α limit are trivial. So assume as induction hypothesis that $H_\alpha^* \subseteq VF_\alpha$ in order to show $H_{\alpha+1}^* \subseteq VF_{\alpha+1}$. Let $\varphi \in H_{\alpha+1}^*$. By definition of $H_{\alpha+1}^*$ this means that

$$H_\alpha^* \cup T \upharpoonright H_\alpha^* \vdash_\omega \varphi \tag{1}$$

We will show for all admissible $X \supseteq VF_\alpha$ that

$$(\mathbb{N}, X) \models H_\alpha^* \cup T \upharpoonright H_\alpha^* \tag{2}$$

Then (1) and (2) will imply that $(\mathbb{N}, X) \models \varphi$ for all admissible $X \supseteq VF_\alpha$, because the ω -rule is valid in all models of the form (\mathbb{N}, X) . This in turn will imply that $\varphi \in VF_{\alpha+1}$, as desired.

We first show that $(\mathbb{N}, X) \models H_\alpha^*$. By induction hypothesis $H_\alpha^* \subseteq VF_\alpha$. Since $VF_\alpha \subseteq VF_{\alpha+1}$ this implies, by definition of $VF_{\alpha+1}$, that $(\mathbb{N}, X) \models H_\alpha^*$ for all admissible $X \supseteq VF_\alpha$.

Now we show that $(\mathbb{N}, X) \models T \upharpoonright H_\alpha^*$. Recall that $T \upharpoonright H_\alpha^*$ denotes the T-schema restricted to all members of H_α^* and their negations.

If $\varphi \in H_\alpha^*$ then $(\mathbb{N}, X) \models \varphi$ by previous argument. Clearly $(\mathbb{N}, X) \models Tr \ulcorner \varphi \urcorner$ because $\varphi \in H_\alpha^* \subseteq VF_\alpha \subseteq X$ for every admissible X . Hence $(\mathbb{N}, X) \models Tr \ulcorner \varphi \urcorner \leftrightarrow \varphi$.

Now assume $\varphi \in (H_\alpha^*)^-$. Hence φ has the form $\neg\psi$ for some $\psi \in H_\alpha^*$. Then $(\mathbb{N}, X) \models \psi$ by the above argument. Since $\psi \in H_\alpha^*$, no admissible $X \supseteq VF_\alpha$ can contain $\neg\psi$ (recall that by induction hypothesis, $VF_\alpha \supseteq H_\alpha^*$). Hence $(\mathbb{N}, X) \models \neg Tr \ulcorner \neg\psi \urcorner$. Therefore, $(\mathbb{N}, X) \models \neg Tr \ulcorner \neg\psi \urcorner \wedge \psi$, which implies $(\mathbb{N}, X) \models Tr \ulcorner \neg\psi \urcorner \leftrightarrow \neg\psi$.

This concludes the proof.

It is not hard to see that some interesting general facts about truth are actually entailed by (are included in) H^* —for example, the statement that all arithmetical (i.e., not truth-theoretic) sentences are either true or false (have a true negation). So the appeal to the ω -rule solves some problems that the previous definition of groundedness encountered. However, not all is well.

It is a well-known fact that the following statements (suitably formalized) are not true in VF :⁴

1. For any sentence x , the double negation of x is true if and only if x is true;
2. A conjunction is true if and only if both conjuncts are true;
3. A disjunction is true if and only if one of its disjuncts is true;
4. If a conditional and its antecedent are true then so is the consequent;
5. No sentence is both true and false;
6. Every sentence is either true or false;
7. A sentence is true if and only if its negation is not true.

Hence, by Proposition 2, none of these sentences is entailed by H^* .⁵

⁴ See for example Field (2008, chap. 11).

⁵ Notice that in (1)–(7), the quantifiers range over all sentences, even those containing the truth predicate.

4 Discussion

According to Horwich, some or perhaps all of the above truth-theoretic generalizations are true. For example, in (Horwich 1998b, p. 77) he clearly endorses (5) and (6). (Their conjunction is equivalent to (7)). However, none of these generalizations turn out to be grounded according to his notion of groundedness—not even if we appeal to the ω -rule. So what can be done?

Perhaps one can simply provide a stronger notion of groundedness without invoking compositional principles for the truth predicate, i.e., using only tools that are acceptable from a minimalist point of view. Since we are admitting the ω -rule, one might think that this project has better chances of success now.⁶ For instance, Meadows (2015) has shown how to characterize the sentences that are true in the minimal Strong Kleene fixed point using an infinitary tableau system that only involves rules for the logical connectives, the ω -rule, and simple introduction and elimination rules for the truth predicate (namely, to infer φ from $Tr\ulcorner\varphi\urcorner$ and $\neg\varphi$ from $\neg Tr\ulcorner\varphi\urcorner$). Hence, at least in the Strong Kleene case, it seems possible to adopt Kripke's theory without invoking compositional principles for the truth predicate. Unfortunately, there are two problems with this proposal. First, Kripke's theory is a non-classical theory of truth, in violation of Horwich's desideratum that classical logic is not to be rejected. This problem can be overcome, however, by appealing to the classical closure of the fixed point, i.e., by declaring every sentence that is not true or false in the fixed point to be false. Either way, a major problem remains: Kripke's Strong Kleene theory (in either version) fails to entail many generalizations that Horwich and classical logicians endorse—e.g., it does not entail (5), (6), or (7).⁷ For that purpose, we would need a stronger valuation scheme. For example, if we require that the admissible expansions in the definition of the successor stage of the VF -construction are maximal consistent, then we do get (1), (2), (4), (5), (6), and (7).⁸ However, this suggestion is problematic as well. First, it is not clear that we can characterize the theory without invoking compositional clauses for the truth predicate.⁹ The second problem is that the theory will be non-classical again. Unfortunately, this time the move from the fixed point to its classical closure

⁶ Thanks to an anonymous referee for pressing that point.

⁷ See Field (2008, chap. 11.3) and Halbach (2011, chap. 15).

⁸ See Field (2008, chap. 11.3). Field calls the resulting theories *strong supervaluational theories*.

⁹ For instance, consider what Field calls *medium supervaluational theories*—the theories that are obtained by requiring that the admissible expansions in the VF -construction are classically consistent (a condition that is also imposed on the strong theories that we are discussing above). These were introduced by Cantini (1990). Meadows (2015) has developed an infinitary tableau system characterizing the minimal Cantini fixed point. Cantini (1990) has developed an infinitary Tait calculus doing the same job, and both Welch (2009) and Beringer and Schindler (2017) have developed infinite two person games with the same function. All of these systems contain specific conditions or rules for the truth predicate corresponding to the consistency requirement—rules that go beyond the simple introduction and elimination rule that we encountered during our discussion of the Strong Kleene fixed point—and it is quite dubitable whether they are acceptable from a minimalist point of view.

is of little help: it can be shown that the desired generalizations (6) and (7) are no longer true in the classical closure.¹⁰

A second response is simply to adopt (some of) the relevant generalizations as truth-theoretic axioms. This is indeed the preferred option of e.g., Halbach and Horsten (2005). This move does not seem to square with Horwich's minimalist position, though. Horwich insists that all facts about truth need to be explained on the basis of what he considers to be the *fundamental* facts concerning the notion of truth—namely, instances of the Equivalence Schema. Hence, if the generalizations in question do not follow from them (in some broad sense of 'following', which may involve the ω -rule), it would appear that they are simply not true according to Horwich's account.

This last point hints at a deeper problem underlying Horwich's account; it would appear that some generalizations *are* simply out of his reach. Even if it were possible, say, to select a maximal consistent set of instances of the Equivalence Schema among the uncountably many ones, some instances of the Equivalence Schema must be excluded—e.g., the instance given by the liar. Now, since some instances of the Equivalence Schema do not hold, it is not possible to account for some instances of, say, the principle of bivalence, (6). If we had both

The proposition that p is true if and only if p

and

The proposition that not p is true if and only if not p

then the law of excluded middle would allow us to derive

The proposition that p is true or the proposition that not p is true

But if p is 'paradoxical' we simply will not have the relevant instances of the Equivalence Schema at our disposal.¹¹ Hence, Horwich's adoption of the principle of bivalence seems to be in conflict with his account that all facts about truth need to be explained on the basis of instances of the Equivalence Schema.

Maybe Horwich could simply deny, despite our intuitions to the contrary, that the principle of bivalence and other truth-theoretic generalizations are actually true. Horwich offers both an account of the meaning of the word 'true' as well as the property of truth, that is truth itself. While the axioms for the property of truth consist in instances of the Equivalence scheme, the meaning of the word 'true' is given by specifying a fundamental acceptance property in terms of which our overall use of the word is best explained. The fundamental acceptance property governing our use of 'true' is our inclination to *accept* instances of the Equivalence Schema. This account of the meaning of 'true' is part of a larger use-theoretic account of meaning that is developed in more detail in (Horwich 1998a). Similarly, Horwich offers an account as to why we accept certain truth-theoretic generalizations in addition to offering an account as to why such generalizations are (allegedly) true (Horwich 2005, pp. 83–84).

¹⁰ As Field (2008, p. 180) points out, no non-trivial fixed point can contain either the liar sentence or its negation; hence (6) cannot be true in a classical model if the truth predicate is interpreted by the fixed point. Hence (7) cannot be true there either, because (7) entails (6) in classical logic.

¹¹ A similar observation was made by Beall and Armour-Garb (2005, p. 93).

Now, while Horwich denies that all instances of the Equivalence Schema are correct about the property of truth, he claims that

this is not an overwhelming difficulty for the supposition that what we *mean* by ‘true’ is captured by the equivalence schema. For although certain instances yield contradictions, it might be argued that anyone who means what we do by ‘true’ has a certain inclination to accept even those instances—an inclination that is overridden by the discovery that they lead to contradictions. Indeed, one might suppose that it is only because we have such an inclination that the ‘liar’ sentence present us with a *paradox!* (Horwich 1998b, p. 136)

Perhaps a similar move could be made regarding certain generalizations about truth. The idea is that our inclination to accept certain generalizations, such as the principle of bivalence, can be explained on the basis of our inclination to accept *all* instances of the Equivalence Schema.¹² But just as the Equivalence Schema actually holds only for some but not all propositions, so does the principle of bivalence only hold for some but not all propositions. It may only hold for the non-truth-theoretic propositions, or only the grounded propositions, or what have you. And, as we have seen, some of these restricted generalizations can be accounted for if one appeals to the ω -rule.

If this route is taken, one does not need to go so far as to invoke the ω -rule. Halbach (2001) and, more recently, Horsten and Leigh (2017) have shown that reflection principles already get one quite far. Reflection principles are schemata of the form

$$Prov_S(\ulcorner\varphi\urcorner) \rightarrow \varphi$$

where $Prov_S$ is some (standard) provability predicate for some theory S . In the above mentioned articles, it is shown that certain interesting generalizations about truth can be obtained by adding iterated reflection principles to disquotational theories of truth. While it is true that one can account for more generalizations using the ω -rule, reflection principles have the obvious advantage that they allow one to stay within the confines of first-order logic.

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¹² However, this argument was criticized by Armour-Garb (2010).

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