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A critical analysis of the benefits and problems of shifting in and out of mathematical register in a Year 9 class

Elizabeth Morland

(PGCE Secondary Mathematics, 2020-2021)

email: morland.elizabeth@gmail.com

Abstract

This case study examines the differences between mathematical and everyday language, the need for both kinds of speech in the classroom, as well as some of the problems that can occur. Lesson observations, written work and interviews were conducted with the students and teachers of a Year 9 middle-attaining set in a UK comprehensive school. This data is used to explore how teachers switch between registers, the benefits and drawbacks of different translations between registers and ambiguity created by the different meanings of the word "regular". The study concludes that, while two registers are in play, the boundaries between mathematical and everyday speech are often blurred. This creates a specific set of problems around how to interpret speech in the classroom and how to encourage students' use of mathematical register.

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A critical analysis of the benefits and problems of shifting in and out of mathematical register in a Year 9 class

Elizabeth Morland

Introduction

One of the early papers I read on mathematical language use starts with an anecdote where Mrs. Miller used the language of ‘bottom number’ instead of ‘denominator’ when teaching Juan about adding fractions. After a test in which ‘denominator’ was not understood, “Mrs. Miller explains that denominator is the formal term for the ‘bottom number.’ Juan exclaims, ‘I know how to find the least common bottom number!’” (Hughes, Powell & Stevens, 2016, p.2). The paper concludes that “in simplifying her language without connecting this informal language to formal mathematics language, she did Juan a disservice.” (ibid.).

I was left with a number of questions. Would it be true to say that Juan *does* know how to find the least common denominator – to what extent is understanding mathematics the same as understanding the language we use to talk about mathematics? Had Juan been in a classroom in Germany we would not fault his mathematical ability if he could not understand the term “lowest common denominator” but understood “kleinster gemeinsamer Nenner”. If it is not in knowing arbitrary terms, how does mathematical knowledge intersect with and rely upon mathematical language? Is it true to say that Mrs. Miller has done “Juan a disservice”? While one of Juan’s test scores was lower than it could have been, his immediate understanding of the question once the term is explained shows that he does have the mathematical understanding. Many children are first introduced to fractions in informal language. While our ultimate goal is to move students eventually towards more formal ways of speaking mathematically, how are we to judge what the right pace for that transition is?

The benefits of formal mathematical language are not to be underestimated. Adler (1998, p.26), exploring mathematical language issues in multilingual South African classrooms, writes about one teacher who would “‘run out of words’ if she were to try to explain mathematics in Tswana”, her main language.

While "denominator" translates concisely into everyday English, most mathematical terms do not. In this paper I will explore some of the complexities about the relationship between mathematical understanding and mathematical register. Drawing on the existing body of educational research into mathematical language use I will lay out what constitutes mathematical register and its importance for teaching mathematics as well as the issues and challenges that arise in its use in the classroom, and then go on to explore some of these issues in a Year 9 class.

Literature Review

Mathematical register and constructing mathematical knowledge

Communication is at the heart of the classroom (Morgan, 2017). Vygotsky's social-cultural theory of learning views language not only as a medium for social interaction but also for thought, inner speech providing a process by which social interactions teach us new ways of thinking (Dawes & Watson, 2017). This paper will focus specifically on the ways language is used differently in mathematical contexts from everyday life. Halliday's influential 1978 paper characterised these differences in his defining of "mathematical register", a concept which goes beyond subject-specific vocabulary:

> A set of meanings that is appropriate to a particular function of language together with the words and structures which express these meanings. We can refer to a 'mathematics register', in the sense of the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes.
>
> (Halliday, 1978, p.195)

Following Vygotsky, mathematical register is not only a way of communicating about maths, but also of constructing mathematical knowledge (Schleppegrell, 2007): learning to speak mathematically and learning mathematics are inseparable. By moving away from everyday language students also "move from everyday, informal ways of construing knowledge into the technical and academic ways that are necessary for disciplinary learning" (ibid., p.140).

The features of mathematical register

Schleppegrell breaks down the linguistic features of mathematical register into two categories: multiple semiotic systems and grammatical patterns (ibid., p.141).

Multiple Semiotic Systems

In addition to spoken and written language, mathematics extensively employs both algebraic symbolism and visual representations (Morgan, 2017). Veel (1999) notes that spoken language notably predominates in the mathematics classroom, because it "provides the link between the symbolic and visual representations for students and is therefore a powerful agent in the learning process" (p.189). However, while links can be made, "the meanings realized by one semiotic cannot be exactly replicated by another" (O'Halloran, 1999, p.1). O'Halloran goes onto describe an example from a secondary-school mathematics classroom where a teacher narrates a verbal problem – "a man is actually at this point here he is climbing a cliff and /ahh doesn't know how high up he is" – accompanied by the diagram in Figure 1 (1999, p.24). Here we can see all four semiotic systems at work: spoken language (teacher's narration); written language ("river"); algebraic symbolism ("h - 10"); and visual representation (diagram). Mathematical meaning is built by cross-referencing between these different mediums (O'Halloran, 1999).

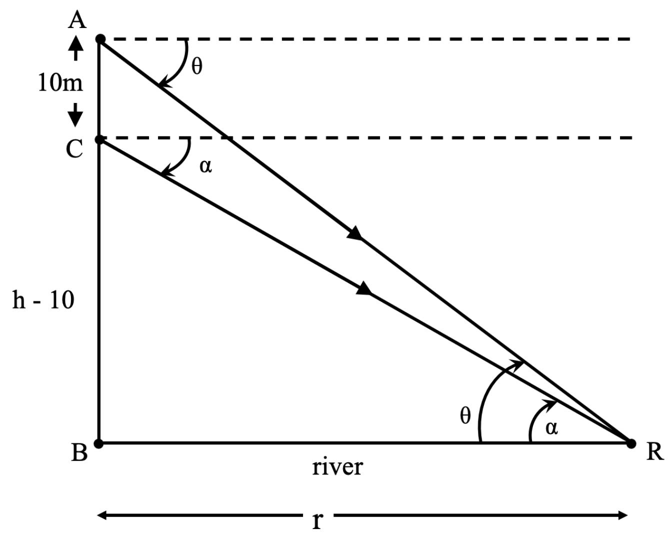

Figure 1: Diagrammatic representation of trigonometric problem

(redrawn from O'Halloran, 1999, p.24)

While O'Halloran's four systems encompass much of mathematical discourse, some aspects of mathematical communication fall outside or between them. For example, Pimm (1987, p.143) discusses the use of pictograms, such as the angle-symbol "∠", which arguably fall between written and diagrammatic mediums. O'Halloran's systems also downplay how physical space and time are used to express mathematical concepts: when the teacher says "a man is actually at *this* point here" they were probably pointing to the diagram – while spoken language provides some link, physical deixis is an important bridge between text and image. O'Halloran's systems also do not encompass the "enactive" element in Bruner's Enactive-Iconic-Symbolic framework (Dawes & Watson, 2017, p.43). There may be an expectation here that in higher mathematical discourse discussions will have moved beyond the enactive mode, but algebra tiles and other manipulatives remain important for conveying mathematical meaning in the classroom.

Grammar and vocabulary

Mathematical language has different standard grammars from everyday language—long, dense noun phrases such as "the volume of the rectangular prism with sides 8, 10 and 12 cm" (Schleppegrell, 2007, p.143); logical language which involves ascribing technical meaning to conjunctions and other small words like “any” (Rowland, 2001); the use of relational processes associated with the verbs “to be” and “to have” (Schleppegrell, 2007). The most obvious difference between mathematical and everyday language, though, is in technical vocabulary with precise formal definitions (Morgan, 2017). The amount of this specialised vocabulary is substantial: The National Centre for Excellence in Teaching Mathematics produced a glossary targeted at Key Stage 1-3 containing over 450 words (NCETM, 2014). This lexicon contains words only used within a mathematical context (e.g. “trapezium”) and also words with different everyday English meanings (e.g. “similar”) (Pimm, 1987).

Mathematical register in the classroom

Understanding and using mathematical register is important in a classroom setting. The National Curriculum states that, “the quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematical justification, argument or proof” (DfE, 2013, p.3). Hence teachers need both to be able to use mathematical register themselves, and to encourage and develop students’ ability to use it.

However mathematical is not the only type of speech that occurs in the mathematics classroom. Teachers typically switch frequently between everyday and mathematical registers (Lane, O’Meara & Walsh, 2019). Everyday language may be used to support learning, providing a bridge to understanding a mathematical concept, and to regulate classroom activity, and students use everyday language for a variety of reasons including lack of proficiency or discomfort with mathematical register or off-task discussion (Morgan & Alshwaikh, 2012). It is because students may not yet be proficient that the bridging use of everyday English is so important. As learners of a new language, students require translation of new mathematical terms into a language that they understand.

Moschkovich (1999) describes the technique of “revoicing”, when a teacher rearticulates something that a student has said in a way that is “closer to the standard discourse practices of the discipline” (p.15). Moschkovich argues that this technique encourages student participation by accepting student contributions while moving them towards more formalised mathematical discussion. However, as with all translations, this change in language also leads to the possibility that a teacher may not have correctly interpreted the student's contribution (Schleppegrell, 2007, p.150). Moschkovich provides the following example:

> Student: 'The rectangle has par . . . parallelogram . . . and the triangle does not have parallelogram.'
>
> Teacher: 'He says that this [a triangle] is not a parallelogram'
>
> (Moschkovich, 1999, p.15)

Here it is unclear whether the student's unconventional "does not have parallelogram" is intended to convey that a triangle *is* not a parallelogram, as the teacher revoices, or alternatively that the triangle has no parallel sides.

The challenges of mathematical register

The use of mathematical register poses significant challenges to students (Morgan, 2017; Rowland, 2001; Schleppegrell, 2017), including that of miscommunication. As Rowland (2001) explains, while language is a medium of communication "it is simplistic to suppose that words and symbols act as pure vehicles of shared meaning" (p.183). Instead, he argues, successful communication is grounded in compatibility rather than sameness. Two individuals using the same word have different private meanings, but communication is possible when these meanings align sufficiently.

Focusing on vocabulary, formative research by Otterburn and Nicholson (1976) found that a large proportion of 15-16 age students taking GCSE-level qualifications showed significant gaps in their understanding of mathematical vocabulary. While 99.7% of students demonstrated a correct understanding of "multiply", this fell to 19% for "similar" and 11% for "trapezium". Follow-up studies in 1989 and 1993 showed similar results (Hardcastle & Orton 1993; Nicholson, 1989). Solely mathematical words, like "trapezium", may be difficult because they are unfamiliar; they are long and difficult to spell and say, particularly as their Latin or Greek etymological roots are no longer familiar to most students (Morgan, 2017). By contrast, borrowed words like "similar" cause issues when everyday meanings do not precisely align with the mathematical usage. Here in particular private meanings differ, especially as students may not be explicitly aware of the dual meanings. Rowland (2001) urges that "it is a challenging but worthwhile teaching task to try to tease out what sort of sameness the pupil is (intuitively) aware of" (p.182). Adams (2003) describes the process by which students develop definitions of mathematical words, moving through informal definitions as a grounding or introduction to understanding a formal definition (e.g. being able to point to a square -> a square is a four sided shape -> a square has four equal sides -> a square is a regular quadrilateral). Students' different definitional understandings may also not be straightforward to "tease out" (p.787).

Adams suggests that while everyday meanings can cause confusion, making "connections between children's prior understanding of [words] and the mathematical meaning" can also be useful in teaching mathematical vocabulary (ibid., p.788). This aligns with the strategy outlined by Gay and White (2002) of a verbal and visual word association grid. Here, along with the word itself and a definition in mathematical language, a personal example of the word is chosen, for example the intersection of streets in Figure 2a. For solely mathematical words, Gay and White acknowledge that "finding a personal example […] could be extremely difficult" (ibid., p.35). In this case they suggest that students could instead list characteristics (Figure 2b). In the bottom right of the grid an example is given, often drawing on a different semiotic system, such as a diagram or symbolic notation. Adams (2003, p.788) advocates the use of examples in teaching the definitions of mathematical words, although he also includes non-examples as another way of clarifying meaning and tackling common misconceptions.

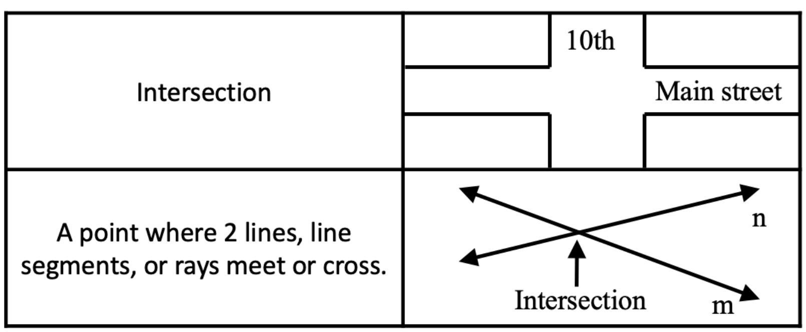

Figure 2a: Verbal and visual word association strategy diagram for intersection

(redrawn from Gay & White, 2002, p.34)

Variable	can stand for anything can be negative or positive or neutral
a variable is a letter that stands for a number	variable $x - 2 = 3$

Figure 2b: Verbal and visual word association strategy diagram for variable
(redrawn from Gay & White, 2002, p.34)

Research Questions

Given the importance of both mathematical and everyday English in the classroom, I was interested in the transitions, translations, connections, and blurred spaces between the two. This seemed a particularly fruitful topic for exploration as these in-between spaces have been characterised as simultaneously productive and as possible sites of misunderstanding.

RQ1 examines the way we shift in and out of mathematical register, whereas RQ2 focuses more specifically on one instance where a shift occurs, when rephrasing or translating between registers. Here I will explore more fully the positive and negative consequences of these translations. In RQ3 I will focus on where a single word has a different meaning in mathematical register from everyday English. Here there may be ambiguity about which register is being used, leading to a shifted set of problems related to prising out the differences of meaning.

RQ1 Why do teachers shift in and out of mathematical register?

RQ2 How is rephrasing statements from mathematical register to everyday English useful in the classroom and in what ways can it be harmful?

RQ3 What specific problems of register use arise from mathematical words that have a different meaning in everyday English?

Methodology

The class I worked with was a middle-attaining Year 9 set of approximately 25 students in a UK comprehensive school. I had multiple reasons for choosing this class. Firstly, the class teaching was shared between two teachers (an early-career teacher, who will be referred to by the pseudonym Teacher A, and the Head of Department, referred to as Teacher B) allowing me to compare their views and methods. Secondly, the class was studying a content block called Reasoning with Geometry: Deduction which covered solving angle problems involving polygons and parallel lines, as well as making conjectures with angles and shapes. These topics involved a large amount of technical vocabulary, including words they had encountered on multiple prior occasions (e.g. names of quadrilaterals) and those they might be less familiar with (e.g. "conjecture"). In addition, the topic drew strongly on the different semiotic systems involved in mathematical register, with copious use of diagrams, the need to provide written justifications for answers as well as some algebraic symbolic manipulation. The class was one I saw regularly but had not taken over teaching: given that a significant portion of my planned data collection was observation, using this class allowed me to undertake my research without disrupting my professional responsibilities.

I took a case-study approach to examine in depth how mathematical register was already being used in the classroom, rather than a specific intervention which might have better lent itself to action research. Some of the main concerns about this approach relate to generalisability and bias (Bell & Waters, 2014, p.12). The first concern is particularly relevant as the research took place during the period when Coronavirus restrictions were in place in schools. By describing the specific instance I was researching, I hope that readers will be able to decide to what extent my findings are applicable to other situations. I will address concerns about bias later in this section.

Data collection

My data collection comprised four different parts as laid out in Table 1. I collected data in multiple ways so that I could triangulate my findings in order to increase their validity (Cohen, Manion & Morrison, 2017, p.256). The order of collection was adapted from my initial plan, in which I had planned to conduct the teacher interviews first, to avoid a situation where, post-interview, the class teachers became hyperaware of their own vocabulary use. Despite this, the teachers' knowledge of my research topic did change their teaching methods, one teacher saying after a lesson "I tried to fit in as much vocabulary for you as possible". This could negatively affect how generalisable my findings are to situations where research was not being conducted.

Data collection method (in the order carried out)	Details
Lesson observations	Closely observed four lessons, one taught by Teacher A and three taught by Teacher B
Taught lesson	Taught lesson involving three different vocabulary related activities. Written work produced in the lesson was collected from all students.
Student interviews	Short ten-minute semi-structured interviews with three students
Teacher interviews	20-minute semi-structured interviews with each teacher

Table 1: Data collection methods in order of occurrence

Lesson Observations

During my four lesson observations, the majority of the students participated on Microsoft Teams, contributing using the chat function. One to two students were in school where the teacher was giving the online lesson, able to see the PowerPoint on a smart board and contribute verbally. I was able to supplement and cross-check the content of my observation notes by using recordings of the lesson, which were produced for students who might need to study asynchronously as part of remote learning.

I included observation as part of my data collection because I wished to observe language use in a way which disturbed it as little as possible. In addition, much of the prior research drew heavily on transcriptions from classroom observation, so having data in that form might allow easier comparisons. One of the main drawbacks of this approach is the possible introduction of bias, particularly as I was a solo observer (Bell & Waters, 2014, p.211). While recordings of the lessons allowed me to go back to certain points to check I had transcribed speech correctly, the moments I chose to focus on may well have been affected by my own expectations.

Taught lesson

The hour-long taught lesson included three different activities (see Table 2) and took place face-to-face with the class teacher, Teacher B, taking observation notes of the session. During online teaching I did not have access to student work and student responses were limited by the constraints of typing answers. I hoped that this lesson would allow me to hear a greater number of student responses as well as collect written work from the whole class, giving me a broader picture of the student group. One drawback was that the lesson's explicit focus on vocabulary was atypical and may have changed the students' language use.

Activity	Description
Word Scale	Placing thirteen words on a scale from mathematical to ordinary
Explanation Evaluation	Looking at four separate explanations of co-interior angles and writing what they liked and disliked about each
Diagram Description	A game of consequences where students described a mathematical diagram in as much detail as possible, after which a different student tried to re-draw it based on the description alone

Table 2: Descriptions of taught lesson activities

Interviews

I conducted interviews with three students and the two class teachers, to gain more insight into how both groups thought about mathematical register. These took a semi-structured form. I had a set list of questions, some shared between student and teacher interviews, which allowed easier direct comparison between responses. However, I retained the flexibility to follow up on points of interest as they arose. By probing further, I may have introduced bias by encouraging or leading the interviewees towards certain conclusions (Cohen, Manion & Morrison, 2017, p.513-514). However, I felt that it provided necessary encouragement, particularly for the student interviewees in an unfamiliar setting. One other possible source of bias was that one student who I approached did not give consent, and therefore was replaced. This means that my data collection may have excluded the most reticent members of the student body. One mitigant was that I was able to collect written work from all members of the class, allowing a more comprehensive view.

Ethical considerations

Throughout conducting my research, I followed BERA's Ethical Guidelines for Educational Research (2018).

The lesson observation and taught lesson parts of my data collection fell within the normal remit of my teaching and training activities. After informing the school and class teachers of the nature of these activities, no further consent was required from participants. Additionally, as part of the requirement for transparency, at the beginning of the taught lesson I explained to the class that this lesson would form part of my research project, detailing the scope and focus of that research.

Explicit consent was required for the interviews. At the beginning of each interview I re-explained the research project, outlined what the interview would involve and asked for consent from each interviewee, making it clear that participation was not mandatory. I further asked for consent to record the interview. One student withheld consent and so was not interviewed.

All data collected was anonymised and stored on devices with password protection or in the case of physical documents kept in locked rooms when unattended.

Finally, I do not believe that participants in this research were subject to any disadvantage or harm. Lesson observations did not interfere with student learning, and the taught lesson was designed to strengthen vocabulary use as well as providing data for the study. The lesson time that students missed for interviews was minimal.

Findings and discussion

In the following section I will address each research question in turn, linking my findings to existing research as well as discussing any limitations to my conclusions.

RQ1 Why do teachers shift in and out of mathematical register?

Figure 4 contains a transcript of three and a half minutes of a lesson taught by Teacher B. It covers the time spent on a single PowerPoint slide (Figure 3), where the aim was to prove or disprove a single mathematical conjecture. The majority of the students were participating via a Teams meeting, with a single student in the classroom.

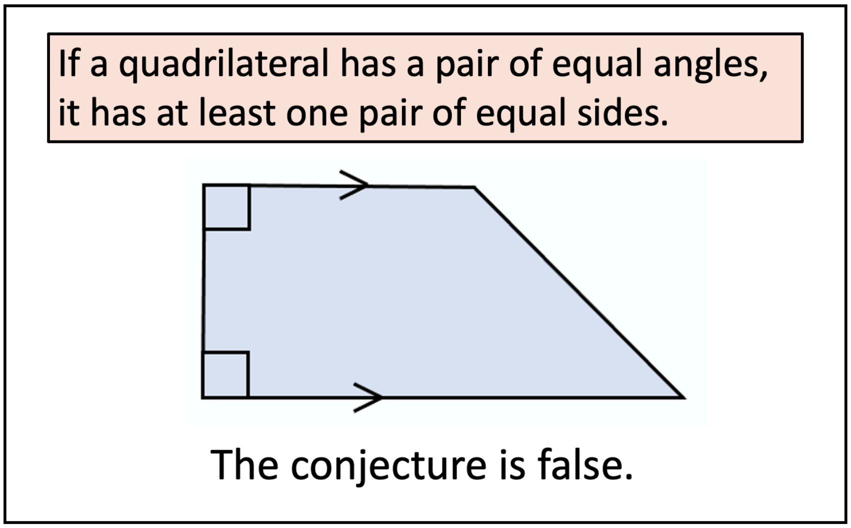

Figure 3: PowerPoint slide from a lesson observation

Through a process of re-reading the transcription I tried to identify different categories of speech that were occurring (see Table 3). While the categorisations were my own, many of my observations aligned with that of the literature, as discussed later in this section. As Veel (1999) noted, spoken language dominated the classroom: compare the length of the transcript with number of words on the slide. This weighting, particularly towards speech by the teacher, was perhaps even more extreme due to the Teams format, which made both student questioning and peer-to-peer discussion much more challenging.

Speech Mode	Description
Instructive	The teacher gives instructions or directions to students
Mathematical	The teacher talks about mathematical content using mathematical register
Narrative	The teacher describes what is occurring in the classroom
Meta	The teacher evaluates or analyses the mathematical discussion or mathematics teaching
Informal	The teacher talks about mathematical content using an informal or everyday register

Table 3: Categorisation of different speech modes

T: Alright, next one. If a quadrilateral has a pair of equal angles, it has at least one pair of equal sides. So we'll go true and false again. Quick vote. If a quadrilateral has a pair of equal angles, it has at least one pair of equal sides.

[pause]

What are we voting for? I've got two votes. Two votes. So think about giving some equal angles. I might be thinking right angles for this one. Ooo, lots of people going true. I guess there are lots, there are lots of quadrilaterals that have equal angles and equal sides. Can someone give me an example of a shape that has equal angles and equal sides?

S: Square?

T: Lovely example, a square has all equal angles and all equal sides. Any others?

S [typed]: rectangle

T: A rectangle? Again, equal angles, equal sides.

S [typed]: rhombus

T: A rhombus? Again, equal angles, equal sides. Again, we're not doing very well. What example helped us in the last one? It's often, it's often that pesky little shape. Can we draw a trapezium with equal angles…?

S: No

T: But no equal sides? I think quite often this is a mistake of the education system. We don't show you enough.

S: Does it have to be equal?

T: Well I think this is… [reveals answer] I don't think many people would consider that shape a trapezium. And I think this is possibly a mistake of the education system is that we quite often see the trapezium that sort of looks like, like um, you know when you see pictures of things at the circus, that's the only thing I can think of, or sort of a triangle with the top lopped off, yeah and… Quite often we don't think about this shape as a trapezium, but a trapezium is a quadrilateral, which this shape is, with one pair of parallel sides, so that is a trapezium. But I don't think that many people think about them. OK. So the conjecture is false.

Figure 4: Transcript of a discussion involving Teacher B of the conjecture "If a quadrilateral has a pair of equal angles, it has at least one pair of equal sides"

(See Table 3 for colour coding key)

Within Teacher B's speech I observed frequent switches between everyday and mathematical registers, as shown by the fragmented use of the mathematical speech mode within the transcript. The process of categorisation also highlighted the blurred boundaries between the two. For example, I categorised "So we'll go true or false again." as instructive speech. This phrase can be interpreted in a general non-mathematical way – students will have encountered true-or-false questions in a variety of non-mathematical contexts. However, in the context of mathematical conjectures, “true” and “false” certainly hold specific meaning, bringing with them acceptable standards of justification: proof and counterexample. My decision not to categorise the phrase as within the mathematical register was certainly subjective.

Everyday language was used for both the regulatory and learning purposes described by Morgan and Alshwaikh (2012). Both the instructive and narrative categories were largely regulatory in their aims, with the first providing students with direction about what they should be doing or focusing on. The narrative mode was particularly important given the nature of the classroom set up. Both votes cast and typed contributions were not visible to the student in the physical classroom, and not always visible to students online. The Teams format often made it much less clear what experiences were being shared by all students, for example in the case of screen-share lagging. However, even in a normal classroom, such shifts out of mathematical register and into a narrative mode are still necessary, for example when repeating near-inaudible student contributions or describing answers displayed on mini-whiteboards.

The other two non-mathematical speech modes I identified, related more closely to learning goals. The informal register was used to describe what could be called in more formal mathematical language a non-rectangular isosceles trapezium. Teacher B’s description that it “sort of looks like […] things at the circus, […] sort of a triangle with the top lopped off” draws both on the students’ wider experience outside of maths, and also more basic mathematical knowledge of triangles, in order to support student understanding. Perhaps the most obvious kind of support, that of a drawing, was not possible due to the Teams set up. Importantly, she highlights the distinction between the everyday and mathematical registers by repeated use of the phrase “sort of”. In another lesson, she again drew attention to differences in register, asking a student “Has your shape got a name? And don’t say, like, Bill.”, using humour to highlight the everyday meaning of “name”.

Attention was also brought to expectations around the use of the mathematical register through the use of meta-language, where the teacher analyses or evaluates the mathematical activity that is taking place. One common example of this is the use of praise, for instance the phrase “lovely example”. Here the teacher switches out of mathematical register in order to comment on the quality of the mathematical discussion taking place. Another example of this occurred in another lesson when a student was asked to name a pair of corresponding angles from a labelled diagram. For the student's first few incorrect responses Teacher A's speech remained within the mathematical register, explaining his mistake. However, after several attempts, she shifted out of mathematical register saying "Now you're just guessing". Mathematical register was being used as the medium of communication between mathematicians – when the teacher judged that the student was not acting in good faith as a mathematician she shifted out of the register.

By comparison, the shift towards mathematical register described in Moschkovich's concept of revoicing occurs exactly in cases where the teacher treats student contributions as having legitimate mathematical content. Once again, the Teams setting may have changed the types of revoicing that occurred, as it was harder and more time consuming for students to provide extended responses via chat. For this reason, teachers tended to use fewer open questions, often asking for Teams "Live Reactions" to a defined set of answers like the true or false format. Most text contributions comprised one to two words. This contrasts with Moschkovich's (1999) observations where more open discussions were taking place and students often responded more extendedly. However, because of these restrictions, revoicing did often occur in order to flesh out student contributions – when asked to give a reason why two angles were equal, a typed student response was "alternate", which Teacher A then expanded to "Angle BDC is equal to 42 degrees because alternate angles are equal".

In another lesson, the following exchange occurred:

> *Teacher B: What's the angle fact?*
>
> *S1: 360 degrees*
>
> *Teacher B: Yes, they add up to 360. Can you expand on that or can someone else expand? Can't just write 360 degrees.*
>
> *S2: angles in a circle add up to 360??*
>
> *S3: angles on a point*
>
> *Teacher B: Ooo, what has [S2] gone for? Ooo, you're so close. I don't like circle, because I don't think circles have any angles in them. I agree [S3]. I prefer angles around a point. I would steer clear from angles around a circle. I think that's something that's potentially picked up from primary school, which is fine. I would say that if we think about a circle as say a two-*

> *dimensional shape. There are no vertices on a circle so we can't have angles. What we mean is a full turn or angles around a point.*

Teacher B expands on S1's contribution by saying "they *add up* to 360". However, overall this discourse differs in a significant way from that described by Moschkovich. She writes that "by focusing on mathematical discourse, teachers can move beyond focusing on errors in English vocabulary or grammar to hear and support the mathematical content of what students are saying" (Moschkovich, 1999, p.12). In contrast, while the mathematical content of S2's statement is clear, Teacher B places importance in explicitly correcting the vocabulary use, rather than accepting the statement and then revoicing within the mathematical register.

This difference may be due in part to the students involved. Moschkovich's study focused on English language learners, whereas all the students in the class I studied used English as their first language. The students I observed were also significantly older (13-14 years old as opposed to 8-9 in Moschkovich's study). This may have led to different expectations about language use in the classroom. I also believe that the particular preference for "point" stems from the requirements of exam mark schemes. An Edexcel GCSE mark scheme states "C1 [communication mark]: (dep M3) for angles ADC, BCD and ABC correct and at least 2 appropriate reasons, eg […] angles at a point add up to 360°. Underlined words need to be shown" (Pearson, 2018, p.9). Importantly, the use of the specific word "point" is required for the communication mark to be awarded. The implication is that you "can't just write 360 degrees" *in an exam*. It would be interesting to consider to what extent the language used in classrooms reflects the register of exam papers rather than the register of academic mathematical discourse and in what ways these two registers differ from each other.

RQ2 How is rephrasing statements from mathematical register to everyday English useful in the classroom and in what ways can it be harmful?

This section will focus on two instances where mathematical terms were translated or rephrased into everyday English. First, I will consider near-synonymous terms used to refer to mathematical theorems, and the possible implications of their use. Then I will discuss the benefits and problems related to different explanations of types of angles in parallel lines.

While I did observe the term “theorem” being used with the class, most commonly with reference to Pythagoras’ Theorem, more commonly, when describing statements such as “co-interior angles add up to 180°” or “alternate angles are equal”, a synonym taken from a more everyday register was used. Teacher A referred to these statements as “reasons” or “rules” whereas Teacher B mostly referred to them as “angle facts”.

The use of multiple different terms caused some confusion. In the first lesson led by Teacher B, following on from Teacher A’s introduction of the topic, students were asked to indicate whether two angles in a diagram were congruent, supplementary or neither, and then whether they knew which “angle fact” was being used, by liking the appropriate comments in the chat, as can be seen in Figure 5.

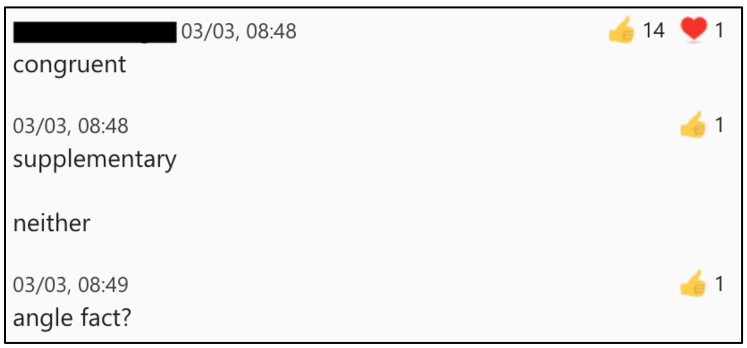

Figure 5: Screenshot from the Microsoft Teams chat box, where students were being asked to vote on the appropriate description for two angles in a diagram by liking the appropriate comment

While over half of students responded correctly to the first question, only one student thought they knew the related angle fact, despite having completed similar questions the previous lesson. Part way through the exercise the following exchange took place:

> *Teacher B: No one knows the angle fact! No one's going to go out there and say they know the angle fact?*
>
> *S: what do you mean by angle facts*
>
> *Teacher B: Good question […] So by angle fact I sort of mean the relationship between them. So lots of ones you’d have done in year seven are like “angles on a straight line add up to 180 degrees”. When you're looking for reasons, and Teacher A might have called them reasons. I like to call them facts because they are sort of stated and they are true. We know they absolutely must be true […] It’s those reasons that we can sort of tell what different angles are worth.*

As the student's question suggests, some of the difficulty is not in the mathematical task itself but the language being used to discuss it. Teacher B herself identifies the fact that Teacher A has been using different language and makes the explicit link between "reasons" and "angle facts". Following this, the number of students that believed they knew the angle fact increased slightly, although this was still far below the number answering the first question. In addition to language difficulties, this could be due to the fact that students might have been identifying whether angles were congruent or supplementary by sight, following the crowd by liking the most popular option, or unwilling to like a comment that might have led to a follow-up question being asked of them.

While all three, "reason", "rule" and "fact", were used to refer to the same theorems the differences in rephrasing might have brought out subtly different meanings. Teacher B's response highlights these differences saying, "I like to call them facts because they are sort of stated and they are true." This suggests that she has made a deliberate choice in order to emphasise the truth of the theorems. In contrast the term "reason" (perhaps another example of exam register), seems to place more weight on their use as justifications, bringing out the way the mathematical theorems are used to support later proofs. There are also specific characteristics of mathematical theorems that may not captured by these translations. For example, while "rule" is a term that students will undoubtedly be familiar with within a school environment, therefore providing an accessible entry point, school rules are chosen rather than derived, may change over time, and rules in one environment (not eating in the science lab) may not apply in other situations (the canteen). It is plausible that using the term "rule" could lead to misconceptions, such as uncertainty about whether rules learnt in one context (the angles in parallel lines topic) may be applied in others (a circle theorems question). This could be a case where, as described by Schleppegrell (2007), the use of more everyday vocabulary leads to more "informal ways of construing knowledge" (p.140). However, these conclusions are highly speculative, as I did not speak to students directly about their understanding of "rules" or equivalent terms.

One other point where rephrasing appeared particularly important was when first introducing or embedding vocabulary. Students had first met the terms alternate, co-interior, and corresponding angles during the summer term in Year 8, when schools were in lockdown and remote learning set ups were more limited. Therefore, at the beginning of the unit very few students could explain or use the terms correctly and so a large amount of class time was spent re-introducing and reinforcing these terms. Co-interior angles were explained in the following way in Teacher A's lesson, accompanying the slide and annotations in Figure 6:

> *Teacher A: How are you going to remember which ones are co-interior? Co-interior. Interior means they are on the inside. They are on the inside of the parallel lines, again it means they are facing each other, and they are also on the same side of your transversal. Transversal is this line that cuts through your parallel lines.*

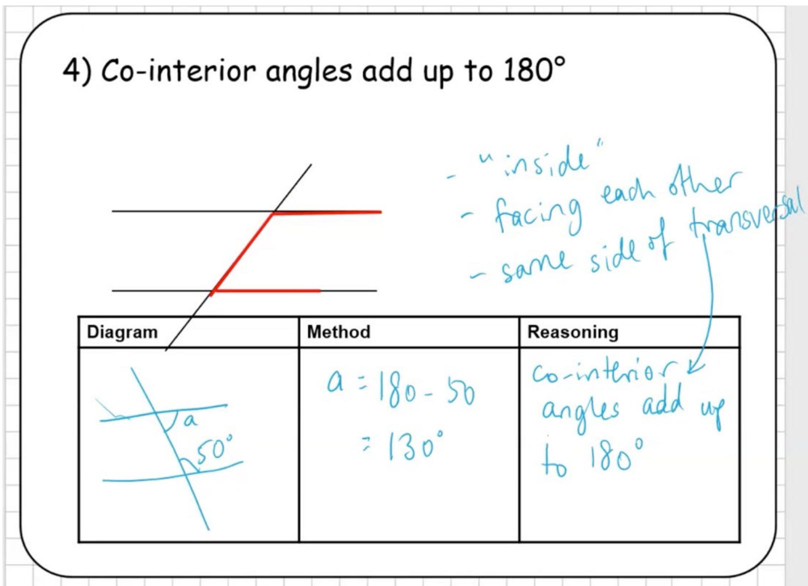

Figure 6: PowerPoint slide with teacher annotations as shown on Microsoft Teams and an interactive whiteboard by Teacher A during a lesson.

Here several different methods are used to explain the mathematical term.

- A definition is given in the mathematical register "Co-interior [means …] they are on the inside of the parallel lines [and …] on the same side of your transversal".
- The word is broken down into its component etymological parts – "Interior means [...] inside".
- Translations of mathematical vocabulary into more everyday English are given – "They are facing each other", "Transversal is this line that cuts through your parallel lines".
- Two diagrams with examples of co-interior angles are provided.
- A property of co-interior angles is given – that they add up to 180°.

Teacher A has used many of the same techniques described by Gay and White (2002). However, during speech these different methods of explanation are not split into neatly into different sections as in Figures 2a and 2b. Instead, the almost concurrent use of formal and informal definitions leads to frequent switches in and out of mathematical register as discussed in RQ1.

One of the benefits of rephrasing into everyday English can be seen in students' and teachers' responses to the definition, given in mathematical register, during the Explanation Evaluation section of my taught lesson (Figures 7a, 7b and 7c). In Figure 7c, the student writes that "the words are a little complex", an opinion shared by many of their peers. In fact, 14 out of 24 students (one student in the class was absent for this lesson) commented on the fact that mathematical terminology used in the definition might be hard to understand, particularly focusing on the word "transversal". Therefore, providing translations of specific pieces of mathematical vocabulary, as Teacher A did in her explanation, seems important for ensuring understanding. In addition, while not a case of translating *out of* mathematical register, the translation between semiotic forms was also deemed very important. 15 out of 24 commented on an explanation without a diagram, that a diagram would be helpful to increase their understanding. This aligns with their teachers' own views (See Figures 7a and 7b). Teacher B, when asked to explain alternate angles in interview said "It's really hard. I would do that with a diagram."

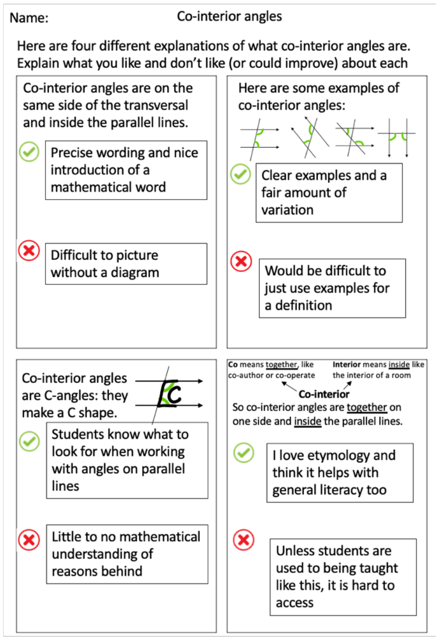

Figure 7a: Explanation Evaluation by Teacher B

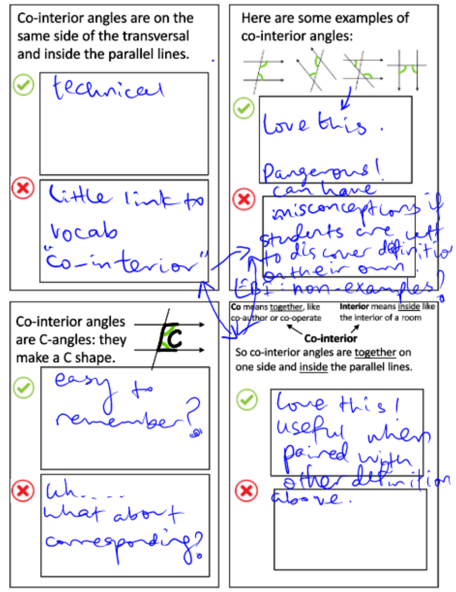

Figure 7b: Explanation Evaluation by Teacher A

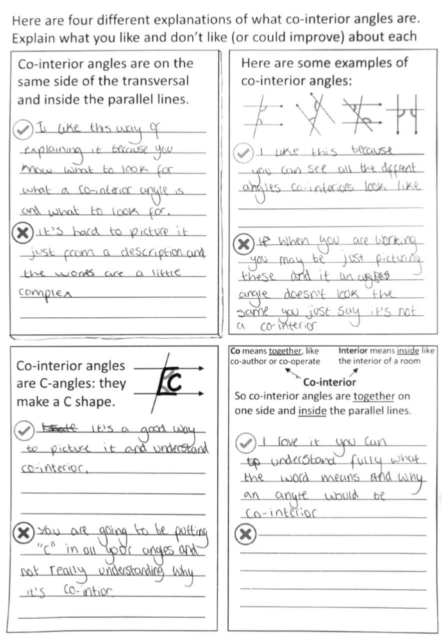

Figure 7c: Explanation Evaluation by a student

The teachers' opinions of the two types of explanation given in a more everyday register differed significantly. Both teachers held strong negative opinions about the third explanation, that of "C-angles", where the angles are described in terms of the shapes they make. During interview, Teacher B described them as "horrible", while Teacher A said, when defining vertically opposite angles: "I'm tempted to say the shape X, but no no no no. […] There are so many misconceptions. No understanding for what is actually means, just looking for shapes. Think about the shape C. It could be co-interior or corresponding". The teachers' objections were of two types. Firstly, that this kind of explanation produces "little to no mathematical understanding" and also that confusions may arise such as about whether C-angles referred to corresponding or co-interior angles. The latter point highlights one of the possible issues with translation into everyday English – that overreliance on translations might mean that students will be unable to correctly use or understand the mathematical register. Students however had a generally more positive view, with 12 out of 24 saying this way would be easy to remember. This could relate to Pimm's (1987, p.142) observation that the term C-angles is a form of pictogram. Therefore, this explanation straddles both written and diagrammatic modes – a combination that students expressed a strong preference for. Three students noted that "not all co-interior angles make a C shape" or similar. This touches on one of the possible misconceptions – the fact that certain orientations and configurations of parallel lines and transversals lead to co-interior angles where the C shape may be less apparent. For example, the one time this kind of translation was used during my observations Teacher B said "We're looking for that F. There's a backwards slanty F there". Here, the adjectives "backwards" and "slanty" demonstrate how corresponding angles do not always form a typical F shape. This misconception may be reenforced by that fact that, outside of this specific mathematical context, the orientation of letter shapes is important.

In contrast, both teachers "loved" the etymological translation of the term "co-interior", and in interview discussed their use of this method with their classes. Teacher A talked about how exploring related maths and everyday English terms shows that maths is "not a standalone subject" breaking down the idea of "you are either a maths person or you're not". Teacher B talked as well about how these methods could help to improve general literacy as well as mathematical understanding. However most strikingly, both teachers displayed genuine joy in finding the "wonderful ways" that words connect. While some students thought this type of explanation might be "too long" and that "a blob of writing can be quite intimidating", many talked positively about "break[ing] down the meaning", "explaining the words deeper" and understanding "why it means that". In Figure 7c, while the student "like[s]" some of the the other definitions the etymological explanation was the only one they "love", mirroring the language of their teachers. These positive reactions perhaps link back to Gay and White's (2002) recommendation that a "personal example" is found (p.35). While students won't have encountered the word "co-interior" outside of the mathematics classroom, making etymological links to words they may know might provide the same sort of connection.

One of the major drawbacks to my findings is that I have largely relied upon students' and teachers' self-reporting of the benefits and drawbacks of different types of rephrasing. It would be interesting to explore whether the C-angles explanation really is more memorable for students and whether explanations of this kind led to more misconceptions in its application. Similarly, it is hard to know whether etymological explanations do increase the understanding of what mathematical words mean. Collecting this type of data was difficult within the context of my own study as all students were exposed to both types of explanation and access to student work was limited.

What was clearer was that, despite a variety of different translations and approaches, students still had great difficulty understanding mathematical vocabulary. Of the three students I interviewed none gave a correct definition of alternate angles, with two stating the property that "alternate angles are equal" rather than giving a definition. Similarly, another student wrote in criticism of a formal definition in Figure 7a that "there are simpler ways you could say it e.g. co-interior angles add up to 180°". This reveals a difficulty, not just with the definition of individual words, but with the concept of definition itself. Students are confusing the definition of a mathematical object with attribution of its properties, an issue that Schleppegrell (2007, p.143) argues may also have a linguistic origin – in the multipurpose use in English of the verb "to be" – "alternate angles *are* on different sides of the transversal" versus "alternate angles *are* equal".

RQ3 What specific problems of register use arise from mathematical words that have a different meaning in everyday English?

While a large variety of words with different meanings in mathematical and everyday English came up throughout my data collection, this section will focus on the problems and use of a single word, “regular”. While easily comparable to discussions in the literature, such as those of the word “similar”, “regular” also has more than one common meaning in English (see Table 4), adding further complexity.

Meaning	Register	Definition
Mathematically regular	Mathematical	Of a polygon: having all its sides equal and all its angles equal
Recurring	Everyday	Recurring or taking place repeatedly at (short) uniform intervals
Normal	Everyday (more predominantly US)	Having the usual, typical, or expected attributes, qualities, parts, etc.; normal, ordinary, standard.

Table 4: Different definitions of “regular”

(adapted from OED Online, 2021)

I included “regular” on the list of thirteen words that the students placed on the scale depending on whether they viewed the word as part of mathematical or ordinary English. The distribution of where students placed “regular” on their scale can be seen in Figure 8 together with two Word Scales produced by students.

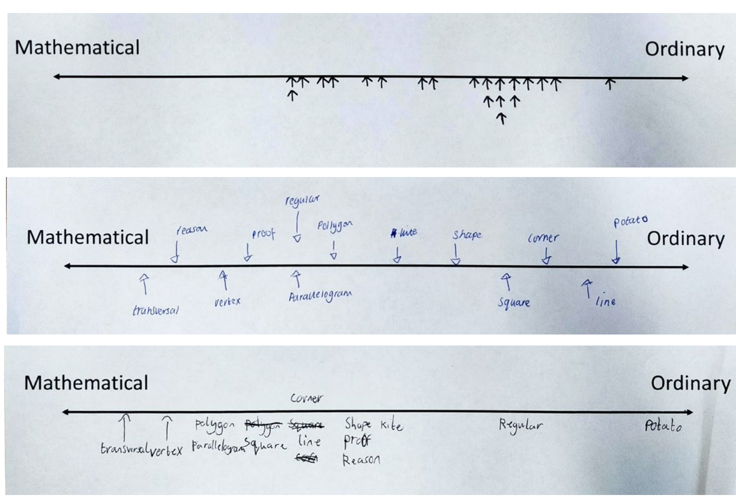

Figure 8: Distribution of where students placed “regular” on their Word Scale and two examples of student Word Scales

In terms of ranking (see Figure 9 below), seven out of the 24 students ranked “regular” as the second most ordinary word, beaten only by “potato”, which had been included as an archetypal non-mathematical word. Overall, the distribution of the rank was bimodal with peaks at 2nd and 8th most ordinary. This is perhaps suggestive of the multiple meanings of “regular”, indicating a split in the class between those who recognized the specifically mathematical meaning of the word, and those who were only aware of its everyday usages. The skew towards the ordinary end of the scale (see Figure 8) may even have been compounded by the synonymity of “regular” and “ordinary”. However, given that students were not asked to explain their choices I cannot be sure of this.

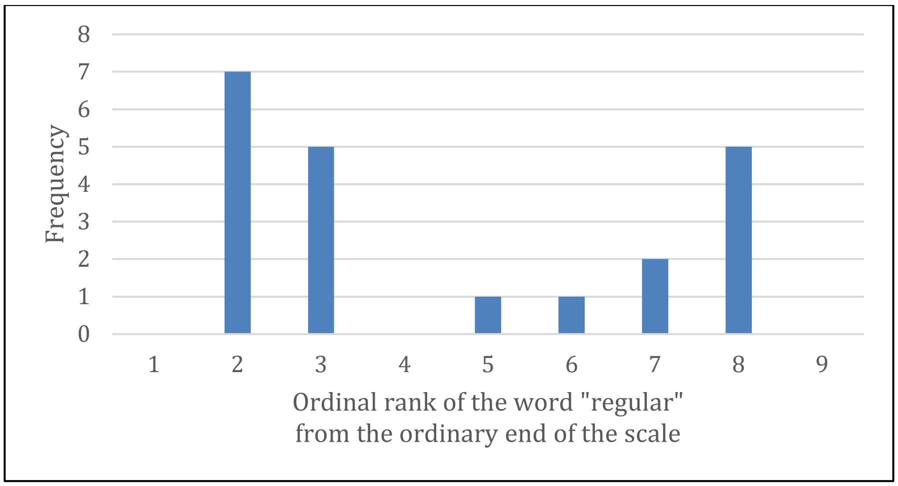

Figure 9: Bar chart showing the ordinal rank of the word “regular” from the ordinary end of the scale

In the discussion that followed, it was the normal rather than recurring definition that was offered by the class. When asked what the mathematical usage was, the following exchange took place:

S: A shape you see more often. Like a regular square

T: A regular square?

S: Yeah, like if you slanted it

Here we can see how the everyday understanding of “regular” as “normal” has perhaps influenced the student’s mathematical understanding, when they say that a regular shape is one you see “more often”. It is also perhaps a misconception embedded by the fact that regular shapes are those that are seen more often in the classroom, perhaps particularly when the vocabulary is first introduced at a primary level.

Similar themes also arose later in the class, after a formal mathematical definition of regular had been discussed, during an activity where students were asked to describe a diagram in words. The only diagram containing what was probably a regular shape (equality of side lengths and angles was not marked), in this case a regular pentagon, was described by students only as “a pentagon” and “a massive pentagon”. This could suggest that given that the archetypal pentagon in their minds is regular, no further description is seen to be needed. However, given that both recipients of the descriptions drew pentagons containing two right angles, perhaps this association is not as strong.

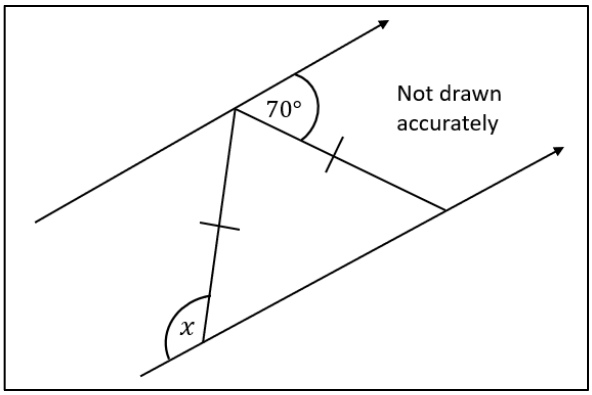

Figure 10: Diagram used in the Diagram Description task

(redrawn from AQA, 2021)

Of the three possible references to regularity, two occurred in descriptions of Figures 10 and the other in a description of a very similar diagram also depicting an isosceles triangle between two parallel lines. While requiring some mathematical deduction, an activity student had not been explicitly asked to engage in, these diagrams in fact contained sufficient information to determine that the triangles were irregular. It is however interesting to consider what additional assumptions the students made when describing the triangles as “regular”, “equal” or “normal” (as presented in Table 5).

Student	Diagram	Description
Student 1	Figure 10	It’s a pair of parall lines with a tryangle in between them. […] It’s a regular tryangle.
Student 2	Figure 10	It is a pair of parrell sides with a triangle inside it. […] It is an equal triangle
Student 3	Similar diagram	Draw a normal triangle but on a slight slant. Also draw two lines facing each other, landscape on an angle.

Table 5: Student descriptions of Figure 10 and a similar diagram

This last descriptor perhaps points us back to the idea of an archetypal "normal" triangle, which the students identify by looking at the diagram as you would a picture, rather than reading it in a specialised mathematical way (Morgan, 2017, p.153). This view is reinforced by the description "but on a slight slant". Here the archetypal triangle not only looks a certain way but also has a particular orientation, with a horizontal base, and therefore the description of the triangle as "normal" needs to be qualified with a "but". However, it is also possible that the students, particularly in descriptions 1 and 2, are reading the diagrams mathematically to some extent. The descriptor "equal" may be recognition of the dashes along two of the triangle's sides which denote equality of length. Alternatively, the student may have been reaching towards the word "equilateral", making the false assumption that the third side is also equal.

During the student and teacher interviews I asked the interviewees to explain the meaning of the word "regular", which elicited the responses in Table 6. While all students have connected "regular" to some sense of equality, Student C gets no further than this and Student A gives an incomplete definition with no reference to angles (although perhaps the trailing "and if it" suggests that they themselves might recognise its incompleteness).

Interviewee	Response to "Explain the meaning of the word 'regular'"
Student A	Regular is if it's got all equal sided length sides and if it… yeah.
Student B	It's when a shape has all the um angles the same but and they also have all the lines the same.
Student C	Regular is like you use it in, I don't know, your general conversation and mathematical like a regular… parallelogram or a regular shape. I think it means that it is all like… I don't know if it's equal? I'm not too sure.

Table 6: Student interview responses to the question "Explain the meaning of word 'regular'"

Throughout the taught lesson, or during interviews, the recurring meaning of regular was not explicitly talked about. Perhaps it was used implicitly in the exchange:

> *T: Is there any difference between a corner and a vertex?*
>
> *S: Corner is used more regularly.*

Although in this context, while the implication is that "corner" is used repeatedly, the intervals are not evenly spaced. It was only after my choice to focus on "regular" that I reflected on this additional everyday meaning that ties more closely to the mathematical concept, given its idea of uniformity. While the everyday usage may have strengthened the connection between "regular" and the idea of equality that we see in all students' responses, it falls short of eliciting a precise definition.

Overall, the mathematical use of word "regular" shared many of the same issues as other words borrowed from everyday English which are discussed in the literature (Adams, 2003; Morgan, 2017; Rowland, 2001). As Adams (2003) points out, finding the connections between students' everyday understanding of a word and its mathematical meaning can be useful in strengthening understanding (p.788), as in the case of the recurring meaning of regular. However, students sometimes did not recognise that regular had a specific mathematical use, causing them to transfer an everyday meaning directly into the mathematical setting. This can create misconceptions, such as Rowland describes in the case of the word "similar" where "candidates understood a vague 'sameness' relation which lacked the precision necessary for successful completion of the question" (Rowland, 2001, p.182). In the same way, I found that students often related mathematical regularity to a vague idea of normalness which, while sometimes an effective way to identify regular shapes, does not encompass a precise enough understanding in all contexts, such as when deciding whether the triangle in Figure 10 is regular. These misconceptions back up the importance Rowland's recommendation to "point out and emphasise, on a regular basis, that mathematical words are not always what they appear" (ibid.). It also highlights how some types of questioning may not reveal misconceptions. For example, if students were asked to identify which of the pentagons in Figure 11 were regular, both the mathematical and normal definitions could both lead to the selection of the left shape.

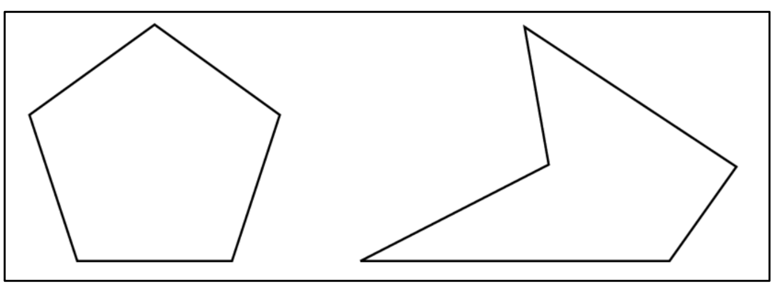

Figure 11: A regular and an irregular pentagon

One of the main drawbacks to my findings was the lack of an iterative approach that would have allowed me to return to areas of interest, informed by prior data collection. This was necessitated by the short time frame for data collection, combined with other teaching commitments, which meant data was analysed after the collection period was over, and therefore my focus on the word “regular” was arrived at after the fact. This meant for example that during the Diagram Description activity only a minority of students had a diagram where regular or seemingly regular polygons were included, limiting the amount of relevant data collected and restricting the students for whom I had data. This latter restriction was particularly problematic as it meant that the students that I interviewed were not the same as those discussed in relation to the Diagram Description. In addition, student comments made during the taught lesson were not ascribed to specific students. This means that it was not possible to triangulate the conceptual understanding of a particular student by looking at more than one type of data. Consequently, it is only possible to ascribe specific misconceptions and beliefs very tentatively to students based on the data collected.

Conclusion

Much of my research started from the basis of there being two separate registers which teachers and students might need to shift in and out of or translate between. However, while the distinctions between mathematical and everyday English were evident, my findings often emphasised how closely intertwined the two registers are in a classroom setting. Language switched frequently between registers, often multiple times within the same sentence. At other times it was difficult to classify what register was being used, either because of a mixture of different registers or ambiguity in the intended meaning. This particularly arose in the case of words with multiple meanings, which some types of questioning might not distinguish between. As a teacher it seems important to be careful when interpreting or revoicing student communication that we allow for the possibility that what a student means and what we understand may differ.

Conducting this research reinforced for me both the importance and difficulty of teaching students to speak mathematically. It did not however point towards many definitive tactics for improving students’ use of mathematical register. While my findings in RQ2 highlighted the importance of examples and possible benefits of using etymology to help students understand the meaning of mathematical vocabulary, the reliance on student’s and teacher’s perceptions combined with the comparatively minimal examination of literature on this topic meant that my conclusions could only be tentative. These limitations were also partly a result of the case study form that this research took. Perhaps an action research study, directly comparing different methods of vocabulary instruction, could provide more actionable recommendations.

What I am able to take away is a greater awareness and understanding of the features and use of mathematical register. While a proficient user, I had not before considered the full range of differences between mathematical and everyday language, and therefore was not fully aware of the potential pitfalls for students learning to speak this way for the first time. Perhaps one of the successes of my taught lesson, separate from its use in my research, was to give students an opportunity to think explicitly about these differences themselves.

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