Calorimetry at a Future Linear Collider

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This thesis describes the optimisation of the calorimeter design for collider experiments at the future Compact Linear Collider (CLIC) and the International Linear Collider (ILC). The detector design of these experiments is built around high-granularity Particle Flow Calorimetry that, in contrast to traditional calorimetry, uses the energy measurements for charged particles from the tracking detectors. This can only be realised if calorimetric energy deposits from charged particles can be separated from those of neutral particles. This is made possible with fine granularity calorimeters and sophisticated pattern recognition software, which is provided by the PandoraPFA algorithm. This thesis presents results on Particle Flow calorimetry performance for a number of detector configurations. To obtain these results a new calibration procedure was developed and applied to the detector simulation and reconstruction to ensure optimal performance was achieved for each detector configuration considered.

This thesis also describes the development of a software compensation technique that vastly improves the intrinsic energy resolution of a Particle Flow Calorimetry detector. This technique is implemented within the PandoraPFA framework and demonstrates the gains that can be made by fully exploiting the information provided by the fine granularity calorimeters envisaged at a future linear collider.

A study of the sensitivity of the CLIC experiment to anomalous gauge couplings that affect vector boson scattering processes is presented. These anomalous couplings provide insight into possible beyond
standard model physics. This study, which utilises the excellent jet energy resolution from Particle Flow Calorimetry, was performed at centre-of-mass energies of 1.4 TeV and 3 TeV with integrated luminosities of $1.5 \text{ab}^{-1}$ and $2 \text{ab}^{-1}$ respectively. The precision achievable at CLIC is shown to be approximately one to two orders of magnitude better than that currently offered by the LHC.

In addition, a study into various technology options for the CLIC vertex detector is described.
Declaration

This dissertation is the result of my own work, except where explicit reference is made to the work of others, and has not been submitted for another qualification to this or any other university. This dissertation does not exceed the word limit for the respective Degree Committee.

Steven Green
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Firstly, I would like to thank Mark Thomson and John Marshall for the excellent supervision that they have both given me throughout the course of this work. They have both been sources of insight, wisdom and encouragement over the past years and I am truly grateful to have had them as my supervisors. I would like to thank Boruo Xu for his friendship and sense of humour that has made sharing an office with him for the past four years a joy. I am extremely grateful to have worked in the Cambridge HEP group throughout my PhD and would like to thank everyone there for creating a welcoming and friendly working environment.

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I would like to thank my parents, family and friends for all of their help getting me to this point. The foundation that my parents provided has given me the platform to get to where I am now and I am eternally grateful for all that they have done for me.

I would like to give special thanks to my wife Emily. She has been an inspirational support to me even since we first met and I lack the words to truly describe how thankful I am for her. Her positivity and cheerful nature have been invaluable to me personally, while her checking of gammer has been equally invaluable to the reader.

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Contents

1 Introduction .......................................................... 1
  1.1 Future Linear Colliders ......................................... 4
    1.1.1 Future Linear Collider Coordinate System ................. 4
    1.1.2 The International Linear Collider ......................... 4
      1.1.2.1 Experimental Conditions at the ILC .................. 5
    1.1.3 The Compact Linear Collider .............................. 6
      1.1.3.1 Experimental Conditions at CLIC .................... 6
      1.1.3.2 Beam-Related Backgrounds at CLIC .................. 8

2 Anomalous Gauge Coupling Theory ................................ 11
  2.1 The Standard Model .............................................. 11
  2.2 Higgs Physics ................................................... 15
    2.2.1 Spontaneous Symmetry Breaking ............................ 15
    2.2.2 Electroweak Interactions .................................. 16
      2.2.2.1 Custodial Symmetry .................................. 18
  2.3 Anomalous Gauge Couplings ..................................... 20
    2.3.1 Effective Field Theory ................................. 20
    2.3.2 Electroweak Chiral Lagrangian ......................... 22

3 Particle Flow Calorimetry for Future Linear Colliders ....... 25
  3.1 Particle Flow Calorimetry ...................................... 25
    3.1.1 Particle Shower Development ............................... 27
  3.2 International Large Detector .................................. 29
    3.2.1 Overview ................................................... 30
    3.2.2 Vertex Detector ............................................ 31
    3.2.3 Time Projection Chamber .................................. 31
    3.2.4 Supplemental Silicon Tracking System .................... 34
    3.2.5 Electromagnetic Calorimeter .............................. 36
    3.2.6 Hadronic Calorimeter .................................... 37
### Contents

3.2.7 Solenoid, Yoke and Muon System ................................. 39
3.2.8 Forward Calorimetry ........................................... 40
3.3 Simulation ......................................................... 42
3.4 CLIC_ILD .......................................................... 43
3.5 Particle Flow Reconstruction ......................................... 45
  3.5.1 PandoraPFA .................................................... 45
3.6 Performance ....................................................... 50
  3.6.1 Jet Energy Resolution ........................................... 50
  3.6.2 Decomposition of the Jet Energy Resolution ..................... 52
  3.6.3 Single Particle Energy Resolution ................................ 53
3.7 Summary of ILD Detector Performance ............................... 54

4 Capacitively Coupled Pixel Detectors for the CLIC Vertex Detector 59
  4.1 Introduction ..................................................... 59
    4.1.1 HV-CMOS ...................................................... 60
    4.1.2 CLIC ASICs .................................................. 62
      4.1.2.1 CLICpix ................................................ 63
    4.1.3 Capacitive Coupling .......................................... 64
  4.2 Device Fabrication ................................................ 64
  4.3 Device Characterisation ............................................ 65
    4.3.1 Source Measurements ......................................... 66
      4.3.1.1 Experimental Setup .................................... 67
      4.3.1.2 Analysis ............................................... 67
      4.3.1.3 Results: Rise Time vs Pulse Height ....................... 69
      4.3.1.4 Results: ToT vs Pulse Height ............................ 70
      4.3.1.5 Results: Cross Couplings ................................ 71
    4.3.2 Test Pulse Calibration ........................................ 74
      4.3.2.1 Experimental Setup .................................... 74
      4.3.2.2 Analysis ............................................... 74
      4.3.2.3 Results ............................................... 75
  4.4 Test Beam Analysis ................................................ 78
    4.4.1 Test Beam Setup ............................................ 80
    4.4.2 Analysis ..................................................... 80
    4.4.3 Results ..................................................... 81
  4.5 Conclusions ..................................................... 82
5 Energy Estimators

5.1 Motivation ................................................................. 83

5.2 Calibration in the Particle Flow Paradigm ........................... 86
  5.2.1 Overview of the Calibration Procedure ......................... 87
  5.2.2 MIP Scale Determination in the Digitiser ....................... 88
  5.2.3 Digitisation Implementation .................................... 88
    5.2.3.1 ECAL Digitisation Implementation ................................. 88
    5.2.3.2 HCAL Digitisation Implementation ............................... 90
    5.2.3.3 HCAL Ring Digitisation Implementation ....................... 92
  5.2.4 MIP Scale Determination in PandoraPFA ......................... 93
  5.2.5 Electromagnetic Scale in PandoraPFA ............................ 94
  5.2.6 Hadronic Scale in PandoraPFA ................................... 96
  5.2.7 Summary ............................................................. 99

5.3 Novel Energy Estimators ............................................. 100
  5.3.1 HCAL Hit Energy Truncation .................................... 102
    5.3.1.1 Legacy Energy Corrections ...................................... 102
    5.3.1.2 Impact on Single Particle Energy Resolution ............... 103
    5.3.1.3 Impact on Jet Energy Resolution ............................... 104
  5.3.2 Software Compensation ......................................... 106
    5.3.2.1 Impact on Single Particle Energy Resolution ............... 110
    5.3.2.2 Impact on Jet Energy Resolution ............................... 111
  5.3.3 Summary ............................................................. 113

5.4 Timing Cuts ............................................................ 114
  5.4.1 Impact on Single Particle Energy Resolution ............... 115
  5.4.2 Impact on Jet Energy Resolution ............................... 116
  5.4.3 Summary ............................................................. 117

6 Calorimeter Optimisation Studies .................................. 119

6.1 Introduction .......................................................... 119

6.2 Electromagnetic Calorimeter Optimisation .......................... 119
  6.2.1 ECAL Cell Size .................................................... 121
  6.2.2 ECAL Longitudinal Sampling Frequency ........................ 124
  6.2.3 ECAL Active Material ............................................. 125

6.3 Hadronic Calorimeter Optimisation .................................. 128
  6.3.1 HCAL Cell Size .................................................... 129
  6.3.2 HCAL Number of Layers ........................................... 132
### 6.3.3 HCal Longitudinal Sampling Frequency .................................. 135
### 6.3.4 HCal Sampling Fraction ....................................................... 138
### 6.3.5 HCal Absorber Material ...................................................... 139
### 6.4 Global Detector Parameters ................................................... 143
    - 6.4.1 The Magnetic Field Strength ........................................... 144
    - 6.4.2 Inner ECal Radius ......................................................... 147
### 6.5 Summary .............................................................................. 149

### 7 The Sensitivity of CLIC to Anomalous Gauge Couplings through Vector Boson Scattering ................................................................. 151
    - 7.1 Motivation ........................................................................... 151
    - 7.2 Event Generation, Simulation and Reconstruction ..................... 155
    - 7.3 Modelling of Anomalous Gauge Couplings ................................. 158
    - 7.4 Data Analysis .................................................................... 160
        - 7.4.1 Limiting Beam Related Backgrounds ............................... 160
        - 7.4.2 Jet Finding ................................................................ 161
            - 7.4.2.1 Optimal Jet Finding Algorithm ................................ 163
        - 7.4.3 Lepton Finding ............................................................. 164
        - 7.4.4 Discriminant Variables ............................................... 166
        - 7.4.5 Jet Energy Resolution at CLIC ..................................... 167
    - 7.5 Event Selection .................................................................... 167
        - 7.5.1 Preselection .................................................................. 168
        - 7.5.2 Multivariate analysis ..................................................... 169
        - 7.5.3 Event Selection Summary ............................................. 172
    - 7.6 Anomalous Coupling Fitting Methodology ................................. 173
        - 7.6.1 Sensitive Distribution ..................................................... 173
        - 7.6.2 $\chi^2$ Surface and Confidence Limit Definition .................. 178
        - 7.6.3 Event Weight Interpolation Scheme ................................. 179
    - 7.7 Results ............................................................................... 181
        - 7.7.1 Systematic Uncertainties .............................................. 181
    - 7.8 Sensitivity at $\sqrt{s} = 3$ TeV ................................................. 183

### 8 Summary ................................................................................. 191

### Bibliography ............................................................................... 197
“I thank and praise you, God of my ancestors: You have given me wisdom and power”
— Daniel 2:23a
Chapter 1

Introduction

“Instruct the wise and they will be wiser still; teach the righteous and they will add to their learning.”

— Proverbs 9:9

The Standard Model has proven to be one of the greatest accomplishments of modern day particle physics. It has been used to make countless predictions of various physics processes across a wide range of energies that have proven to be consistent with experimental measurements. The final piece of the Standard Model to be discovered was the Higgs boson, which was found by the ATLAS [1] and CMS [2] experiments at the Large Hadron Collider (LHC) in 2012.

Despite the remarkable descriptive power of the Standard Model, there are a number of features in the universe that it does not provide a description for. How does gravity fit into the Standard Model? Why is there an excess of matter over antimatter in the observable universe? What is "dark matter" and "dark energy"? How does "dark matter" couple with the particles in the Standard Model? What are the properties of the Higgs field in the Standard Model? While the LHC and previous generations of particle collider experiment have had enormous success in validating the Standard Model and searching for new physics, it is clear that there is more work to be done.

The linear collider experiments are a set of proposals for the next generation of particle collider experiments. These experiments are TeV scale $e^+e^-$ colliders with an emphasis on precision measurements. The physics programme for the linear collider is designed to complement and extend the work done at the LHC and to develop our understanding
of particle physics. One of the primary goals of the linear collider experiments is to study the Higgs field of the Standard Model. A detailed description of the Higgs field could help in the description of "dark matter" as many extensions of the Standard Model Higgs field contain particles that fit the properties of "dark matter". The linear collider experiments will also provide a detailed description of the properties of the top quark. This will complement the Higgs study because the strongest couplings for the Higgs in the Standard Model occurs with the top quark. Another goal of the linear collider experiments is to provide high precision measurements of the electroweak sector in the Standard Model. As the electroweak sector is the only place in the Standard Model where CP violation can occur, a detailed description will help to determine why there is an excess of matter over antimatter in the universe. Furthermore, the linear collider will expand the search for many Standard Model extensions such as supersymmetry (SUSY).

As well as searching for beyond Standard Model physics, precision measurements at the linear collider will guide the future direction of experimental particle physics. Precision measurements have helped to guide the course of particle physics experiments in the past; LEP electroweak data, which gave indirect information about the lightness of the Higgs boson, was used to build the physics case for the LHC. By colliding electrons and positrons, which are fundamental particles, the experimental conditions found at the linear collider will be far cleaner than those at the LHC, which makes it easier to perform precision measurements. High precision measurements are made possible at the linear collider due to the use of particle flow calorimetry, a revolutionary technique in calorimetry that offers exceptional energy resolution for jets. This paradigm shift means the linear collider detectors are significantly different from those found in previous generations of particle colliders. As the detector design is continually evolving, the ongoing research in this area is vital for determining the overall success of these proposed experiments.

This thesis is organised as follows. Chapter 2 contains a summary of the Standard Model as well as an outline of the physics of interest related to the analysis presented in chapter 7. Chapter 3 contains a discussion of the Particle Flow calorimetry paradigm. This is followed by a review of the detectors that will be used at the future linear colliders and how they are tailored for Particle Flow calorimetry. Next there is a discussion of PandoraPFA and the role it has in event reconstruction and this chapter concludes with a discussion of the metrics used to quantify detector performance. The material presented in chapters 2 and 3 is intended to add context to the work presented in the remainder of this thesis.
Chapter 4 presents a study into a novel technology option for the Compact LIinear Collider (CLIC) vertex detector. Laboratory and test beam measurements were made of the novel vertex detector technology option and the analysis of those new measurements is presented in this chapter. The author of this thesis recorded the laboratory measurements, participated in the recording of the test beam measurements and performed the analysis of the data while on long term attachment working with the CLICdp group at CERN. The experimental setup and software for making the laboratory and test beam measurements were provided by the CLICdp collaboration.

Chapter 5 contains numerous studies related to the treatment of energy deposits in the linear collider simulation. This begins with an outline of the calibration procedure for the linear collider detector simulation. It is then followed by a number of novel software techniques aimed at improving the energy resolution of a calorimeter designed for particle flow calorimetry. Finally, chapter 5 concludes with a study of the timing requirements applied in the software trigger that will be used at the linear collider experiments. The creation and implementation of the calibration procedure for use with the linear collider reconstruction is new work. The software compensation technique, was inspired from research done by the CALICE collaboration [3], however, the PandoraPFA implementation and training of the technique is new research. This research was made into a publication describing the use of software compensation in PandoraPFA [4]. The legacy energy corrections, including the HCal hit energy truncation, were developed by Professor Mark Thomson and implemented in PandoraPFA by Dr John Marshall.

Chapter 6 presents an optimisation study of the linear collider calorimeters. The starkest contrast in detector design, when comparing particle flow calorimetry to tradition calorimetry, is the design of the calorimeters. As the linear collider experiments will be the first experiments purposely built with particle flow calorimetry in mind, this study will be vital for guiding detector construction. The entirety of this chapter contains new research.

Chapter 7 contains a study into anomalous gauge couplings that are sensitive to massive gauge boson quartic vertices at the CLIC experiment. This study is of particular interest as it provides a detailed probe of the electroweak symmetry breaking sector of the Standard Model as well as showing CLICs sensitivity to a possible extension to the Standard Model. The analysis presented in this chapter is new research. The detector simulation, event reconstruction and anomalous gauge coupling weight determination for the signal sample were performed by the author, while the detector simulation and event reconstruction for the background samples were performed by the CLIC collaboration.
The thesis concludes with a summary in chapter 8.

1.1 Future Linear Colliders

There are two proposed future linear colliders; the International Linear Collider (ILC) and the Compact Linear Collider (CLIC). These colliders are both $e^+e^-$ colliders with focus upon precision measurements, however, they operate at different collision energies, which presents each experiment with its own unique challenges. One benefit of a linear collider is that it is possible to stage the experiment at several different energies throughout the experiment’s lifetime.

1.1.1 Future Linear Collider Coordinate System

Figure 1.1 shows the coordinate system for the future linear colliders with respect to the ILD detector that will be discussed in chapter 3. The origin of the system is defined as the interaction point for the collider and the $e^+$ and $e^-$ beams travel along the $\pm z$ axes.

![Figure 1.1: Coordinate system for the future linear collider shown with respect to the ILD detector.](image)

1.1.2 The International Linear Collider

The ILC, shown in figure 1.2, initially plans to operate at a centre-of-mass energy of 250 GeV to study the Higgs boson in detail through the Higgstrahlung process.
(e^+ e^- \rightarrow ZH) [5]. The analysis of this process makes it possible to examine all the decays of the Higgs boson with high precision. The next phase of operation will increase the collision energy to 500 GeV. This will extend the study of the Higgs, making it possible to observe the Higgs coupling with the top quark and to determine self interactions of the Higgs. Furthermore, at this energy, it will be possible to search for evidence for SUSY and extended Higgs states. Finally, there is an option to increase the centre-of-mass up to 1 TeV, further extending the search for SUSY and composite Higgs models.

Figure 1.2: Schematic layout of the ILC, indicating all the major subsystems (not to scale). Figure taken from [5].

1.1.2.1 Experimental Conditions at the ILC

The experimental conditions at the ILC are relatively mild in comparison to CLIC and the LHC. The largest source of background for the ILC is $\gamma\gamma \rightarrow \text{hadrons}$ events where the photons are produced via beamstrahlung. At ILC energies the cross-section for this background is order of hundreds of nb, which gives approximately $\gamma\gamma \rightarrow \text{hadrons}$ event per bunch crossing. A number of secondary $e^+ e^-$ pairs are also produced in each bunch crossing, but there are confined to a narrow region around the beam axis.
1.1.3 The Compact Linear Collider

The CLIC experiment, shown in figure 1.3, plans to operate with maximum collision energy of 3 TeV [6, 7]. CLIC will also operate at intermediate energy stages, however, these energies are to be determined by the ongoing work at the LHC. Figure 1.3 shows two beam acceleration scheme [8] that CLIC will use to obtain the large electric fields required to accelerate the electrons and positrons to high energies. The two beam accelerator scheme works by decelerating one of the beams, the drive beam, in a controlled manner and using the radiofrequency power produced by that deceleration to accelerate the second beam, the main beam. The upper half of the diagram shows the accelerator structure for the drive beam, which consists of an accelerator followed by a delay loop and combiner rings that control the frequency of the electron and positron bunches. The lower half of the diagram shows the accelerator structure for the main beam, which contains damping and predamping rings, bunch compressors and a booster linac. The damping and predamping rings are used to reduce the spread in both position and momentum of the beam particles, the bunch compressors are used to reduce the longitudinal spread of the bunches and the booster linac is used to accelerate the beam particles prior to entering the main linac.

The large collision energy of CLIC gives it a greater physics reach to search for extensions to the Standard Model, e.g. SUSY, that would be inaccessible at ILC-like energies. Although the exact energies for the staging of the CLIC experiment are not certain, CLIC will operate at a low collision energy during staging, 380 GeV, to study the Higgs and the top quark. One of the most important channels for studying the top quark at CLIC will be top pair production, i.e. $e^+e^- \rightarrow t\bar{t}$. The threshold energy for top pair production is twice the mass of the top quark, $2 \times m_t \sim 346$ GeV, however, the peak cross-section occurs at a slightly larger energy [6]. Therefore, 380 GeV is an ideal energy for CLIC to operate at for studying the top quark via top pair production. The higher energy stages of the CLIC experiment will provide access to different channels for studying Higgs couplings, as shown by figure 1.4.

1.1.3.1 Experimental Conditions at CLIC

The CLIC experiment will operate in a unique environment in comparison to either the ILC or previous generations of lepton colliders. It is vital that this is properly accounted
Figure 1.3: CLIC layout at 3 TeV. Figure taken from [7].

Figure 1.4: Cross section for production mechanisms of the Standard Model Higgs boson as a function of the collision energy. The cross sections were calculated assuming a Higgs mass of 120 GeV. Figure taken from [6].

for when determining the physics potential that CLIC has to offer. The following aspects of the CLIC experiment present the largest challenges to the physics potential:

- **The high bunch charge density.** The small beam size at the impact point for CLIC, which is shown and compared to the ILC in table 1.1, produces extremely large electromagnetic fields. These fields can interact with the opposite beam
particles causing them to radiate photons in an effect known as beamstrahlung. Beamstrahlung acts to reduce the collision energy of the $e^+e^-$ pairs.

- **Beam related backgrounds.** Beamstrahlung photons can subsequently interact to produce background events that must be accounted for. Dominant backgrounds of this form that cannot be easily vetoed in the reconstruction include incoherent pair production of $e^+e^-$ and $\gamma\gamma \rightarrow \text{hadrons}$. While these backgrounds are also problematic for the ILC experiment, the lower collision energy means it is has a much smaller impact on performance.

- **Fast readout technology.** The CLIC bunch train consists of 312 bunches with a repetition rate of 50 Hz. Each bunch is separated by 0.5 ns, therefore, it will be necessary to integrate over multiple bunch crossing when reading out the detectors. This places tight constraints on all detector electrical readout speeds and time resolutions.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>ILC $\sqrt{s} = 500 \text{ GeV}$</th>
<th>CLIC $\sqrt{s} = 3 \text{ TeV}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_x [\text{nm}]$</td>
<td>$\sim 474$</td>
<td>$\sim 45$</td>
</tr>
<tr>
<td>$\sigma_y [\text{nm}]$</td>
<td>$\sim 5.9$</td>
<td>$\sim 1$</td>
</tr>
<tr>
<td>$\sigma_z [\mu\text{m}]$</td>
<td>$\sim 300$</td>
<td>$\sim 44$</td>
</tr>
<tr>
<td>$N \times 10^{10}$</td>
<td>2</td>
<td>0.372</td>
</tr>
</tbody>
</table>

**Table 1.1:** Beam parameters for the ILC [5] and CLIC [6]. The parameters shown are: $\sigma_x$, $\sigma_y$ and $\sigma_z$, the bunch dimensions at the impact point; and $N$, the number of particles per bunch. The linear collider coordinate system is defined in section 1.1.1.

### 1.1.3.2 Beam-Related Backgrounds at CLIC

The primary sources of background for the CLIC experiment are as follows:

- $e^+e^-$ pair creation from the interaction of a beamstrahlung photons with the opposing beam. The different mechanisms for pair creation are as follows:
  
  - **Coherent pair production:** the interaction of a real beamstrahlung photon with the electromagnetic field from the opposing beam.
  
  - **Trident pair production:** the interaction of a virtual beamstrahlung photon with the electromagnetic field from the opposing beam.
– **Incoherent pair production**: the interaction of a real or virtual beamstrahlung photon with the individual particles in the opposing beam.

- $\gamma\gamma \rightarrow \text{hadrons}$ events from the interaction of real or virtual beamstrahlung photons with each other.

- Beam halo muons that arise from interactions of the beam particles during collimation. The dominant mechanisms producing beam halo muons are photon conversions into muon pairs ($\gamma e^- \rightarrow \mu^+\mu^- e^-$) and annihilation of positrons with atomic $e^-$ into muon pairs ($e^+ e^- \rightarrow \mu^+\mu^-$) [9].

These backgrounds must be properly addressed to get a true measure of the physics potential CLIC has to offer. Coherent and trident pair production are not dominant sources of background because they have low transverse momenta and are collinear with the outgoing beam, as figure 1.5 shows. This is not the case for incoherent pair production of $e^+e^-$, which is dominant in the forward regions of the detector, and $\gamma\gamma \rightarrow \text{hadrons}$, which is dominant in the tracker and the calorimeters (with the exception of low radii in the calorimeter endcaps) [6,10]. Beam halo muons are not a major source of background either as they can be easily removed during the reconstruction since they produce a clear signal in the detector. An algorithm was developed within the PandoraPFA framework for this purpose and it was found to be highly effective at removing the beam halo muon backgrounds [6].

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure15.png}
\caption{Angular distribution of number of particles for beam induced backgrounds for CLIC at $\sqrt{s} = 3$ TeV. Figure taken from [6].}
\end{figure}

The dominant beam-related background found at the CLIC experiment is $\gamma\gamma \rightarrow \text{hadrons}$. Table 1.2 shows that $\gamma\gamma \rightarrow \text{hadrons}$ backgrounds deposit more energy within
the detector than the incoherent pair production. Each bunch crossing for CLIC at $\sqrt{s} = 3$ TeV contains an average of $3.2 \gamma\gamma \rightarrow \text{hadrons}$ events and $3 \times 10^5$ incoherent pairs, however, the vast majority of incoherent pairs are produced with low transverse momenta and are collinear with the outgoing beam.

<table>
<thead>
<tr>
<th>Subdetector</th>
<th>Incoherent Pairs [TeV]</th>
<th>$\gamma\gamma \rightarrow \text{hadrons}$ [TeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECal Endcaps</td>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>ECal Barrel</td>
<td>-</td>
<td>1.5</td>
</tr>
<tr>
<td>HCal Endcaps</td>
<td>16</td>
<td>6</td>
</tr>
<tr>
<td>HCal Barrel</td>
<td>0</td>
<td>0.3</td>
</tr>
<tr>
<td>Total Calorimeter</td>
<td>18</td>
<td>19</td>
</tr>
<tr>
<td>Central Tracker</td>
<td>-</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 1.2: Summary of the background conditions at $\sqrt{s} = 3$ TeV for the CLIC_ILD detector model. The numbers correspond to the background for an entire CLIC bunch train. The reconstructed calorimeter energies are integrated over 300 ns from the start of the bunch train. The backgrounds in the HCal from incoherent pairs are pessimistic as no attempts to mitigate the effect of neutrons from incoherent pair interactions in the BeamCal have been made. Table taken from [6].
Chapter 2

Anomalous Gauge Coupling Theory

"'Meaningless! Meaningless!' says the Teacher. 'Utterly meaningless! Everything is meaningless.'"
— Ecclesiastes 1:2

Presented in chapter 7 is an analysis of the sensitivity of the CLIC experiment to the anomalous gauge couplings $\alpha_4$ and $\alpha_5$ through the vector boson scattering process. Here, a brief description of the Standard Model of particle physics and a deeper discussion of the anomalous coupling theory studied in chapter 7 is given.

2.1 The Standard Model

The Standard Model is a non-abelian gauge theory of the $SU(3) \times SU(2)_L \times U(1)$ symmetry group. It provides a description of three of the four fundamental forces of nature: the electromagnetic, weak and strong nuclear forces [11,12]. The Standard Model contains a total of 24 fermion fields: six flavours of quark each with three colours and six leptons. A summary of the properties of these particles is given in table 2.1 and 2.2. As these fields, $\psi$, are spin-$\frac{1}{2}$, they obey the Dirac equation

$$L = \overline{\psi}(i\gamma \partial - m)\psi,$$  \hspace{1cm} (2.1)

where $L$ is the Lagrangian density and $m$ is a mass term. The derivative term, $\partial = \gamma^\mu \partial_\mu$, represents a summation over the partial derivative, $\partial_\mu = (\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$, of the field $\psi$ and the gamma matrices, $\gamma^\mu$. Each of the gauge transformations of the Standard Model
are defined by a unitary operator $U$, which acts to transform the vector space, $\Psi$, formed from a combination of fermion fields, $\psi$, in the following way

$$\Psi \rightarrow \Psi' = U\Psi .$$ (2.2)

<table>
<thead>
<tr>
<th>Generation</th>
<th>Particle</th>
<th>Mass [MeV]</th>
<th>Spin</th>
<th>$Q/e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$e^-$</td>
<td>0.548579909070 ± 0.0000000000016</td>
<td>1/2</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>$\nu_e$</td>
<td></td>
<td>-</td>
<td>1/2</td>
</tr>
<tr>
<td>2</td>
<td>$\mu^-$</td>
<td>105.6583745 ± 0.0000024</td>
<td>1/2</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>$\nu_\mu$</td>
<td></td>
<td>-</td>
<td>1/2</td>
</tr>
<tr>
<td>3</td>
<td>$\tau^-$</td>
<td>1776.86 ± 0.12</td>
<td>1/2</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>$\nu_\tau$</td>
<td></td>
<td>-</td>
<td>1/2</td>
</tr>
</tbody>
</table>

**Table 2.1:** The mass, spin and electric charge ($Q$) of the leptons found in the Standard Model [13]. Neutrino masses have not been included in the above table as precise measurements are yet to be made. However, oscillations between different neutrino flavour states have been observed, which indicates that the flavour and mass eigenstates differ and that the neutrinos have a non-zero mass. The current upper bound on neutrino mass measurements is 2 eV.

<table>
<thead>
<tr>
<th>Generation</th>
<th>Particle</th>
<th>Mass [MeV]</th>
<th>Spin</th>
<th>$Q/e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$u$</td>
<td>2.2$^{+0.8}_{-0.4}$</td>
<td>1/2</td>
<td>+2/3</td>
</tr>
<tr>
<td></td>
<td>$d$</td>
<td>4.7$^{+0.5}_{-0.4}$</td>
<td>1/2</td>
<td>-1/3</td>
</tr>
<tr>
<td>2</td>
<td>$c$</td>
<td>1270 ± 30</td>
<td>1/2</td>
<td>+2/3</td>
</tr>
<tr>
<td></td>
<td>$s$</td>
<td>98$^{+8}_{-4}$</td>
<td>1/2</td>
<td>+2/3</td>
</tr>
<tr>
<td>3</td>
<td>$t$</td>
<td>173210 ± 510 ± 710</td>
<td>1/2</td>
<td>+2/3</td>
</tr>
<tr>
<td></td>
<td>$b$</td>
<td>4180$^{+40}_{-30}$</td>
<td>1/2</td>
<td>-1/3</td>
</tr>
</tbody>
</table>

**Table 2.2:** The mass, spin and electric charge ($Q$) of the quarks found in the Standard Model [13]. Each of the particles in the above table corresponds to three fermion fields, one for each of the three colours of the SU(3) symmetry.

In the Standard Model, the Lagrangian density describing the fermion fields is invariant under a SU(3), SU(2)$_L$ and U(1) gauge transformations. The SU(2)$_L$ gauge symmetry acts on doublets formed of pairs of left handed chiral components of the fermion fields,
\[ \psi_L = \frac{1}{2}(1 - \gamma_5)\psi, \] while the right handed components, \( \psi_R = \frac{1}{2}(1 + \gamma_5)\psi, \) transform trivially as singlets \([14]\). Similarly, the SU(3) symmetry acts on triplets formed of the fermion fields for each flavour of quark. All fields transform under the fundamental representation of U(1). The invariance of the Standard Model Lagrangian to these gauge transformations is established by introducing 12 gauge fields, summarised in table 2.3, through the covariant derivative of the fermion fields

\[ \partial^\mu \rightarrow D^\mu = \partial^\mu + ig_1 Y B^\mu + ig_2 T \cdot W^\mu + ig_3 X \cdot G^\mu, \] (2.3)

where \( B^\mu \) is the gauge field for the U(1) symmetry, \( W^\mu \) (\( W^\mu_j, j = 1, 2, 3 \)) are the fields of the SU(2)\( _L \) symmetry and \( G^\mu \) (\( G^\mu_j, j = 1, \ldots, 8 \)) are the fields of the SU(3). \( Y \) is the weak hypercharge, which relates to the chirality and flavour of the fermion field that it is associated to. The three coefficients \( g_1, g_2 \) and \( g_3 \) are coupling constants related to the three gauged symmetry groups in the Standard Model. Mixing of the gauge fields for the U(1) and SU(2) symmetry of the form

\[ Z_\mu = \cos \theta_W W^3_\mu - \sin \theta_W B_\mu, \] (2.4)
\[ A_\mu = \sin \theta_W W^3_\mu + \cos \theta_W B_\mu, \] (2.5)
\[ W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \pm iW^2_\mu), \] (2.6)

where

\[ \cos \theta_W = \frac{g_2}{g_1 + g_2} \text{ and } \sin \theta_W = \frac{g_1}{g_1 + g_2}, \] (2.7)

gives the electroweak gauge bosons; \( W^\pm, Z \) and \( \gamma \). This mixing ensures that the \( W^\pm \) and \( Z \) bosons become massive, while the \( \gamma \) remains massless. The \( G^\mu_j \) fields are the eight massless gluons of the strong force. \( T \) and \( X \) are the generators for the SU(2) and SU(3) symmetries, which are typically chosen as

\[ T_i = \frac{1}{2} \tau_i, \] (2.8)
\[ X_i = \frac{1}{2} \lambda_i, \] (2.9)
\[ \lambda_8 = i \sigma_2, \] (2.10)

where \( \tau \) and \( \lambda \) are the Pauli and the Gell-Mann matrices, respectively. The gauge fields
Table 2.3: The mass, spin and electric charge ($Q$) of the gauge bosons found in the Standard Model [13]. The $\gamma$ and $g_s$ theoretically have zero mass, which is consistent with measurements. The upper bound on the $\gamma$ mass has been measured at $10^{-18}$ eV, while gluon masses of up to a few MeV have not been precluded. The upper bound on the magnitude of the charge of the $\gamma$ is measured at $10^{-35}$.

<table>
<thead>
<tr>
<th>Force</th>
<th>Particle</th>
<th>Mass [GeV]</th>
<th>Spin</th>
<th>$Q/e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electromagnetic</td>
<td>$\gamma$</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Weak Nuclear</td>
<td>$W^\pm$</td>
<td>80.385 ± 0.015</td>
<td>1</td>
<td>$\pm 1$</td>
</tr>
<tr>
<td></td>
<td>$Z$</td>
<td>91.1876 ± 0.0021</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Strong Nuclear</td>
<td>$g$ ($\times 8$ colours)</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Higgs</td>
<td>H</td>
<td>125.1 ± 0.3</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

of the Standard Model, $B_\mu$, $W_\mu$ and $G_\mu$, transform under the gauge transformations as

$$ K_\mu \rightarrow K'_\mu = U K_\mu U^\dagger + \frac{i}{g} (\partial^\mu U) U^\dagger, \quad (2.11) $$

where $K_\mu$ is any of $B_\mu$, $W_\mu$ and $G_\mu$ and $g$ is the coupling constant associated to the relevant gauged symmetry group. As the $B_\mu$, $W_\mu$ and $G_\mu$ gauge fields are spin-1, they are described by the Proca Lagrangian density

$$ \mathcal{L} = -\frac{1}{4} F_{\mu\nu}^{\mu\nu} F_{\mu\nu} + \frac{1}{2} m_K^2 K_{i\mu} K^i_\mu, \quad (2.12) $$

where

$$ F_{\mu\nu}^{\mu\nu} = \partial^\mu K^\nu_i - \partial^\nu K^\mu_i - g f_{ijk} K^\mu_j K^\nu_k, \quad (2.13) $$

and $f_{ijk}$ are the fully anti-symmetric structure constants of the group, $K^\mu_i$ is the $i^{th}$ gauge field of the group and $m_K$ is a mass term for the gauge boson. The structure constants are defined from the commutation relations between generators of the symmetry group

$$ [T_i, T_j] = if_{ijk} T_k. \quad (2.14) $$

These structure constants govern the self-interactions for the gauge bosons. There is only one structure constant for the U(1) symmetry, which is zero, as the U(1) symmetry is abelian. The SU(2) symmetry structure constants are $f_{ijk} = \epsilon_{ijk}$, where $\epsilon_{ijk}$ is the Levi-Civita tensor. Due to the symmetries that are present in the Standard Model, $m_K = 0$ for all the gauge fields, however, it is clear that this is not the case.
2.2 Higgs Physics

Mass terms are generated in the Standard Model by introducing a Higgs field that undergoes spontaneous symmetry breaking. This allows the gauge bosons, as well as the quarks and leptons, to obtain a mass, while still respecting the gauge symmetries found in the Standard Model.

2.2.1 Spontaneous Symmetry Breaking

To illustrate spontaneous symmetry breaking, consider a complex scalar field $\psi$ with the Klein-Gordon Lagrangian

$$L = \partial^\mu \psi^* \partial_\mu \psi - m^2 |\psi|^2 = \partial^\mu \psi^* \partial_\mu \psi - V(\psi), \quad (2.15)$$

where $m$ is a mass term and $V(\psi)$ is the potential of the field $\psi$. This Lagrangian density is invariant under the global symmetry $\psi \rightarrow e^{i\alpha} \psi$. By adding extra terms to the Lagrangian, which retain the invariance to this global symmetry, it is possible to modify the interactions of this scalar field. Consider modifying the potential of the scalar field to the following

$$V(\psi) = m^2 |\psi|^2 + \lambda |\psi|^4. \quad (2.16)$$

If $m^2 > 0$, the potential has a minimum at zero, however, if $m^2 < 0$ then the minima exist on a circle in the complex $\psi$ plane, which is centred at $(0, 0)$ and has radius $v = \sqrt{-m^2/\lambda}$.

To quantise this theory it is necessary to expand about the minimum of the potential. However, in the case of $m^2 < 0$ there are an infinite number of choices of minima to expand about. Irrespective of the choice of minimum used to expand the field about, the symmetry $\psi \rightarrow e^{i\alpha} \psi$ is broken. Fluctuations about the minimum along the degenerate direction leave the potential unchanged, which is a consequence of the breaking of the $\psi \rightarrow e^{i\alpha} \psi$ symmetry; this is known as spontaneous symmetry breaking. Goldstone’s theorem [15] implies that, for Lorentz-invariant theories, spontaneous symmetry breaking always leads to the existence of massless particles known as Goldstone bosons. If the complex scalar field $\psi$ is expanded about the non-zero minima, $\psi$ takes the form

$$\psi = \frac{1}{\sqrt{2}} (v + \psi_1 + i\psi_2), \quad (2.17)$$
where $\psi_1$ and $\psi_2$ are real fields and $v = \sqrt{-m^2/\lambda}$. Applying this parameterisation to the Lagrangian yields a mass term of $\sqrt{-m^2}$ for the $\psi_1$ field. However, there is no corresponding mass term for the $\psi_2$ field, which indicates that it is massless as predicated by Goldstone’s theorem

$$L = \frac{1}{2} \partial^\mu \psi_1 \partial_\mu \psi_1 + \frac{1}{2} \partial^\mu \psi_2 \partial_\mu \psi_2 - m^2 |\psi_1|^2 + \ldots .$$  \hfill (2.18)

Spontaneous symmetry breaking is the origin of gauge boson mass terms when applied to local symmetries instead of global ones. For example, consider the global symmetry, $\psi \rightarrow e^{i\alpha} \psi$ that exists in equation 2.15. If this global symmetry is promoted to a local symmetry by letting $\alpha \rightarrow \alpha(x)$ and $\partial^\mu \rightarrow D^\mu = \partial^\mu + iA^\mu$, where $A^\mu$ is the gauge field that transforms as $A^\mu \rightarrow A^\mu - \partial^\alpha \alpha(x)$, the Lagrangian becomes

$$L = (D^\mu \psi)^* (D_\mu \psi) - m^2 |\psi|^2 - \lambda |\psi|^4 .$$  \hfill (2.19)

If the $\psi$ field is expanded about a non-zero minimum in the potential, i.e. $m^2 < 0$ and $v = \sqrt{-m^2/\lambda}$, as was done in equation 2.17, then a gauge boson mass term, $+\frac{1}{2} A^\mu A_\mu$, is generated from the $(D^\mu \psi)^*(D_\mu \psi)$ term.

### 2.2.2 Electroweak Interactions

The electroweak sector of the Standard Model is that related to the SU(2)$_L \times$ U(1) symmetry [16]. In this sector, spontaneous symmetry breaking must occur in such a way as to give three massive gauge bosons, $W^\pm$ and $Z$, and one massless gauge boson, the $\gamma$. This can be achieved through a Higgs field, $H$, that transforms as a doublet under the SU(2)$_L$ symmetry. The Lagrangian for this field is

$$L_{\text{Higgs}} = (D_\mu H)^\dagger D^\mu H - V(H) .$$  \hfill (2.20)

The Higgs potential, $V(H)$, is

$$V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2 ,$$  \hfill (2.21)
where $\mu$ and $\lambda$ are constants. The covariant derivative of this Higgs field must satisfy the SU(2)$_L \times$ U(1) gauge symmetry meaning it takes the form

$$D_\mu H = (\partial_\mu + ig_1 Y B_\mu + ig_2 \frac{\tau^i}{2} W^i_\mu) H ,$$  

(2.22)

where $g_1$ and $g_2$ are coupling constants for the U(1) and SU(2)$_L$ gauged symmetries, respectively, $Y = \frac{1}{2}$ is the weak hypercharge of the Higgs and $\tau^i$ are the Pauli matrices. $B_\mu$ and $W^i_\mu$ are the gauge fields for the U(1) and SU(2)$_L$ gauged symmetries, respectively.

Consider spontaneously breaking the symmetry in the Higgs sector by expanding the Higgs field about a non-zero vacuum expectation value (vev)

$$\langle H \rangle = \left( \begin{array}{c} 0 \\ \frac{v}{\sqrt{2}} \end{array} \right) ,$$  

(2.23)

where the minima of the field is defined as

$$\frac{v}{\sqrt{2}} = \sqrt{\frac{\mu^2}{2\lambda}} ,$$  

(2.24)

where $v$ real. In that case, the kinematic term in the Higgs Lagrangian, $D^\mu H^\dagger D_\mu H$, contains mass terms for the gauge bosons

$$D^\mu H^\dagger D_\mu H \subset \frac{v^2}{2} (ig_1 Y B^\mu + ig_2 \frac{\tau^i}{2} W^{i\mu})(ig_1 Y B_\mu + ig_2 \frac{\tau^i}{2} W^{i\mu}) .$$  

(2.25)

If there is mixing of the SU(2)$_L$ and U(1) fields of the form

$$Z_\mu = \cos \theta_W W^3_\mu - \sin \theta_W B_\mu ,$$  

(2.26)

$$A_\mu = \sin \theta_W W^3_\mu + \cos \theta_W B_\mu ,$$  

(2.27)

$$W^\pm_\mu = \frac{1}{\sqrt{2}} (W^1_\mu \mp iW^2_\mu) ,$$  

(2.28)

then the following gauge boson mass terms are generated

$$\frac{(gv)^2}{4} W^{+\mu} W^{-\mu} + \frac{(g^2 + g'^2) v^2}{8} Z_\mu Z^\mu .$$  

(2.29)
The gauge boson masses generated by spontaneous symmetry breaking of the Higgs field are

\[ m_W = \frac{gv}{2}, \]
\[ m_Z = \frac{v\sqrt{g^2 + g'^2}}{2} = \frac{m_W}{\cos\theta_W}, \]
\[ m_A = 0, \]

where \( \theta_W \) is the Weinberg angle. This mixing produces a massless gauge boson, the \( \gamma \), and three massive gauge bosons, the \( W^\pm \) and \( Z \). By acquiring a non-zero vev, the Higgs field breaks the SU(2)_L \times U(1) symmetry that was present in the Lagrangian to the U(1)_em symmetry of electromagnetism.

The ratio of the masses of the \( W^\pm \) and \( Z \) bosons is predicted when spontaneous symmetry breaking occurs in the Higgs sector. This prediction sets the \( \rho \) parameter to unity, where the \( \rho \) parameter is defined as

\[ \rho = \frac{m_W^2}{m_Z^2 \cos\theta_W^2} = 1. \]

This is a consequence of the Higgs potential containing custodial symmetry [13]. As the \( \rho \) parameter has been experimentally measured to be 1.00040 ± 0.00024 [17], it is clear that any extension to the Standard Model should retain this result.

### 2.2.2.1 Custodial Symmetry

The Standard Model Higgs field is defined by the Lagrangian

\[ \mathcal{L}_{Higgs} = (D_\mu H)^\dagger D^\mu H - V(H), \]

where

\[ V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2, \]

and \( \mu \) and \( \lambda \) are constants. By construction the Higgs sector of the Standard Model is invariant under local SU(2)_L \times U(1) gauge transformations. However, a larger global
symmetry also exists in this sector, which can be seen by examining the Higgs doublet \[ H = (\psi, \psi^0) = (\psi_1 + i\psi_2, \psi_3 + i\psi_4). \] (2.34)

All the terms in the Higgs potential involve \( H^\dagger H = \psi_1^2 + \psi_2^2 + \psi_3^2 + \psi_4^2 \), which is invariant under any rotation of these four components and hence under an SO(4) global symmetry. In general, \( \text{SO}(4) \cong \text{SU}(2) \times \text{SU}(2) \), where \( \cong \) denotes an isomorphism. In the case of the Higgs sector \( \text{SO}(4) \cong \text{SU}(2)_L \times \text{SU}(2)_R \) where the \( \text{SU}(2)_L \) symmetry is the gauged symmetry of the Standard Model. This symmetry can be manifested using an alternative parameterisation \[ \Phi = (i\tau_2 H, H) = \begin{pmatrix} \psi^0 & \psi^+ \\ -\psi^+ & \psi^0 \end{pmatrix}. \] (2.35)

In this parametrisation the Higgs Lagrangian, \( L_{\text{Higgs}} \), becomes

\[
L_{\text{Higgs}} = \frac{1}{2} \text{Tr}[(D_\mu \Phi)^\dagger D^\mu \Phi] + \mu^2 \text{Tr}[\Phi^\dagger \Phi] - \lambda \text{Tr}[\Phi^\dagger \Phi \Phi^\dagger \Phi],
\]

which is invariant under transformations of the form

\[
\Phi \rightarrow U_L \Phi U_R^\dagger,
\]

where \( U_L \) and \( U_R \) are transformations of the \( \text{SU}(2)_L \) and \( \text{SU}(2)_R \) symmetry groups respectively.

When the Higgs field acquires a non-zero vev the \( \text{SU}(2)_L \times \text{SU}(2)_R \) symmetry of the Higgs potential is broken to an \( \text{SU}(2)_C \) symmetry, which is known as custodial symmetry \[ 20 \]. As \( \text{SO}(3) \cong \text{SU}(2) \), symmetry breaking in the Higgs sector is equivalent to an \( \text{SO}(4) \) symmetry being broken to an \( \text{SO}(3) \) symmetry. This becomes clear when inspecting the form of the Higgs potential after symmetry breaking. After expanding the Higgs field about the non-zero vev that is defined in equation 2.23, the terms in the Higgs potential involve \( H^\dagger H = (\psi_3 - v)^2 + \psi_1^2 + \psi_2^2 + \psi_4^2 \). Since \( H^\dagger H \) is only invariant to rotations between the \( \psi_1, \psi_2 \) and \( \psi_4 \) fields, the \( \text{SO}(4) \) global symmetry of the Higgs potential has been broken by spontaneous symmetry breaking to an \( \text{SO}(3) \) symmetry.

The Higgs field, \( H \), transforms a singlet under this \( \text{SU}(2)_C \) custodial symmetry, while the \( \text{SU}(2)_L \) gauge boson fields, \( W^i_{\mu} \), transform as a triplet. It is the transformation of the
$W^\pm_\mu$ fields under the SU(2)$_C$ symmetry that enforces the relationship between the masses of the $W^\pm$ and $Z$ gauge bosons and that $\rho$ should equal unity. It should be noted that the SU(2)$_L \times$ SU(2)$_R$ symmetry only exists in the Higgs sector of the Standard Model. The SU(2)$_R$ symmetry in the Standard Model is broken by Yukawa couplings of the Higgs to quarks and leptons and by a non-zero coupling to the U(1) gauge symmetry of the Standard Model, $g_1$. However, this breaking of the SU(2)$_R$ symmetry is weak, which means the deviations of $\rho$ from unity are minimal [20].

2.3 Anomalous Gauge Couplings

The existence of a light Higgs boson, as discovered by the LHC [1,2], suggests the presence of new physics beyond the Standard Model because of the naturalness principle [21]. The naturalness principle suggests that the mass of the Higgs boson, $\sim 125$ GeV, should be similar to the Plank mass, $\sim \mathcal{O}(10^{19})$ GeV [22], however, this is not the case. Such a disparity could only be resolved with a large amount of fine-tuning, which is undesirable, or beyond Standard Model physics involving new particles that protect the Higgs mass from large radiative corrections. Therefore, examining the electroweak sector for deviations from Standard Model predictions is a natural place to start a search for new physics. An example of a search for new physics in this sector is presented in chapter 7. This search uses an effective field theory (EFT) framework to parameterise deviations from the Standard Model through anomalous gauge couplings. The EFT framework is discussed in section 2.3.1 and the electroweak chiral Lagrangian, which introduces the anomalous gauge couplings, is discussed in section 2.3.2.

2.3.1 Effective Field Theory

There are a number of features in the observable universe that cannot be accounted for using the Standard Model of particle physics. However, the Standard Model provides a very good description of the interactions between particles at the energies being probed at modern particle collider experiments. Any underlying theory governing the interactions of particles must, therefore, behave like the Standard Model over these energies, or distance scales. Above such energies the theory will deviate from the Standard Model in order to account for the full underlying theory. Effective field theories (EFTs) work from
this premise by assuming that the complete theory has a momentum scale, \( \Lambda \), below which Standard Model behaviour is replicated \([23,24]\).

Quantum field theories must be renormalizable to ensure that non-infinite predictions of the coefficients in the Lagrangian can be made and tested \([25]\). Infinities arise from non-renormalizable theories due to divergent integrals from loop diagrams that assume the theory being applied is valid at all energy and length scales. Effective field theories act to avoid such problems by only integrating up to the momentum scale \( \Lambda \) and not above it. At the energy scale being considered, any infinities arising from the loop calculations in the EFT can be absorbed into a finite number of parameters. This methodology avoids the assumption that the theory in question is applicable to all energy scales and allows measurable predictions to be made.

As the Standard Model should be replicated at the low energy scale, it is appropriate when creating an EFT Lagrangian to append new operators to the Standard Model Lagrangian to account for areas of new physics. This gives the general form for an EFT Lagrangian as \([23]\)

\[
\mathcal{L}_{EFT} = \mathcal{L}_{SM} + \sum_{\text{dimension } d>4} \sum_i \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)},
\]

where \( \mathcal{L}_{SM} \) is the Standard Model Lagrangian, \( c_i^{(d)} \) are free parameters, \( \mathcal{O}_i^{(d)} \) is the \( i^{th} \) unique operator with dimension \( d \) in the EFT and \( \Lambda \) is the EFT momentum scale. The sum runs over all unique operators with dimension greater than four. The presence of the \( \Lambda^{d-4} \) in the denominator is required to ensure correct dimensionality of the new terms being added to the Lagrangian.

New physics is introduced by the operators \( \mathcal{O}_i^{(d)} \), but suppressed by the momentum scale \( \Lambda \). It is assumed that \( \Lambda \) is large with respect to the momentum scales that have been examined at pre-existing particle collider experiments, therefore, any new physics is suppressed. Under this assumption, new operators with dimension less than, or equal to, four can be vetoed from the EFT as their effects would be readily observed at pre-existing particle collider experiments, due to the \( \Lambda^{4-d} \) coefficient. At energies below the momentum scale, \( \Lambda \), it is possible to find the dominant new physics terms in the EFT and consider these as corrections to the Standard Model. Above this scale the EFT breaks down as operator \( \mathcal{O}_i^{(d)} \) in \( \mathcal{L}_{EFT} \) has a non-negligible coefficient. In the extremal limit, \( \Lambda \to \infty \), the Standard Model is recovered as new physics is too far out of reach to have any impact on observables.
2.3.2 Electroweak Chiral Lagrangian

The introduction of a Higgs field undergoing spontaneous symmetry breaking is able to produce mass terms in the Lagrangian for the $W^\pm$ and $Z$ bosons. However, it is possible to introduce these terms by parameterising the Higgs field using the gauge boson fields of the SU(2)$_L$ Standard Model symmetry [26]. In this approach, the pattern of spontaneous symmetry breaking mirrors that found in the Higgs sector of the Standard Model i.e. a global SU(2)$_L \times$ SU(2)$_R$ symmetry is broken to an SU(2)$_C$ symmetry. This will ensure that the $\rho$ parameter, introduced in section 2.2.2, retains a value of unity which is consistent with experimental measurements. The Standard Model spontaneous symmetry breaking pattern can be replicated using a field, $\Sigma(x)$, which transforms under the SU(2)$_L \times$ SU(2)$_R$ global symmetries as

$$\Sigma \rightarrow U_L \Sigma U_R^\dagger,$$

where $U_L$ and $U_R$ are transformations of the SU(2)$_L$ and SU(2)$_R$ symmetry groups, respectively, and $\Sigma(x)$ is

$$\Sigma(x) = \exp\left(-i \frac{v}{\Lambda} \sum_{a=1}^{3} \pi^a \tau^a\right),$$

where $\pi^a$ are the three would-be Goldstone bosons that exist when the SU(2)$_L \times$ U(1) symmetry is broken to U(1)$_{em}$ [27]. The SU(2)$_L$ and U(1) symmetries of the Standard Model are gauged in the usual way by defining the covariant derivate of the $\Sigma$ field

$$D_\mu \Sigma(x) = \partial_\mu \Sigma(x) + ig_2 W^a_\mu \tau^a \Sigma(x) - ig_1 B_\mu \tau^3 \Sigma(x),$$

where $g_1$ and $g_2$ are coupling constants for the U(1) and SU(2)$_L$ symmetries respectively and $\tau^a$ are the Pauli spin matrices. The lowest order derivative term for this $\Sigma$ field that could appear in the Lagrangian is

$$L_\Sigma = \frac{v^2}{4} \text{Tr}(D^\mu \Sigma D_\mu \Sigma) = -\frac{v^2}{4} \text{Tr}(V_\mu V^\mu),$$

where $V_\mu = (D_\mu \Sigma)\Sigma^\dagger$. This terms respects all the symmetries present in the Higgs sector of that Standard Model, including the custodial symmetry in the limit $g_1 \rightarrow 0$. Furthermore, by expanding this field about a non-zero vev, the SU(2)$_L \times$ SU(2)$_R$ global symmetry is broken to an SU(2)$_C$ symmetry exactly as it is in the Standard Model. For example, if this field is expanded about the point $\Sigma = 1$, i.e. the unitary gauge, mass
terms for the electroweak gauge bosons are generated that match those produced from spontaneous symmetry breaking of the Higgs field as described in section 2.2.1

\[
\frac{v^2}{4} \text{Tr}[V^\mu V_\mu] = -\frac{(gv)^2}{4} W_\mu W^{-\mu} - \frac{(g^2 + g'^2)v^2}{8} Z_\mu Z^\mu
\]  

(2.43)

\[m_A = 0,\]

\[m_W = \frac{gv}{2},\]  

(2.44)

\[m_Z = \frac{v\sqrt{g^2 + g'^2}}{2} = \frac{m_W}{\cos \theta_W}.\]

So far, all that has been done is a parameterisation of the Higgs field, however, it was shown by Longhitano [27] that there are several relevant operators involving the \(\Sigma\) field that are SU\( (2)_L \times U(1)\) invariant. As these operators obey the same symmetries as those found in the Standard Model they should be considered. This can be done using EFT approach, as discussed in section 2.3.1. Of the operators introduced by Longhitano, only two involve quartic massive gauge boson vertices and preserve the custodial symmetry [28]. These are

\[\alpha_4 \text{Tr}[V^\mu V_\nu] \text{Tr}[V^{\nu'} V_{\mu'}] \quad \text{and} \quad \alpha_5 \text{Tr}[V^\mu V_\mu]^2.\]  

(2.45)

These terms contribute to the massive gauge boson quartic vertices shown in figure 2.1. The Standard Model already contains triple and quartic vertices involving the electroweak gauge bosons, shown in figure 2.2, and these are also present in this EFT approach. These vertices originate from the kinematic terms in the Proca Lagrangian density \(\mathcal{L}_{\text{kin}} = -\frac{1}{4} B_{\mu \nu} B^{\mu \nu} - \frac{1}{4} W_{\mu \nu} W^{\mu \nu}\). Of the vertices showing sensitivity to \(\alpha_4\) and \(\alpha_5\), only the vertex shown in figure 2.1c is not present in the Standard Model.

Both terms shown in equation 2.45 contain dimension 8 operators [23] and, with respect to the EFT approach (i.e. equation 2.38) their coefficients are proportional to \(\Lambda^{-4}\), where \(\Lambda\) is the momentum scale of the new physics being modelled. In the limit that the momentum scale of new physics is beyond experimental reach, i.e. \(\Lambda \to \infty\), these terms do not contribute to measurable observables and the Standard Model is recovered. It should be noted that in this case, the Standard Model has been parameterised using the \(\Sigma\) field; so, in the limit \(\Lambda \to \infty\), the gauge boson mass terms generated from \(\mathcal{L}_\Sigma\) do not vanish.
Figure 2.1: Gauge boson self-coupling vertices that are sensitive to the anomalous gauge couplings $\alpha_4$ and $\alpha_5$.

Figure 2.2: Gauge boson self-coupling vertices in the Standard Model.
Chapter 3

Particle Flow Calorimetry for Future Linear Colliders

“How much better to get wisdom than gold, to get insight rather than silver!”
— Proverbs 16:16

Particle flow calorimetry can provide extremely good jet energy resolutions at a future linear collider. Jet energy resolution is crucial at the linear collider as many of the interesting processes will be characterised by multi-jet final states. Many of these multi-jet final states will be produced from the hadronic decays of W and Z bosons and one of the key goals of the future linear collider is to be able to separate these decays. However, separation of these decays can be achieved only by placing a tight requirement on the jet energy resolution; \( \sigma_E/E \lesssim 3.5\% \) for 50-500 GeV jets at the ILC and up to 1.5 TeV at CLIC [29]. The use of particle flow calorimetry will also be highly beneficial for quantifying final states of interest that contain charged leptons and missing momentum.

3.1 Particle Flow Calorimetry

The premise of particle flow calorimetry is to use the sub-detector that offers the best energy resolution to measure the energy of any given particle, which corresponds to energy measurements being made in the ECal for photons, the HCal for neutral hadrons and, crucially, the tracker for charged particles. The starkest contrast of this approach to that
of traditional calorimetry occurs in the measurement of the energy of charged particles. In particle flow calorimetry the momenta of a charged particle is measured using the curvature of the path it traverses as it bends in a magnetic field. The energy of the charged particle can be estimated from the momentum measurement by assuming the mass of the particle is negligible. In traditional calorimetry the energy of a charged particle would be measured using the calorimeters, predominantly the hadronic calorimeter (HCal). The tracker energy resolution for a single charged particle of energy $E_{X} \pm$ (GeV) is typically $10^{-4} \times E_{X}^2$, while for the HCal it is $\sim 0.55 \times \sqrt{E_{X}}$ [29]. The energy resolution offered by the tracker is significantly better than that offered by the HCal for energies up to $\sim \mathcal{O}(300 \text{ GeV})$. This means that particle flow calorimetry has the potential to offer a much better energy resolution for charged particles below $\sim \mathcal{O}(300 \text{ GeV})$, than that of the traditional calorimetry approach. Particle flow calorimetry offers gains in performance for collision energies well beyond 300 GeV as the average long-lived particle energy for physics processes of interest is typically much less than 300 GeV. Furthermore, it also leads to a significant improvement in the measurement of jet energies as, after the decay of short-lived particles, approximately 60% of the energy of a jet is carried in the form of charged particles. The measurement of jet energies in the particle flow paradigm is summarised in table 3.1. The benefits to the energy resolution for both charged particles and jets offered by the particle flow approach to calorimetry is the driving factor behind why it is planned for use at the linear collider experiments.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Charged Particles ($X^\pm$)</td>
<td>Tracker</td>
<td>$\sim 0.6E_j$</td>
<td>$10^{-4} \times E_{X}^2$</td>
</tr>
<tr>
<td>Photons ($\gamma$)</td>
<td>ECal</td>
<td>$\sim 0.3E_j$</td>
<td>$0.15 \times \sqrt{E_{\gamma}}$</td>
</tr>
<tr>
<td>Neutral Hadrons ($X^0$)</td>
<td>HCal</td>
<td>$\sim 0.1E_j$</td>
<td>$0.55 \times \sqrt{E_{X^0}}$</td>
</tr>
</tbody>
</table>

**Table 3.1:** The approximate energy fractions and resolutions for charged particles ($X^\pm$) of energy $E_{X} \pm$ (GeV), photons ($\gamma$) of energy $E_{\gamma}$ (GeV) and neutral hadrons ($X^0$) of energy $E_{X^0}$ (GeV) in a jet with total energy $E_j$ (GeV). The energy resolution for photons and neutral hadrons reflects the performance of a linear collider-like ECal and HCal, respectively. Taken from [29].

Particle flow calorimetry is challenging to put into practice as it requires a precise reconstruction for all long-lived particles within a detector. Charged particle energy measurements are made using the curvature of the track they traverse as they bend in the magnetic field, but they also produce calorimetric energy deposits, as shown in figure 3.1. If both of these energy measurements are used, the energy of all charged particles would...
be double counted. Therefore, to avoid this, any calorimetric energy deposits originating from charged particles are not included in the final energy measurement. However, this methodology makes it possible to double count and omit energy measurements if the origin of a calorimetric energy deposit is misidentified. For example:

- If a calorimetric energy deposit, made by a charged particle, is not associated to a track, the calorimetric energy deposit will be double counted: firstly when the track energy is accounted for and secondly when the calorimetric energy deposit is incorrectly reported as the energy of a neutral particle.

- If a calorimetric energy deposit, made by a neutral particle, is incorrectly associated to a track, that calorimetric energy deposit is not accounted for.

These effects, collectively known as "confusion", degrade the energy resolution of a particle flow detector. Therefore, it is crucial to make correct associations between charged particle tracks and their calorimetric energy deposits to minimise the effect of confusion. These associations can only be successfully made if the calorimeters used have fine segmentation, such as those found at the linear collider experiment, so that it becomes possible to separate the energy deposits from nearby showering particles. Even with this segmentation, making the association of charged particle tracks to calorimetric energy deposits is highly non-trivial. At the linear collider experiment, these associations are made using sophisticated pattern recognition algorithms, provided by PandoraPFA [30]. The fine segmentation of the linear collider calorimeters allows PandoraPFA to reconstruct the four-momenta of all particles entering the calorimeter and to report the energy of all reconstructed particles using energy measurements from the optimal sub-detectors.

### 3.1.1 Particle Shower Development

There are two types of calorimetric energy deposit that can be produced when a particle showers in a calorimeter; electromagnetic and hadronic. Electromagnetic showers are produced when either photons, electrons or positrons interact with matter in a calorimeter, while hadronic showers are produced when hadrons interact with the calorimeter.

Electromagnetic showers are well understood because, at linear collider-like energies, the dominant mechanism for energy loss is bremsstrahlung for electrons or positrons and pair production for photons, i.e. $\gamma \rightarrow e^+e^-$ [32]. This is true for particle energies greater than a critical energy, $E_C \sim 10$ MeV. Below this energy ionisation and excitation of atomic nuclei are the dominant energy loss mechanisms. Above $E_C$, the average distance
any particle in the electromagnetic shower travels before undergoing bremsstrahlung or pair production is approximately one radiation length ($X_0$).

A simple model of an electromagnetic shower can be developed by assuming that above $E_C$ energy loss occurs only through bremsstrahlung and pair production, while below $E_C$ energy loss occurs only by ionisation and excitation. Assuming that when bremsstrahlung or pair production take place the parent particle energy is split equally between the daughter particles, after every $X_0$ the total number of particles in the shower doubles and the average particle energy halves. In this case, the peak number of particles occurs when the average particle energy falls below $E_C$, which occurs after $t_{max}$ radiation lengths, where $t_{max} = \ln(E/E_C)/\ln(2)$ and $E$ is the energy of the incident showering particle. For example, the peak number of particles produced in a 10 GeV electromagnetic shower occurs after $\sim 10$ radiation lengths and at that point the electromagnetic shower contains $\sim 1000$ particles. These shower particles will be largely confined to a cylinder centred on the incident particle direction with a radius given by the Molière radius for the material the particles is showering in. In the nominal ILD ECAL the absorber material is tungsten, which had a Molière radius of 0.93 cm or roughly two cell widths. Therefore, in the nominal ILD ECAL a 10 GeV electromagnetic particle shower would produce a peak occupancy of the order of 50 to 100 particles per cell.

The interaction of hadrons with matter in a calorimeter is extremely complex in comparison to electromagnetic showers. There are a wide variety of energy loss mechanisms that play a role in hadronic showers such as ionisation by charged particles,
electromagnetic sub-showers produced when hadronic particles decay and invisible energy loss mechanisms such as nuclear recoil and nuclear binding energy losses [33]. As there is no clearly dominant mechanism, there is no simple analytic model equivalent for hadronic showers.

3.2 International Large Detector

The current detector concepts for the linear collider experiments have been designed to make particle flow calorimetry possible. While there are a number of different concepts that are under consideration for both the ILC and CLIC, one of the most prominent, and the focus of this work, is the International Large Detector (ILD). The ILD detector, shown in figure 3.2, achieves very high spatial resolution for all sub-detector systems thanks to its highly segmented calorimeters and central tracking system, both of which are encompassed within a 3.5 T magnetic field. PandoraPFA [29, 31] provides the sophisticated pattern recognition software that is required for particle flow calorimetry. A variant of the ILD detector model has also been adapted for use at CLIC and will be discussed in section 3.4.

![Figure 3.2: (a) Quadrant view of the ILD detector concept. The interaction point is in the lower right corner of the picture. Dimensions are in mm. (b) An artistic view of the ILD detector concept. Figures taken from [34].](image-url)
3.2.1 Overview

The tracking system for the ILD detector consists of a vertex detector, a Time Projection Chamber (TPC) and a number of supplementary silicon detectors. The vertex detector is designed to give precise information about displaced vertices with respect to the impact point (IP), which is crucial for the study of short lived particles such as the D and B mesons. The vertex detector is located close to the IP and surrounding it is the TPC, which is the central tracker for ILD. The TPC provides detailed measurements of the trajectory of charged particle tracks passing through it, up to 224 measurements per track. This information is used for determining the curvature of the charged particle track and hence the momentum of the charged particle that traversed it. Finally, the purpose of the supplementary silicon detectors is to provide additional, high precision, spatial measurements to aid track fitting and extend coverage of the detector down to low polar angles.

The calorimetric system for the ILD detector is comprised of an electromagnetic calorimeter (ECal), a hadronic calorimeter (HCal) and a number of forward calorimeters (FCal). The primary function of the ECal is to induce electromagnetic particles to shower within it and to measure the energy of these particle showers. Similarly, the HCal is designed to induce and measure the energy of hadronic particle showers. The ECal surrounds the tracking system in the ILD detector and is itself surrounded by the HCal. The function of FCal is to extend the coverage of the calorimeter system to low polar angles and to provide measurements of the luminosity of the colliding $e^\pm$ beams.

The outermost elements of the ILD detector are the solenoid, iron yoke and muon system. The solenoid generates a magnetic field of 3.5 T, which is essential for determining the energy of charged particles in the particle flow paradigm. The iron yoke is used to return the magnetic field generated by the solenoid. The yoke is instrumented by the muon system to provide additional information, which supplements the calorimetric energy measurements made by the ILD calorimeters.
3.2.2 Vertex Detector

The main goal of the ILD vertex detector is to achieve a resolution on the impact parameter of charged particle tracks of

\[ \sigma_b < 5 \oplus \frac{10}{p \sin(\theta)^{3/2}} \mu m, \]

(3.1)

where \( \sigma_b \) is the resolution on the track impact parameter, \( p \) is the momentum of the track in units of GeV and \( \theta \) is the angle between the track and the vertex detector plane. The first term in this parameterisation is the transverse impact parameter’s resolution and the second is a multiple-scattering term. This makes precisely tagging secondary vertices from charm and bottom mesons possible. Typically these mesons have relatively short proper lifetimes, \( \tau \), such that \( c\tau \approx \mathcal{O}(300 \mu m) \). To achieve this impact parameter resolution, a spatial resolution of better than 3 \( \mu m \) is required near the interaction point (IP). Furthermore, a low material budget of less than 0.15 % of a radiation length per layer is required to ensure that few electromagnetic showers are initiated within the vertex detector. A low pixel occupancy is essential for determining the trajectory of individual tracks in the detector. Furthermore, consideration will have to be given to the mechanical structure of the detector, power consumption and cooling.

There are a number of different pixel technology options under consideration for the vertex detector for the ILD detector. This is an active area of ongoing research and development for the linear collider collaboration. The current design of the vertex detector consists of three concentric layers of double-sided ladders with the first layer containing 10 ladders, the second 11 ladders and the third 17 ladders as shown in figure 3.3. Every ladder has two silicon pixel sensors on each side and the ladder thickness is approximately 2 mm. The radii covered by the detector range from 16 mm to 60 mm from the IP. Table 3.2 shows the layout, coverage, pitch, spatial resolution and readout times for each of the vertex detector layers. The first vertex layer is optimised for spatial resolution, while the second layer is optimised for a fast read-out time. The remaining layers are optimised to reduce power consumption.

3.2.3 Time Projection Chamber

The central tracking system for the ILD detector is a TPC, which is shown in figure 3.4. The TPC consists of a cylindrical gas volume with a central electrode providing an axial
### Table 3.2:

Parameters for the ILD vertex detector. In this table $R$ is the radial position of the vertex layer and $\theta$ is the polar angle with respect to the beam direction. Table taken from [34].

<table>
<thead>
<tr>
<th>Layer</th>
<th>$R$ [mm]</th>
<th>Coverage [z]</th>
<th>Coverage $[\cos(\theta)]$</th>
<th>Pitch $[\mu m^2]$</th>
<th>Spatial Resolution $[\mu m]$</th>
<th>Readout Time [\mu s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layer 1</td>
<td>16</td>
<td>62.5</td>
<td>0.97</td>
<td>$17 \times 17$</td>
<td>2.8</td>
<td>50</td>
</tr>
<tr>
<td>Layer 2</td>
<td>18</td>
<td>62.5</td>
<td>0.96</td>
<td>$17 \times 85$</td>
<td>6.0</td>
<td>10</td>
</tr>
<tr>
<td>Layer 3</td>
<td>37</td>
<td>125.0</td>
<td>0.96</td>
<td>$34 \times 34$</td>
<td>4.0</td>
<td>100</td>
</tr>
<tr>
<td>Layer 4</td>
<td>39</td>
<td>125.0</td>
<td>0.95</td>
<td>$34 \times 34$</td>
<td>4.0</td>
<td>100</td>
</tr>
<tr>
<td>Layer 5</td>
<td>58</td>
<td>125.0</td>
<td>0.91</td>
<td>$34 \times 34$</td>
<td>4.0</td>
<td>100</td>
</tr>
<tr>
<td>Layer 6</td>
<td>60</td>
<td>125.0</td>
<td>0.90</td>
<td>$34 \times 34$</td>
<td>4.0</td>
<td>100</td>
</tr>
</tbody>
</table>

**Figure 3.3:** Vertex detector design for ILD. Figures taken from [35].

Electric field. When a charged particle passes through the TPC, it ionises the gas and the ionised molecules drift in the axial electric field. The direction of the electric field is chosen such that the electrons drift towards the endplates where they are collected. The position of the ionisation point can then be calculated using the drift time of the electrons in the TPC. Combining these TPC hits together makes reconstruction of the charged particle track possible. TPCs have an advantage over silicon tracking in that they continuously track any charged particle passing through them, while silicon detectors are only sensitive within each silicon layer. This compensates for the poor single point resolution that TPCs have, $\sim \mathcal{O}(1 \text{ mm})$, in comparison to silicon detectors, $\sim \mathcal{O}(1 \mu m)$, and makes TPCs a viable option for the ILD detector. Furthermore, TPCs have a very low material budget. This benefits calorimetry as it minimises energy losses prior to the
particle energy entering the calorimeters, which means the calorimetric energy deposits give a better reflection of the true particle energy.

In the $xy$ plane the ILD TPC has a single point resolution that is better than 100 $\mu$m and a double hit resolution of $\sim 2$ mm, while in the $xz$ plane the single point resolution is $\sim 1$ mm and the double hit resolution is $\sim 6$ mm. The gas used for the TPC will be Ar:CH$_4$:CO$_2$ (95:3:2) [35]. Several readout technology options designed to measure the ionisation current are currently under development. For all potential options it is envisaged that the readout pads would be $\approx 1 \times 6$ mm$^2$ giving a total of approximately $10^6$ pads on each TPC endplate.

![Figure 3.4](image)

**Figure 3.4:** (a) Drawing of the proposed end-plate for the TPC. In the insert a back frame, which is designed to support the readout modules, is shown. (b) Conceptual sketch of the TPC system showing the main parts of the TPC (not to scale). The central electrode generates the axial electric field, the endplates collect the ionisation electrons, the field strips help to maintain a uniform electric field across the TPC and the voltage divider strips maintains the voltage difference between the anode and cathode. The field strips are held at fixed voltages such that they replicate the electric field produced by the electrodes. This reinforcing of the electric field configuration minimises non-uniformities in the electric field. The field cage of the TPC is not shown. Figures taken from [34].
3.2.4 Supplemental Silicon Tracking System

There are four components that make up the supplemental silicon tracking system in the ILD detector, shown in figure 3.5, which are:

- Silicon Inner Tracker (SIT) and Silicon External Tracker (SET). These are both barrel components, which are positioned immediately inside and outside the TPC. The SIT helps form associations between hits in the vertex detector and the TPC, while the SET helps with extrapolation of TPC tracks into the calorimeter.

- Endplate of the TPC (ETD). This sensor is identical to the SET, but is positioned in front of the ECAL endcap calorimeter. The ETD extends the coverage of the supplemental silicon tracking system envelope.

- Forward tracker (FTD). This detector consists of seven silicon disks that extend the coverage of the tracking down to small angles that are not covered by the TPC.

![Figure 3.5: (a) A 3D detailed GEANT4 simulation description of the silicon system. (b) A quadrant view of the ILD silicon envelope made of the four components SIT, SET, ETD and FTD as included in the full MOKKA simulation. Figures taken from [34].](image)

The coverage of the SIT, SET, ETD and FTD is given in table 3.3. These detectors are designed to give high precision space points that can be used in track fitting. Furthermore, the ETD and SET are of particular use for extrapolating the charged particle tracks into the calorimeters. This is key for particle flow calorimetry, which relies upon correct...
Table 3.3: Coverage of the supplementary silicon tracking systems in the ILD detector. In this table $\theta$ is the polar angle with respect to the beam direction. Taken from [34].

<table>
<thead>
<tr>
<th>Tracking System</th>
<th>Coverage [cos($\theta$)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIT</td>
<td>0.910</td>
</tr>
<tr>
<td>SET</td>
<td>0.789</td>
</tr>
<tr>
<td>ETD</td>
<td>0.799 - 0.985</td>
</tr>
<tr>
<td>FTD</td>
<td>0.802 - 0.996</td>
</tr>
</tbody>
</table>

association of charged particle tracks and clusters of calorimeter hits. Analogously to the vertex detector, these detectors require low material budget and low occupancy. The FTD, due to its proximity to the beam axis, is particularly prone to high occupancies.

The SIT, SET and ETD are silicon pixel sensors with 50 $\mu$m pitch embedded in 200 $\mu$m thick silicon. The FTD consists of seven silicon tracking disks, the first two being pixel detectors and the remaining five being strip detectors. The pixel detector disks are formed of 16 petals, as shown in figure 3.6. Within these petals the pixel size varies from $26 \times 29 \mu m^2$ to $26 \times 67 \mu m^2$. Strip detectors are used for the outermost tracking disks as the occupancy considerations do not demand a high granularity detector i.e. a pixel detector. These detector disks will have a pitch of 50 $\mu$m. The active sensor and readout ASIC design for each of these detectors is an active area of development for the linear collider.

**Figure 3.6:** A half-disk for the FTD showing the petal concept. The rightmost zoom image shows a detail of the end-of-petal area that houses the read-out electronics. The leftmost image shows the region at $R = 8$ cm where both the column width and the $R$-dimension of the pixels changes. Figures taken from [34].
3.2.5 Electromagnetic Calorimeter

The nominal ILD detector contains a finely segmented electromagnetic sampling calorimeter (ECal). The ILD ECal has been specifically designed with particle flow calorimetry in mind. To that extent the spatial resolution of particle showers within the ECal takes as much, if not more, precedence than the energy resolution.

There are a number of design requirements for the ECal:

- The ECal must be compact in size to reduce the overall cost of the detector.
- Fine segmentation of the ECal is required so that nearby particle showers can be separated. This is an essential requirement for particle flow calorimetry.
- Electromagnetic showers should be contained within the ECal.

Based on these requirements tungsten is used as the absorber material for the ILD ECal as it has a small radiation length ($X_0$), a small Molière radius and a large ratio of radiation length to nuclear interaction length. A comparison of these properties for other ECal absorber material candidates is shown in table 3.4. The small radiation length in tungsten allows for a large number of radiation lengths, $\approx 24 X_0$, to be compacted within a relatively short distance, $\approx 20$ cm, in the nominal ILD ECal. This is sufficient for containing all but the highest energy electromagnetic showers. The small Molière radius in tungsten will lead to compact electromagnetic showers. This makes separation of nearby showers easier. Finally, the large ratio of the radiation length to the nuclear interaction length in tungsten will lead to greater longitudinal separation between electromagnetic and hadronic showers, again making shower identification easier.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\lambda_I$ [cm]</th>
<th>$X_0$ [cm]</th>
<th>$\rho_M$ [cm]</th>
<th>$\lambda_I/X_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fe</td>
<td>16.8</td>
<td>1.76</td>
<td>1.69</td>
<td>9.5</td>
</tr>
<tr>
<td>Cu</td>
<td>15.1</td>
<td>1.43</td>
<td>1.52</td>
<td>10.6</td>
</tr>
<tr>
<td>W</td>
<td>9.6</td>
<td>0.35</td>
<td>0.93</td>
<td>27.4</td>
</tr>
<tr>
<td>Pb</td>
<td>17.1</td>
<td>0.56</td>
<td>1.00</td>
<td>30.5</td>
</tr>
</tbody>
</table>

Table 3.4: Comparison of the nuclear interaction length $\lambda_I$, radiation length $X_0$ and Molière radius for iron, copper, tungsten and lead. Table taken from [29].

The active material in the nominal ILD ECal is silicon, however, a scintillator strip option is also being considered. Figure 3.7 shows a cross section through a layer of the
ECal for both of these options. It contains a total of 30 longitudinal readout layers, which is sufficient to provide a good energy resolution. The tungsten thickness for the innermost 20 layers is 2.1 mm, while for the final 10 layers it is 4.2 mm. This configuration of absorber material thickness is chosen to reduce the number of readout channels and hence the cost, while maintaining a high sampling rate for particle showers at the start of the ECal. It should be noted that this ECal offers no gains in terms of energy resolutions in comparison to pre-existing particle collider experiments, as shown in table 3.5. This is the case because the focus of this calorimeter is split between imaging the particle showers and recording their energy as opposed to purely focusing on the energy measurement. Each of the ECal layers is divided up into square cells, of 5 mm side length, which makes separation of nearby particle showers possible. This cell size was chosen as a balance between being able to resolve nearby particle showers and reducing the overall cost of the calorimeter, which scales with the number of readout channels. An optimisation study of the various ECal parameters for the ILD detector can be found in section 6.2.

![Cross section through an ECal layer for the (a) silicon and (b) scintillator options. Figures taken from [34].](image)

**3.2.6 Hadronic Calorimeter**

A finely segmented hadronic sampling calorimeter (HCal) is used in the nominal ILD detector. The design requirements for the ILD HCal mirror those of the ECal, which can
be found in section 3.2.5, with one exception; the HCal is designed to contain hadronic showers as opposed to electromagnetic showers. Steel is used as the absorber material for the HCal as it has durable mechanical properties that allow the HCal to be constructed without the need for auxiliary supports. When required, auxiliary supports create dead regions in the detector that would harm performance. Furthermore, steel is relatively inexpensive and has a small nuclear interaction length, meaning it is possible to achieve a compact calorimeter design at low cost. The nominal ILD HCal contains approximately $6 \lambda_I$, which when combined with the $1 \lambda_I$ in the ECAL is enough to contain the majority of hadronic showers at ILC like energies.

The active material in the nominal ILD HCal is scintillator. In total, the HCal contains 48 readout layers, which provides an extremely good energy resolution. This can be seen when comparing the HCal energy resolution between different experiments, as shown in table 3.6. An individual layer in the HCal is comprised of 20 mm of steel absorber material with 3 mm of scintillator active material. Each layer in the HCal is segmented into square cells of side length 30 mm. This cell size was chosen as a balance between reducing the cost of the detector, which is proportional to the number of readout channels, and achieving the required spatial resolution to make particle flow calorimetry possible. The segmentation of the ILD HCal gives excellent spatial resolution and sufficiently good energy resolution to make the use of particle flow calorimetry a reality. An optimisation study of the various HCal parameters for the ILD detector can be found in section 6.3.

The ILD HCal is intrinsically non-compensating, which means that it has a different response to electromagnetic and hadronic showers. The origin of this different response

<table>
<thead>
<tr>
<th>Experiment</th>
<th>ECAL Energy Resolution $\frac{\sigma_E}{E}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMS [36]</td>
<td>$\sim 2.8% \sqrt{E(\text{GeV})} \oplus 0.3% \oplus 12% \sqrt{E(\text{GeV})}$</td>
</tr>
<tr>
<td>ATLAS [37]</td>
<td>$\sim 10.1% \sqrt{E(\text{GeV})} \oplus 0.1%$</td>
</tr>
<tr>
<td>LHCb [38]</td>
<td>$\sim 9% \sqrt{E(\text{GeV})} \oplus 0.8%$</td>
</tr>
<tr>
<td>OPAL [39]</td>
<td>$\sim 6.3% \sqrt{E(\text{GeV})}$</td>
</tr>
<tr>
<td>ALEPH [40]</td>
<td>$\sim 17.9% \sqrt{E(\text{GeV})} \oplus 1.9%$</td>
</tr>
<tr>
<td>ILC (ILD Silicon Option) [34]</td>
<td>$\sim 16.6% \sqrt{E(\text{GeV})} \oplus 1.1%$</td>
</tr>
</tbody>
</table>

Table 3.5: Comparison of the ECAL energy resolutions for various experiments.
is the fundamentally different mechanisms governing the propagation of electromagnetic and hadronic showers. One key difference between the mechanisms is that hadronic showers have an invisible energy component, which occurs due to effects such as neutrons coming to rest in the detector and nuclear binding energy losses [4]. In general, this leads to a lower response from a calorimeter to a hadronic shower than an electromagnetic shower. A number of different software techniques have been developed for the linear collider experiment that attempt to correct this non-compensating response. For more details see chapter 5. The ILD ECal has a compensating response due to the use of tungsten as the absorber material [41], therefore, no additional treatment of energies is required.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>HCal Energy Resolution $\sigma_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMS [42]</td>
<td>$\sim \sqrt{\frac{E}{\text{GeV}}} \oplus 4.8%$</td>
</tr>
<tr>
<td>ATLAS [43]</td>
<td>$\sim \sqrt{\frac{E}{\text{GeV}}} \oplus 3.0% \oplus \frac{1.6%}{E(\text{GeV})}$</td>
</tr>
<tr>
<td>LHCb [38]</td>
<td>$\sim \sqrt{\frac{E}{\text{GeV}}} \oplus 9.0%$</td>
</tr>
<tr>
<td>OPAL [39]</td>
<td>$\sim \sqrt{\frac{E}{\text{GeV}}} \oplus 1.6%$</td>
</tr>
<tr>
<td>ALEPH [40]</td>
<td>$\sim \sqrt{\frac{E}{\text{GeV}}} \oplus 1.8%$</td>
</tr>
<tr>
<td>ILC (ILD Silicon Option) [34]</td>
<td>$\sim \sqrt{\frac{E}{\text{GeV}}} \oplus 1.8%$</td>
</tr>
</tbody>
</table>

Table 3.6: Comparison of the HCal energy resolutions for various experiments.

### 3.2.7 Solenoid, Yoke and Muon System

Surrounding the ILD calorimeter system is the solenoid that generates a 3.5 T magnetic field. The magnetic field produced by the coil is crucial for bending charged particles so that their momentum can be determined from the curvature of the path they traverse. Furthermore, the bending of charged particles leads to greater separation of calorimetric energy deposits between charged and neutral particles, which will reduce the effects of confusion when using particle flow calorimetry.

The magnetic field in the ILD detector is returned by an iron yoke that surrounds the solenoid. Iron is chosen for the yoke material as it has a very large permeability. The yoke is instrumented by a muon system in the barrel and forward regions of the detector. The goal of this instrumentation is to identify muons escaping the calorimeters.
and to act as a tail catcher for the calorimeters. The muon system consists of 10 layers, spaced 140 mm apart, followed by 2 and 3 layers spaced 600 mm apart in the barrel and endcap regions of the detector respectively, as shown in figure 3.8. There is also an additional sensitive layer for the barrel region placed immediately outside the HCal to help with association energy deposits between the calorimeters and the yoke. As the majority of particles at ILC like energies will be contained within the calorimeters, the energy and spatial resolution of the muon system are not critical to performance. It is for that reason that the number of layers is lower and the layer thicknesses wider in the yoke than in the calorimeters. The nominal ILD model uses 30 mm wide and 1 m long scintillator strips as the readout technology for the yoke.

![Figure 3.8: The sensitive layers of the ILD muon system. Figure taken from [34].](image)

### 3.2.8 Forward Calorimetry

Forward calorimetry in the ILD detector consists of three additional sampling calorimeters:

- The LumiCal, which is located within the octagonal hole in the ECAL endcap. This will give a precise measurement of the luminosity of the linear collider beam. The LumiCal uses Bhabha scattering, $e^+e^- \rightarrow e^+e^-(\gamma)$, as a gauge process for the luminosity measurement. Using this approach the luminosity can be measured with precision of less than $10^{-3}$ at $\sqrt{s} = 500$ GeV [35].

- The LHCal, which is positioned within the square hole of the HCal endcap. This hadronic calorimeter is designed to extend the coverage of the HCal down to small polar angles.
• The BeamCal, which is located just in front of the final focusing quadrupole. This calorimeter will perform a bunch-by-bunch estimate of the luminosity based on the energy deposited in the calorimeter.

The layout of these calorimeters is shown in figure 3.9 and their coverage is summarised in table 3.7. Each of the forward calorimeters will have to deal with high occupancies due to the presence of background processes, e.g. beamstrahlung, which makes fast readout crucial. Furthermore, the BeamCal experiences a large flux of low energy electrons due to its proximity to the beam pipe, which results in a large radiation dose. This makes radiation hard sensors essential for the BeamCal.

![Figure 3.9](image_url)

**Figure 3.9:** The very forward region of the ILD detector. LumiCal, BeamCal and LHCal are carried by the support tube for the final focusing quadruple, QD0, and the beam pipe. Figure taken from [34].

<table>
<thead>
<tr>
<th>Forward Calorimeter</th>
<th>Polar Angle Coverage [mrad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>LumiCal</td>
<td>31 – 77</td>
</tr>
<tr>
<td>LHCal</td>
<td>29 – 122</td>
</tr>
<tr>
<td>BeamCal</td>
<td>5 – 40</td>
</tr>
</tbody>
</table>

**Table 3.7:** Coverage of the forward calorimeters in the ILD detector.

Each of these forward calorimeters is constructed using tungsten as the absorber material. The small Molière radius of tungsten ensures that narrow electromagnetic showers are formed within them, which makes separation and identification of showering particles easier.

The layout of these calorimeters is as follows:
• The LumiCal is a silicon tungsten sampling calorimeter that contains 30 readout layers. This gives the LumiCal a total depth of \( \approx 24X_0 \).

• The LHCal is also a silicon tungsten sampling calorimeter, which contains 40 readout layers. The total depth of the LHCal is \( \approx 4\lambda_I \).

• The BeamCal is a tungsten based sampling calorimeter. The sensitive detector material for the BeamCal is an ongoing area of research as, due to the extremely high occupancy from the beam induced backgrounds, a very fast readout is required. The exact layer configuration of the BeamCal will depend upon the choice of sensitive detector material and hence is yet to be specified.

The segmentation within the layers, i.e. the cell size, in these forward calorimeters is yet to be fully optimised.

3.3 Simulation

Detector model simulation for all studies presented in this work was performed using MOKKA [44], a GEANT4 [45,46] wrapper providing detailed geometric descriptions of detector concepts for the linear collider. The MOKKA simulation of the ILD detector includes the following [34]:

• The vertex detector is simulated using silicon as the sensitive material. Support material and the cryostat are also included.

• The supplementary silicon tracking systems are included. Again, material has been added to the simulation to represent the support material for these systems. Furthermore, an estimation has been made of the material budget for power and readout cables from the vertex detector, SIT and FTD and material has been added to the simulation to represent these. The material added to represent the power and readout cables comes in the form of an aluminium cylinder running inside the TPC field cage and a cone around the beam pipe.

• The TPC is simulated as a cylindrical volume of a gas mixture surrounded by a field cage. A conservative estimate of the endplate is included in the simulation to account for the support structure, electronics and cooling pipes for the TPC.
• As well as including the silicon tungsten sampling calorimeter, the simulation of the ILD ECal contains additional material to represent the instrumented region of the sensor and a heat shield as shown in figure 3.7.

• Simulation of the ILD HCal has a number of realistic features including detailed modelling of the electronics, detector gaps and the implementation of Birk’s law [47] for the scintillator sensitive detector elements. Birk’s law accounts for the non-linear relationship between the energy deposited per unit length and the amount of scintillation light produced when a charged particle passing through a scintillator.

• The muon system, which is the instrumentation of the iron yoke, uses scintillator as the active material in the simulation. A square cell size of side length 30 mm is assumed. This is in contrast to the nominal ILD model, but as the tail-catcher plays a minimal role in event reconstruction at ILC like energies this difference should have negligible impact.

• The forward calorimeters, the LumiCal, LHCal and BeamCal, are all included in the simulation. Tungsten is used as the absorber material for each of the calorimeters. The LumiCal and LHCal use a silicon readout material, while the BeamCal uses a diamond readout.

The simulation and reconstruction of the large event samples used in the studies presented in the work was performed using the ILCDIRAC [48, 49] grid production tools.

3.4 CLIC_ILD

The increased collision energy of the proposed CLIC accelerator means the use of the nominal ILD detector model would be inappropriate. Therefore, a new detector model, CLIC_ILD [6, 50], based upon the nominal ILD detector model was created to cope with the experimental conditions found at the CLIC experiment. Figure 3.10 shows the longitudinal and transverse cross sections of the CLIC_ILD detector. The main differences between the nominal ILD detector and CLIC_ILD are:

• The higher energies found at the CLIC experiment lead to more intense beam induced backgrounds, which is especially problematic for detectors close to the IP where the occupancies will be extremely high. For this reason the inner vertex detector in CLIC_ILD is moved 15 mm further out from the IP.
• The HCal thickness is increased from $6 \lambda_I$ to $7.5 \lambda_I$. This ensures that higher energy hadronic showers found at the CLIC experiment are contained within the calorimeters.

• The HCal absorber material for the barrel is tungsten as opposed to steel. This reduces the overall thickness of the HCal and keeps the coil size, one of the driving cost factors for the detectors, similar for the nominal ILD and CLIC_ILD detectors. Steel is used as the absorber material for the HCal endcaps as there are no spatial requirements relating to the coil size and this will lower the detector cost. Furthermore, the shower development time in steel is faster than in tungsten. This makes effective time stamping of energy deposits easier, which is crucial for the CLIC experiment for vetoing beam induced backgrounds.

• The magnetic field strength in the CLIC_ILD detector is increased to 4 T. This was found to benefit the reconstruction, particularly at high energies, as it leads to greater separation of charged particle tracks. Furthermore, it was possible to achieve this increase in field strength using the nominal ILD coil design.

• The CLIC_ILD detector contains masking, graphite layers placed in front of the BeamCal, to prevent particles produced by the beam-induced interactions from backscattering into the main detector. It is the increased collision energy that makes
backscattering of particles a more problematic effect for the CLIC experiment than it is for the ILC experiment.

The CLIC_ILD detector will be used for the study of anomalous gauge couplings presented in chapter 7.

3.5 Particle Flow Reconstruction

Particle flow calorimetry relies upon correct associations being made between calorimetric energy deposits and charged particle tracks. Even with a finely segmented detector, such as the ILD detector described in section 3.2, correctly making these associations is a highly non-trivial task and must be done using advanced pattern recognition software. This is provided by the PandoraPFA particle flow algorithm [30]. PandoraPFA is applied in the linear collider reconstruction using MARLIN [51], a c++ framework specifically designed for the linear collider.

3.5.1 PandoraPFA

PandoraPFA’s inputs are calorimeter hits and charged particles tracks and it produces as output reconstructed particles known as particle flow objects (PFOs). The pattern recognition in PandoraPFA is applied in eight main stages as described in the original PandoraPFA paper [29] and the CLIC focused PandoraPFA paper [31]:

1. Track selection. The input track collections are examined to determine whether $V^0$ decays or kinks are present. Two charged tracks originating from a point displaced from the IP indicates the presence of a $V^0$ decay. The vertex of any $V^0$ decay is required to be at least 5 mm from the IP. The $V^0$ decays that PandoraPFA attempts to identify are $\gamma \rightarrow e^+e^-$, $K_S^0 \rightarrow \pi^+\pi^-$, $\Lambda \rightarrow p\pi^-$ and $\bar{\Lambda} \rightarrow \bar{p}\pi^+$. These decays are distinguished from each other by performing a mass hypothesis test using the charge and momenta of the tracks identified as coming from a $V^0$ decay. A charged particle decaying into a single charged particle and a number of neutral ones indicates a kink. The $V^0$ decay and kink information will be propagated in the reconstruction to the final PFO creation stage.

2. Calorimeter hit treatment. The treatment of calorimeter hits by PandoraPFA is of paramount importance to the work presented in chapters 5 and 6. Therefore, full
details of the calorimeter hit selection procedure are presented here. This selection
procedure is broken down into several steps:

- The various collections of, post digitisation, calorimeter hits are passed into the
  Pandora framework and converted into Pandora calorimeter hits.

- To minimise any dependency on the detector geometry each calorimeter hit
  is assigned to a pseudo-layer, which is illustrated in figure 3.11. Pseudo-
  layers are defined using the amount of material found between any given point
  in the calorimeters and the IP. This makes them the natural construct for
  parameterising particle shower development along the direction of travel of
  showering particles. Therefore, all further topological association algorithms in
  PandoraPFA work using the pseudo-layer definition.

- A minimum ionising particle MIP equivalent energy cut is applied to the
  calorimeter hits. If a calorimeter hit contains less than 0.5 (0.3) of the energy
  of a normally incident MIP passing through the calorimeter cell in the ECal
  (HCal) then it is not used in the reconstruction.

- If a calorimeter hit is sufficiently far away from other hits it is flagged as an
  isolated hit. Such hits are most likely due to low energy neutrons produced in
  hadronic showers that can travel a significant distance from the original shower
  before depositing energy. Due to the distance they travel, these hits are very
difficult to associate to the correct particle shower. Furthermore, as such hits
  are unlikely to be the seed for a particle shower, they are not used by the initial
  clustering algorithm.

- Any calorimeter hit that contains an energy consistent with a MIP signal and
  where one Pandora calorimeter hit at most exists in the neighbouring cells
  within the same layer is flagged as a MIP consistent hit. This information is
  used in the identification of MIPs in the reconstruction.

- The energy contribution for each calorimeter hit ultimately depends on whether
  the cluster the calorimeter hit has been associated to is deemed to have originated
  from an electromagnetic or hadronic particle shower. Different scale factors are
  applied to the energy for electromagnetic and hadronic showers to account for
  the non-compensating response of the calorimeters. These scale factors are used
  throughout the reconstruction, including the final reconstructed particle energy,
  once the particle shower type has been identified. For energy comparisons prior
to the shower type being identified, the uncorrected calorimeter hit energy is used. Further details on how these calibration constants are determined can be found in chapter 5.

3. Clustering. This begins by using the projection of the charged particle tracks onto the front face of the ECAL as seeds for the initial clustering phase. Calorimeter hits are looped over on a per layer basis, working from the inner to the outer pseudo-layer, and if they fall within a cone of fixed dimensions surrounding a cluster direction they are associated to the cluster. If no association can be made to any pre-existing calorimeter hit clusters then the calorimeter hit is used to seed a new cluster.

4. Topological cluster merging. The initial clustering algorithm is designed to be conservative to avoid mixing together energy deposits from several particles. The fragments produced by the initial clustering are then merged together by various algorithms whose logic is determined by a number of well-motivated topological rules, such as those shown figure 3.12.

5. Statistical re-clustering. Comparisons between the cluster energy and any associated track momenta are made to determine whether they are consistent. If a large discrepancy is observed then statistical re-clustering is initiated. This involves running a number of differently configured algorithms to change the cluster configuration to determine if a new optimal configuration of tracks and clusters can be found. The definition of a consistent track-cluster association is that the difference between the track momenta, $p_{\text{Track}}$, and cluster energy is less than $3\sigma$ where $\sigma = 60% \times \sqrt{p_{\text{Track}}}$.

The definition of $\sigma$ is the approximate energy resolution for hadronic showers in the ILD calorimeters.

This step relies upon the reported cluster energies being accurate. To ensure this is the case, a well defined calibration procedure is applied for all detector models considered in this work, for more details see chapter 5. At this point in the reconstruction, the energy resolution of the calorimeters impacts the way that the pattern recognition is performed. The better the energy resolution of the calorimeters, the fewer the number of mistakes that are made when pairing up clusters of calorimeter hits to charged particle tracks.

6. Photon identification and recovery. Topological likelihood data is used to identify clusters of calorimeter hits that are consistent with photons. This is possible due to the clear transverse and longitudinal profiles observed for electromagnetic showers.
7. Fragment removal. Neutral clusters originating from a nearby charged particle cluster are identified and merged back into the parent charged particle cluster. These algorithms take into account the changes in the compatibility of the track and cluster associations when merging any neutral clusters into charged clusters.

8. Formation of particle flow objects. Finally, reconstructed particles are produced. The energy for charged particles is taken from the track momenta, while neutral particle energies are taken from the calorimeter cluster measurements. Furthermore, the different electromagnetic and hadronic scales are applied to the output neutral particle energies depending on whether the neutral cluster is consistent with a photon.

Figure 3.11: Schematic showing the definition of the pseudo-layer assignment for calorimeter hits. The solid lines indicate the positions of the physics ECal layers and the dashed lines show the definition of the virtual pseudo-layers. (a) The $xy$-view shows the ILD ECal stave structure. (b) The $yz$ view shows a possible layout for the ECal barrel/endcap overlap region. The pseudo-layers are defined using projection back to the IP. Figures taken from [29].

The application of the pattern recognition algorithms in PandoraPFA when combined with a highly segmented detector make particle flow calorimetry a reality. In turn this provides excellent jet energy resolution for studying many interesting physics processes at the linear collider experiments.
Figure 3.12: The main topological rules for cluster merging: (a) looping track segments; (b) track segments with gaps; (c) track segments pointing to hadronic showers; (d) track-like neutral clusters pointing back to a hadronic shower; (e) back-scattered tracks from hadronic showers; (f) neutral clusters which are close to a charged cluster; (g) a neutral cluster near a charged cluster; (h) cone association; and (i) recovery of photons which overlap with a track segment. In each case the arrow indicates the track, the filled points represent the hits in the associated cluster and the open points represent hits in the neutral cluster. Charged clusters are defined by having an associated charged particle track, while neutral clusters have no associated tracks. Figures taken from [29].
3.6 Performance

The fundamental principle of particle flow calorimetry is to measure the energy of a particle passing through a detector in whichever sub-detector offers the best energy resolution. For particle collider experiments, this involves measuring the momenta of charged particles using the curvature of the track they create in the detector. This offers extremely good energy resolution in comparison to the traditional calorimetric approach.

As many physics processes of interest at the linear collider involve multi-jet final states [52], good jet energy resolution is a crucial aspect of detector performance. As shown in chapter 7, the sensitivity of the linear collider experiments to areas of new physics can be determined using reconstructed jet energies. Furthermore, parameters derived from the energy measurements of jets are extremely useful for identification of physics channels of interest. Therefore, a key metric for describing detector performance is the jet energy resolution. Jet energy resolution in particular can benefit from the application of particle flow calorimetry because \( \approx 70\% \) of the energy of jets is carried in the form of charged particles. As particle flow calorimetry aims to measure the energy of charged particles using the tracker, it has the potential to offer extremely large benefits when measuring jet energies in comparison to the traditional calorimetric approach.

3.6.1 Jet Energy Resolution

The jet energy resolution in these studies was determined through the simulation of off-mass shell Z boson events decaying to light quarks (u, d, s). PYTHIA version 6.4 [53], which had been trained on fragmentation data from the OPAL experiment [54], was used to generate these events. The decay of tau leptons appearing in the events was simulated using TAUOLA [55]. Detector simulation and event reconstruction was carried out as described in sections 3.3 and 3.5 respectively.

As the Z boson in these events is produced at rest, the typical decays form two mono-energetic jets that are produced back-to-back as shown in figure 3.13. Only events where \( |\cos(\theta)| < 0.7 \), where \( \theta \) is the polar angle of the quarks, are used in the jet energy resolution calculation. This ensures that little energy is lost down the beam axis. Using these events, the jet energy resolution was calculated as follows:

\[
\frac{\text{RMS}_{90}(E_j)}{\text{Mean}_{90}(E_j)} = \frac{\text{RMS}_{90}(E_{jj})}{\text{Mean}_{90}(E_{jj})} \times \sqrt{2},
\]  

(3.2)
where $E_{jj}$ is the total reconstructed energy. The variables Mean$_{90}(E_{jj})$ and RMS$_{90}(E_{jj})$ are the mean and root mean squared (RMS) of the $E_{jj}$ distribution respectively. They are calculated across the range of $E_{jj}$ with the smallest RMS containing at least 90% of the data. This definition is used to remove the effect of outliers in the distribution \[29\]. If all associations between charged particle tracks and calorimeter clusters were correctly made, the reconstructed jet energy distribution would be Gaussian. However, the effect of confusion on certain events will distort this distribution and broaden the tails significantly. If the full range were to be used in the jet energy resolution calculation, the effect of these tails is overinflated. When the distribution of reconstructed jet energies is truncated to the narrowest range that contains at least 90% of the data, the effect of these tails can be negated. This removes events where confusion is dominant, which makes the jet energy resolution metric far more robust and representative of the bulk of the data.

![Figure 3.13: A 500 GeV di-jet Z→uds event display for the nominal ILD detector.](image)

An example of the application of this metric can be found in figure 3.14. In this example RMS$(E_{jj})$, the RMS calculated using the full range, is 5.8 GeV, while RMS$_{90}(E_{jj})$, the RMS using the reduced range, is 4.1 GeV. This corresponds to a reduction in the jet energy resolution from 4.1% to 2.9%, which clearly shows an overemphasis of the tails of the distribution when using the full jet energy range.

In the subsequent analysis a range of di-jet energies were considered ranging from the Z mass, 91 GeV, to the nominal running energy of the ILC, 500 GeV. Each event
Figure 3.14: Definition of jet energy resolution. Reconstructed jet energy for 200 GeV di-jet \(Z\rightarrow\text{uds}\) events for nominal ILD detector. The solid vertical line shows the mean of the distribution and the horizontal arrows indicate the mean \(\pm\) the root mean square (RMS) of the distribution. The red and blue lines show the mean and RMS calculated using the full range and 90\% of the full range with the smallest RMS respectively.

The sample contained 10,000 events generated isotropically so that, given the polar angle cut, approximately 7,000 events contribute to the jet energy resolution calculation.

### 3.6.2 Decomposition of the Jet Energy Resolution

It is possible to gain additional insight into the detector performance by cheating the pattern recognition. Cheating the pattern recognition involves using Monte-Carlo (MC) information to correctly cluster calorimeter hits together and associate them to charged particle tracks. This has the effect of removing confusion from the reconstruction. By comparing the detector performance obtained from the standard and cheated reconstructions, it is possible to decompose the detector performance into two terms; one related exclusively to the intrinsic energy resolution of the detector and another related to the pattern recognition confusion. The additional information this provides is extremely useful for characterising changes to the overall detector performance.

The intrinsic energy resolution contribution to the jet energy resolution is determined by fully cheating the pattern recognition; in this case all confusion is negated. The total confusion is defined as the quadrature difference between the jet energy resolution using the standard reconstruction and this fully cheated reconstruction. Furthermore, it is possible to cheat the pattern recognition associated with individual types of particles. This is particularly useful for studies related to the ECal as, by cheating the photon
pattern recognition, it is possible to isolate the confusion associated with photons. The photon confusion is defined as the quadrature difference between the jet energy resolution using the standard reconstruction and the reconstruction where the photon pattern recognition is cheated. Examples of the calculation of the various confusion terms defined above are given in table 3.8.

<table>
<thead>
<tr>
<th>Reconstruction</th>
<th>Jet Energy Resolution [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Reconstruction (No MC Information)</td>
<td>$a = 2.97 \pm 0.05$</td>
</tr>
<tr>
<td>Cheating Entire Reconstruction</td>
<td>$b = 1.69 \pm 0.02$</td>
</tr>
<tr>
<td>Confusion</td>
<td>$\sqrt{a^2 - b^2} = 2.45 \pm 0.05$</td>
</tr>
<tr>
<td>Cheating Photon Reconstruction</td>
<td>$c = 2.73 \pm 0.04$</td>
</tr>
<tr>
<td>Photon Confusion</td>
<td>$\sqrt{a^2 - c^2} = 1.18 \pm 0.06$</td>
</tr>
</tbody>
</table>

Table 3.8: Example calculation of the confusion contributions to the jet energy resolution. These jet energy resolutions are for 250 GeV jets using the nominal ILD detector model and are calculated using the range of jet energies with the smallest RMS containing at least 90% of the data.

A common feature that is observed in these calibration studies is that as the intrinsic energy resolution of a calorimeter improves, the effect of confusion is reduced. This occurs as a better energy resolution means more precise comparisons can be made between the energy of a cluster of calorimeter hits and the momentum of any charged particle tracks associated to it. Comparisons such as these are made by PandoraPFA to determine whether the track cluster associations that have been made are consistent. If a large discrepancy is observed between the cluster energy and track momenta, the clustering of calorimeter hits is modified until a consistent association can be made. For more details on this comparison see chapter 3. This consistency check vastly reduces the number of errors made when clustering calorimeter hits and associating charged particle tracks to those clusters i.e. the confusion. Therefore, improving the precision of this consistency check, by improving the energy resolution, reduces the effect of confusion.

3.6.3 Single Particle Energy Resolution

The energy resolution for individual particles is crucial for a number of physics studies of interest to the linear collider, such as photon energy resolutions in the study of anomalous triple and quartic gauge couplings [56–58]. Therefore, photon and $K_L^0$ energy resolutions,
alongside the jet energy resolution, will be considered in these optimisation studies. As both photon and $K_L^0$ are uncharged, their energy measurements will be made using the calorimeters as opposed to the tracker. Photons are a natural choice of particle to consider as they are particularly relevant for several physics studies and, as they are largely contained within the ECAl, they will be highly sensitive to changes in the ECAl performance. $K_L^0$s were used as, analogously to photons and the ECAl, their energies are primarily measured using the HCAl. In general, neutral hadron energy resolutions are less crucial to physics studies, however, they do make crucial contribution to the jet energy resolution that should not be overlooked. The reported photon energy resolutions were determined using events containing a single 100 GeV photon, while the $K_L^0$ energy resolutions were determined using events containing a single 50 GeV $K_L^0$. These energies were chosen to be as large as possible, to maximise sampling of the calorimeter response, while minimising the effect of energy leakage from the ECAl to the HCAl for the photons and leakage of energy out of the rear of the HCAl for the $K_L^0$ events.

The energy resolution for these single particle samples is determined using a Gaussian fit to the reconstructed energy distributions. To aid convergence, the fit was applied to the narrowest range of the reconstructed energy distribution containing at least 75% of the data. The single particle energy resolution is defined as the standard deviation divided by the mean of the fitted Gaussian. For each energy resolution calculation, a total of 10,000 events were used to populate the reconstructed energy distribution. For clarity, a cut of $|\cos(\theta)| < 0.7$ was applied to veto events where particles travelled down the beam pipe or where they passed through the barrel/endcap overlap region. An example of the reconstructed energy distributions for 100 GeV photons and 50 GeV $K_L^0$s, alongside the Gaussian fits used to determine the energy resolutions, are shown in figure 3.15. The errors quoted on single particle energy resolutions are determined by propagating the errors reported from the Gaussian fit into the resolution calculation.

### 3.7 Summary of ILD Detector Performance

The following section outlines the nominal ILD detector performance using the metrics outlined in section 3.6.

The reconstructed energy distributions for particles whose energies are measured using calorimeters will be Gaussian. This is the case for sampling calorimeters as the active material in each calorimeter hit essentially counts the number of charged particle
tracks passing through it, or possible the number of photons for scintillator options. An estimation of the total energy deposited in a calorimeter hit, including the absorber material, can be made based upon this number of tracks or photons. For more details on how this estimation is made see chapter 5. Finally, the energy of the entire particle shower is estimated by grouping together calorimeter hits and summing their energy. As each calorimeter hit’s energy is an independent random measurement the particle shower energy will, by the central limit theorem, have a Gaussian distribution.

The energy of a calorimeter hit is obtained by counting the number of charged particle tracks or photons found in the active material of the calorimeter, therefore, Poisson statistics govern the distribution of calorimeter hit energies. If the mean of the distribution of the energy of a cluster of calorimeter hits is \( \lambda = N \), where \( N \) is the mean number of objects that are measured in the calorimeters, the standard deviation of that distribution is \( \sigma = \sqrt{\lambda} = \sqrt{N} \) and the energy resolution is \( \sigma/\lambda = 1/\sqrt{N} \). As the total shower energy, \( E_{\text{Reco}} \), is proportional to \( N \), the energy resolution for a particle shower in an ideal calorimeter is \( \sigma_{\text{Reco}}/E_{\text{Reco}} = a/\sqrt{E_{\text{Reco}}} \). In reality, it is typical to express the energy resolution of a calorimeter in the following form

\[
\frac{\sigma_{\text{Reco}}}{E_{\text{Reco}}} = a\frac{1}{\sqrt{E_{\text{Reco}}}} + b + c\frac{1}{E_{\text{Reco}}},
\]

Figure 3.15: The reconstructed energy distribution for (a) 50 GeV \( K^0_L \) and (b) 100 GeV photons. The red line shows a Gaussian fit used to parameterise the detector performance. The fit was applied to the truncated range of the reconstructed PFO energy distribution containing at least 75% of the data with the narrowest RMS. The nominal ILD model was used in this simulation.
where the $b$ term is a constant term that accounts for a variety of instrumental effects that do not depend on energy, e.g. mechanical imperfections, and the $c$ term accounts for electrical noise [32]. Here, $\oplus$ denotes the quadrature sum of variables.

Prototypes of the various ILD calorimeter options have been constructed and validated using test beam measurements. The energy resolution of the ILD ECal, determined from test beam measurements, was parameterised as $16.6\% / \sqrt{E_{Reco}(\text{GeV})} \oplus 1.1\%$ for the silicon option and $12.9\% / \sqrt{E_{Reco}(\text{GeV})} \oplus 1.2\%$ for the scintillator option [34]. The electrical noise was deemed sufficiently small that the $c$ term in the parameterisation could be neglected in both cases. These results were determined using an $e^-$ test beam with energies ranging up to $\approx 40$ GeV. This parametrisation is compared to the full ILD detector simulation in figures 3.16a and 3.16b for the silicon and scintillator ECal options respectively. The test beam parameterisation of the energy resolution for the silicon ECal option is almost identical to the energy resolution observed in the full simulation. At very high energies, $\approx 500$ GeV, the ECal is no longer sufficient to fully contain the photons and so leakage of energy into the HCal leads to a minor degradation in the simulated energy resolution. This accounts for the worse energy resolution seen in the full simulation when compared to an extrapolation of the test beam parameterisation at high energies. The test beam parameterisation of the energy resolution for the scintillator ECal option is marginally better than that observed in the full simulation, which is most likely due to an imperfect implementation of the scintillator ECal within the full detector simulation. The photon energy resolutions seen in the full ILD simulation are similar for the silicon and scintillator ECal options.

Similarly, the energy resolution, determined from test beam measurements, for the nominal ILD HCal was parameterised as $57.6\% / \sqrt{E_{Reco}(\text{GeV})} \oplus 1.6\%$ [3]. A comparison between this test beam parameterisation and the full ILD simulation, using the silicon ECal option, is shown in figure 3.16c. The test beam measurements were made using $\pi^\pm$s with energies ranging from 10 to 80 GeV, while the full ILD simulation used $K_{LS}^0$s ranging from 10 to 100 GeV. The deviation between the test beam parameterisation and the full ILD simulation, which grows as the $K_{LS}^0$ energy increases, is most likely due to the treatment of energy deposits leaking out of the back of the HCal. In the test beam studies, to minimise the effect of leakage, events were only considered if the particle showers started developing at the front of the HCal. In the full simulation studies, all particle showers were used, which means some energy will have leaked out of the back of the calorimeters and been deposited in the uninstrumented solenoid region of the detector, resulting in a degradation in the energy resolution.
Figure 3.16d shows the jet energy resolution as a function of jet energy for the full ILD simulation. Alongside this, the intrinsic energy resolution and confusion contributions to the jet energy resolution are also presented. The jet energy resolution at low energies is dominated by the intrinsic energy resolution of the detector, while at high energies it is dominated by the effect of confusion. This is to be expected because the intrinsic energy resolution of the calorimeters is approximately proportional to $1/\sqrt{E_{Reco}}$. On the other hand, confusion grows with energy because increasing energy leads to more dense event topologies, which makes pattern recognition more challenging. The total jet energy resolution for the ILD detector are sufficiently small, $\sigma_{E_j}/E_j \lesssim 3.8\%$ [6, 29, 34], across the energy range considered to make separation of the hadronic decays of the W and Z bosons possible, which is one of the key requirements for the future linear collider.

In summary, the test beam parameterisations of the photon and neutral hadron energy resolutions for the ILD calorimeters are in good agreement with the full detector simulation across the energy range considered here. This consistency proves that the linear collider simulations are realistic, which is crucial for strengthening the arguments presented in this thesis.
Figure 3.16: (a) The energy resolution as a function of photon energy for the silicon ECAl option. The black markers indicate the energy resolutions for the full ILD simulation. The solid red line shows the test beam parameterisation of the ECAl energy resolution and the blue shaded region indicates the uncertainty on the test beam parameterisation. (b) The energy resolution as a function of photon energy for the scintillator ECAl option. The black markers indicate the energy resolutions for the full ILD simulation. The solid red line shows the test beam parameterisation of the ECAl energy resolution and the blue shaded region indicates the uncertainty on the test beam parameterisation. (c) The energy resolution as a function of neutral hadron energy. The black markers indicate the energy resolutions for the full ILD simulation, with the silicon ECAl option, which was determined using $K^0_s$. The red solid line shows the test beam parameterisation of the HCal energy resolution, which was determined using $\pi^\pm s$. The blue shaded region indicates the uncertainty on the test beam parameterisation. (d) The jet energy resolution ($\text{RMS}_{90}$) as a function of jet energy using the nominal ILD model, with the silicon ECAl option. The intrinsic energy resolution and confusion contributions these the jet energy resolutions are also presented. The black dotted vertical line on the single particle energy resolutions shows the highest energy particles used in the test beam measurements. The test beam parameterisation data was taken from [59].
Chapter 4

Capacitively Coupled Pixel Detectors for the CLIC Vertex Detector

“The beginning of wisdom is this: Get wisdom. Though it cost all you have, get understanding.”
— Proverbs 4:7

4.1 Introduction

Identification of heavy-flavour quarks and tau-leptons at any of the currently proposed linear collider experiments will rely upon precise reconstruction of secondary displaced vertices that are produced when these particles decay [6]. Furthermore, the ability to accurately associate any daughter tracks produced in such decays to the secondary vertices is essential. At CLIC, this can only be realised using a vertex detector with a very high spatial resolution, of approximately 3 μm, and good geometric coverage, extending to low polar angles θ. The vertex detector must also have a low material budget (less than 0.2 % X₀ per layer) in order to prevent additional decay vertices from material interactions, and allow efficient track reconstruction despite the high presence of beam-induced background particles. Tracking in the vertex detector will be aided by the use of time-stamping of individual hits, to an accuracy of 10 ns, to identify particles produced from the physics event of interest.
As there are currently no technology options that fulfil all of the criteria for the CLIC vertex detector, the CLIC experiment has developed an extensive R&D program where new technologies for the vertex detector are considered. High-voltage complementary metal-oxide-semiconductor (HV-CMOS) sensors, which are capacitively coupled to a separate readout application-specific integrated circuit (ASIC) are one such option. The performance of prototype detectors based upon this technology and the impact of mechanical tolerances present in their manufacture are presented in this chapter.

4.1.1 HV-CMOS

Pixel detectors can be broadly classified in two groups: hybrid detectors, where a separate sensor and readout chip are bonded together; and fully-integrated devices, where the collection diode is implanted in the same piece of silicon as the readout circuitry. Fully-integrated devices have traditionally not been suitable for applications with high timing requirements due to the relatively slow charge collection time and limited on-pixel functionality. However, recent developments in CMOS technologies [60] have led to new detector designs that may overcome some of these issues.

HV-CMOS is a processing technology whereby the n-MOS and p-MOS transistors forming the on-pixel electronics are placed entirely within a deep n-well, as shown in figure 4.1. By varying the voltage applied at the gate terminal, n-MOS and p-MOS transistors are able to control the current flowing between the source and drain terminals. The gate voltage produces an inversion layer between the source and drain terminals that acts as a conduit, allowing current to flow between the source and drain as shown in figure 4.2. The voltage at the gate, with respect to the body, controls the width of the inversion layer and hence the magnitude of this current. Logic operations can be performed directly on-pixel using various configurations of n-MOS and p-MOS transistors.

For the HV-CMOS, the deep n-well housing the on-pixel electronics acts as the charge collection diode as well as shielding the circuitry from the p-substrate. This shielding allows for the application of a moderate bias voltage to the sensor bulk that produces a depletion region, which facilitates fast charge collection via a drift current. In contrast, traditional monolithic active pixel sensors (MAPS) have a much smaller depletion region meaning charge collection occurs primarily through the slower mechanism of diffusion. Furthermore, in conventional MAPS there is potential for competition in charge collection between the n-well collecting diode and the p-MOS transistors used to perform logic operations since the p-MOS transistors are embedded within an n-well. This only occurs
Figure 4.1: Schematic cross section of an HV-CMOS sensor: the deep n-well is the charge-collecting electrode and also contains additional CMOS circuitry such as a preamplifier. Image taken from [61].

Figure 4.2: Schematic cross section of an n-MOS transistor. p-MOS transistors have a similar cross section where the n and p doped regions are switched.

as the n-well collection diode is separated from the n-wells housing the p-MOS transistors. HV-CMOS technology does not suffer from this issue as the deep n-well collecting diode
houses the p-MOS transistors, meaning charge is only collected at a single well in the sensor bulk.

HV-CMOS technology thus offers the possibility of fast charge collection with integrated on-pixel functionality, but several limitations still exist. As the on-pixel electronics have to be placed inside the deep n-well and the n-wells of neighbouring pixels have to be isolated from each other, there is a limited physical area of the pixel that can be used for transistor layout, which limits the available on-pixel functionality. In addition to this, it is not possible to implement full CMOS logic inside the deep n-well as coupling between p-MOS transistors and the collection diode will lead to noise injection at the charge collection node. While it is possible to embed p-MOS transistors within a p-well to shield them from the deep n-well, so-called 'quadruple-well technology', to give access to full CMOS logic this option is not readily available for prototyping. By restricting the complexity of on-pixel electronics and using a separate readout ASIC, it is possible to overcome many of these issues. When coupled with the fast charge collection time and removal of competition in charge collection, this makes HV-CMOS technology highly desirable for use in the CLIC vertex detector.

4.1.2 CLIC ASICs

As HV-CMOS technology is such a promising option for use at the CLIC vertex detector, prototype devices based on this technology have been developed for testing. Two ASICs have been developed: the capacitively coupled pixel detector version 3 (CCPDv3), a sensor chip based on HV-CMOS technology, and the CLICpix, a readout chip providing additional on-pixel logic operations. The pixel pitch of the chips, both the CCPDv3 and the CLICpix, is 25 µm, which should be sufficient to meet the requirements for the CLIC vertex detector. Each of the prototype ASICs consists of a matrix of 64 × 64 pixels. The CCPDv3 is fabricated in a 180 nm HV-CMOS process, where 180 nm refers to the smallest size building block that can be used for creating the integrated circuits on a silicon wafer. This is comparable to device fabrication at the LHC, which typically uses a 130 and/or 250 nm CMOS processes [62,63]. In comparison, the CLICpix is fabricated in a 65 nm process, which makes it possible to have more complex on-pixel circuitry incorporated into it than would be possible in previous generations of pixel detectors. A schematic of these devices can be found in figure 4.3.
4.1.2.1 CLICpix

CLICpix is a hybrid pixel readout chip that has been developed for the CLIC vertex detector. Each CLICpix pixel contains a charge-integrating amplifier connected to a discriminator, as shown in figure 4.3. The discriminator remains high for as long as the input signal is over a given threshold, and this output is then used as the input for further logic operations. The additional logic operations record the time of arrival and magnitude of the collected charge, using a Time over Threshold (ToT) measurement. The ToT is stored in a 4-bit on-pixel counter.

The CLICpix operates using a shutter-based readout, where the entire matrix is kept active while the shutter is open and when closed the matrix is read out in its entirety. This is designed to match the expected beam structure for the CLIC experiment, as the accelerator will deliver bunch trains of $e^+$ and $e^-$ that are separated by 20 ms. Each bunch train contains 312 bunches with a spacing of 0.5 ns, giving a total train length of 156 ns. Furthermore, the shutter-based readout is well suited to power-pulsing, where the power to the front-end electronics is turned off between bunch crossings. This helps to significantly reduce the power consumption of the detector.

The threshold voltage, the voltage required for the discriminator to register an output, seen by each CLICpix pixel is slightly different due to variations in the manufacturing process. If these variations are not accounted for then the behaviour of the device across the matrix will not be uniform. To minimise the impact of these fluctuations, each CLICpix pixel contains a 4-bit local adjustment to the threshold voltage, which is calibrated to unify the response across the matrix. The threshold 'equalisation' is achieved by performing two threshold scans across the matrix, once with all four bits.
set to 0 (no local threshold adjustment), and a second time with all four bits set to 1 (maximum local threshold adjustment). For each scan, the baseline voltage of each pixel is determined. By applying a linear interpolation between the 0000 and 1111 cases, each pixel can be tuned to a common point, such that all pixels respond at the same global threshold.

### 4.1.3 Capacitive Coupling

Solder bump-bonding is typically used to connect the sensor and readout ASIC in hybrid pixel detectors. This procedure uses small spheres of solder to connect each pixel on the sensor to the corresponding pixel on the readout ASIC. There are several drawbacks to the use of this procedure for pixel detectors: it is expensive and sets limits on the thickness of both ASICs that is required for mechanical stability. An alternative procedure for connecting the sensor and readout ASICs involves using a thin layer of glue to form a capacitive connection between the two. This procedure reduces the cost and material budget with respect to bump-bonding, making it highly desirable for use in the CLIC vertex detector. In order to make this viable, it is necessary to implement an amplifier in the CCPDv3 pixel, shown in figure 4.3, to boost the signal and overcome the intrinsically small coupling capacitance.

### 4.2 Device Fabrication

There are two issues related to device manufacture that have to be considered when using capacitive coupling to connect the sensor and readout ASIC: the uniformity of the glue layer and the spatial alignment of the sensor and readout pads. The former has been investigated in [64], while the latter is the focus of this study. In order to characterise the impact on detector performance of any misalignment between the CCPDv3 and the CLICpix pads, a number of assemblies have been constructed that purposefully contain misalignments, as shown in figure 4.4. Table 4.1 contains a summary of the samples produced.

The full details of the gluing procedure can be found in [64], along with a study of the absolute precision of the manufacturing procedure. For devices constructed in an
Figure 4.4: Alignment schematic of the CCPDv3 and CLICpix detectors studied. The red dotted line represents the CCPDv3 pad and the solid black line represents the CLICpix top metal layer. From left to right: centred pixels, 1/4 offset (6.25 µm) and 1/2 offset (12.5 µm).

<table>
<thead>
<tr>
<th>Assembly</th>
<th>Alignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>SET 9</td>
<td>Centred</td>
</tr>
<tr>
<td>SET 10</td>
<td>1/4 Offset</td>
</tr>
<tr>
<td>SET 12</td>
<td>Centred</td>
</tr>
<tr>
<td>SET 13</td>
<td>Centred</td>
</tr>
<tr>
<td>SET 15</td>
<td>Centred</td>
</tr>
<tr>
<td>SET 16</td>
<td>1/2 Offset</td>
</tr>
</tbody>
</table>

Table 4.1: A list detailing the alignment of the CCPDv3 and CLICpix coupling pads for the devices considered in this study.

identical fashion to those considered here, the glue layer thicknesses were less than 1 µm and the precision on the pad positioning was less than 2 µm.

4.3 Device Characterisation

A series of laboratory experiments were used to characterise the devices produced for this alignment study. The devices were also tested in realistic experimental conditions using the CERN SPS test beam. Due to the complexities of testing devices in a test beam, extensive laboratory tests were performed first to characterise as many properties of the assemblies as possible. The laboratory experiments performed were:

- **Radioactive source measurements.** The goal of this measurement is to measure the response of the CCPDv3 and CLICpix when a radioactive source is used to deposit charge within the CCPDv3 sensor.
• **Test pulse calibration of the CLICpix chip.** The goal of this measurement is to calibrate the response of the CLICpix sensor. This is achieved by examining the CLICpix response when injecting a quantity of charge directly into the input of the chip, which bypasses the CCPDv3 and glue layer.

During the laboratory measurements each device was mounted on a circuit board as shown in figure 4.5. A field-programmable gate array (FPGA) board was used to configure the devices and to read and process the signals coming from the devices. The FPGA and device mounted circuit boards were connected via an interface board.

![CCPDv3-CLICpix assembly](image)

**Figure 4.5:** Hybrid glue assembly of a CLICpix chip with a CCPDv3 active sensor on top. Figure taken from [65].

### 4.3.1 Source Measurements

A radioactive source was used to deposit charge within the CCPDv3 sensor and the response of the CCPDv3 and CLICpix examined. The CCPDv3 sensor converts the deposited charge into a voltage, which in turn passes through the capacitive glue layer and into the CLICpix chip. Measurements were made of the output voltage produced by the CCPDv3 and the response of the CLICpix readout chip, in units of ToT. As the exact amount of charge deposited by the radioactive source is unknown, calibration of the CCPDv3 is not possible. Instead, this experiment focuses on examining the shape of the voltage produced by the CCPDv3 and determining the response of the CLICpix chip as a function of this voltage. As the CCPDv3 signal passes through the capacitively coupled glue layer before entering the CLICpix chip, this study characterises the properties of the gluing layer as well as the sensor and readout chips.
4.3.1.1 Experimental Setup

The radioactive material used in this study was Sr$^{90}$. Sr$^{90}$ undergoes $\beta^{-}$ decay to form Y$^{90}$, which in turn undergoes $\beta^{-}$ decay to form the stable isotope Z$^{90}$. Each $\beta^{-}$ decay produces an $e^{-}$ and a $\bar{\nu}_e$, and the $e^{-}$ goes on to deposit charge in the CCPDv3 sensor. The Sr$^{90}$ source used had an activity of 29.6 MBq. The decay energy of the Sr$^{90}$ and Y$^{90}$ $\beta^{-}$ decays are 0.546 MeV and 2.28 MeV, respectively [66]. This decay energy will be shared between the electron, anti-neutrino and recoiling daughter nuclei in the $\beta^{-}$ decays and gives an upper bound on the energy of the electrons used to characterise these assemblies.

The radioactive source was positioned directly above the back-side of the CCPDv3 sensor, and measurements were made of both the ToT output from the CLICpix and the CCPDv3 analogue signal for individual pixels on the sensor. The CCPDv3 pulse shape was recorded on a fast sampling oscilloscope that was also used to trigger the CLICpix readout. The on-pixel event counter, which is located in the CLICpix chip, was used to veto events where multiple hits occurred within the active shutter period. The CCPDv3 sensor was biased to 60 V during this experiment. Examples of CCPDv3 output voltage pulses when using the Sr$^{90}$ source can be seen in figure 4.6. The analogue output has a baseline voltage of $\approx 1.15$ V with signal saturation occurring around a height of 700 mV.

4.3.1.2 Analysis

The quantities of interest related to the CCPDv3 output voltage are the pulse height, defined as the peak of the voltage pulse, and the rise time, defined as the time it takes for the CCPDv3 to reach the pulse height. For ease of analysis the baseline voltage is subtracted from the CCPDv3 output voltage and the pulse shape inverted before the following analysis is applied to extract the variables of interest.

The pulse height is defined using a Gaussian fit to the peak of the voltage pulse. This method is used to minimise the dependency of the pulse height on small fluctuations in the output voltage. The peak of the voltage pulse is defined as the region where the change in the CCPDv3 voltage output is greater than 90% of the maximum change in the CCPDv3 voltage output.
The rise time is defined as the time taken for the signal to go from 10% to 90% of the maximum change in the CCPDv3 voltage output. This definition makes the rise time metric more robust against fluctuations in the CCPDv3 voltage output.

Examples of the calculation of these metrics for a representative pulse is shown in figure 4.7. Due to the design of the CCPDv3 matrix, it is only possible to record the CCPDv3 voltage output for 15 pixels running along one edge of the $64 \times 64$ matrix. Therefore, in the subsequent analysis, data was taken for each of these accessible pixels and combined.
4.3.1.3 Results: Rise Time vs Pulse Height

The mean rise time as function of pulse height for the CCPDv3 output voltage is shown in figure 4.8. This was determined by binning the measurements in pulse height and determining the mean rise time for measurements in each bin. The pulse height was binned using a bin width of 4 mV ranging from 0 to 700 mV. A minimum of 10 measurements per bin were used for the calculation of the average rise time. The error bars on this figure show the standard error in the mean rise time. Data was only included in this analysis if the on-pixel event counter registered a single hit in the time window used to take data.

The data in figure 4.8 shows that the rise time for the CCPDv3 front-end is approximately 300 ns across all samples. The rise time is largely independent of pulse height for all but the smallest signals. For very small pulse heights (< 100 mV) rise times are significantly larger, which suggests that the deposited charge takes a longer time to be
collected. This may be due to charge transport occurring via diffusion rather than drift. A gradual reduction in the rise time is observed as the total deposited charge, which is proportional to the pulse height, increases. This is expected as larger charge deposits in the sensor bulk lead to a greater rate of charge collection by the CCPDv3 and a smaller time taken for the pulse height to reach the peak. As the intrinsic performance of the CCPDv3 sensors in the devices tested is very similar, comparisons of the misaligned samples will be made more straightforward. As nearly identical intrinsic performance was observed for the CLICpix readout chips, as will be shown in section 4.3.2, any performance differences observed between these devices will be entirely due to the capacitive glue layer and the pad alignment.

4.3.1.4 Results: ToT vs Pulse Height

The mean ToT measured in the CLICpix as a function of the CCPDv3 output voltage pulse height is shown in figure 4.9. Determination of the mean and error bars for the ToT as a function of pulse height measurement is identical to that described in section 4.3.1.3 for the rise time as a function of pulse height measurement.

For samples where the CCPDv3 and CLICpix are centred, the distribution of the mean ToT against pulse height shows that the ToT increases with pulse height up to
values of approximately 400 mV and for larger pulse heights the mean ToT saturates at $\approx 15$. It is expected that the $\frac{1}{4}$- and $\frac{1}{2}$-offset samples will have a lower ToT than the centred samples due to the lower effective capacitance between the CCPDv3 and CLICpix pads. The greater the offset, the smaller the effective capacitance to the target CLICpix pad will be, and so the lower the recorded ToT. This is can be seen when comparing the centred samples to SET 10, the $\frac{1}{4}$-offset sample, and SET 16, the $\frac{1}{2}$-offset sample. In addition to the charge injected by the radioactive source there will also be background noise present from a variety of effects such as manufacturing defects in the silicon and thermal noise. This additional charge will increase the mean ToT recorded by the CLICpix and is the most likely reason as to why the mean ToT does not smoothly tend to zero as the pulse height decreases.

4.3.1.5 Results: Cross Couplings

Capacitive coupling of the sensor to the readout ASIC can also lead to unwanted signals being induced on neighbouring pixels, due to non-zero stray capacitances. Cross-coupling is the transfer of signal from a sensor pad to the readout ASIC on an adjacent pad, which will occur if there is a non-negligible capacitance between the two pads. Signals are still transferred between the aligned sensor and readout pads, however, if the cross-capacitance...
is large enough unwanted additional hits in the neighbouring pads will be created. This issue is particularly relevant for this study as any misalignment between the sensor and readout pads will result in an increase in the cross-capacitance along the direction of the misalignment.

Any effects of cross-coupling can be studied using the same setup as was used in section 4.3.1.4 for the ToT against pulse height analysis, but in contrast considering the ToT on the adjacent CLICpix pixel along the direction of the misalignment. The mean ToT on the adjacent pixel is shown as a function of the pulse height for all devices where the CCPDv3 and CLICpix are aligned in figure 4.10a and for the misaligned samples in figure 4.10b.

The distributions of the mean ToT on the adjacent CLICpix as a function of pulse height are governed largely by cross-coupling effects. These effects will make this distribution look similar in shape to that of the mean ToT in the target CLICpix as a function of pulse height, shown in figure 4.9. However, the gradient of the adjacent ToT distribution will be shallower than the adjacent ToT distribution as the cross-capacitance is smaller than the aligned capacitance. The exception to this is the $1/2$-offset sample where the cross-capacitance and aligned capacitance will be comparable. In addition to cross-coupling, this distribution will be affected by electrical noise and charge being deposited in the neighbouring HV-CMOS pixels.

For the centred samples, cross-coupling seems to have a small effect as the correlation between the adjacent ToT and pulse height is minimal. For the $1/4$-offset sample there is a stronger correlation between the adjacent ToT and pulse height, which indicates that cross-coupling is having a more dominant effect. The gradient of the adjacent ToT vs pulse height is, however, very shallow as the cross-capacitance for this device will be relatively small. A much stronger cross-capacitive effect can be see in the $1/2$-offset sample, which is expected given it has a larger cross-capacitance than either the centred or $1/4$-offset sample. The adjacent ToT distribution for the $1/2$-offset sample almost mirrors the aligned ToT distribution in terms of both shape and width of the distribution. There are some small differences between the shape of the aligned and adjacent ToT vs pulse height distribution for the $1/2$-offset sample, but this is understood to be from the column structure of the CLICpix readout ASIC, more details of which can be found in section 4.3.2. Overall, these results indicate that, as expected, a misalignment between the CCPDv3 and CLICpix pads increases the effect of cross-coupling along the direction of the misalignment.
Figure 4.10: The mean ToT measured on the adjacent CLICpix pixel, along the direction of the offset, as a function of CCPDv3 voltage pulse height for (a) the centred and (b) the misaligned devices.
4.3.2 Test Pulse Calibration

In order to fully understand the charge transfer to the CLICpix, a calibration of the CLICpix front-end electronics response was performed. This was achieved by directly injecting a voltage pulse of fixed height directly into a capacitor held in each CLICpix pixel. This capacitor will then inject a known amount of charge into the pixel, and by varying the height of the pulse applied the response of the CLICpix to different amounts of charge can be quantified. This experiment extends the characterisation of the CLICpix chip beyond what was found using the radioactive source measurements, as applying the voltage directly to the CLICpix fully isolates the response of the chip from any effects relating to the glue layer or CCPDv3.

4.3.2.1 Experimental Setup

To prevent any influence from neighbouring pixels during the test pulse measurements the matrix was pulsed in stages. Charge was injected into 1 out of every 16 pixels while masking the others to ensure issues related to power consumption were not encountered. This was repeated 15 more times using different mask configurations until the entire matrix had been sampled. This procedure was carried out 100 times to determine the average ToT response on a per-pixel level. The pulse height injected into the CLICpix was varied from 2 to 180 mV in steps of 2 mV in order to fully characterise the response up to saturation of the ToT output. An example of the mean ToT plotted against the injected pulse height is shown in figure 4.11.

4.3.2.2 Analysis

The functional form of the ToT against pulse height plot will be described using a surrogate function \[ y = ax + b - \frac{c}{x - t}, \] (4.1)

where \( y \) is the ToT, \( x \) is the pulse height in mV and \( a, b, c \) and \( t \) are fit parameters. Application of the fit helps to condense the large amount of data recorded for an individual pixel down to a small number of parameters, which makes categorisation of the response of the CLICpix matrix clearer. At large pulse heights the linear relationship dominates,
Figure 4.11: The CLICpix ToT as a function of injected pulse height. The black markers show the mean ToT and the error bars show the standard error.

while for low pulse heights the inversely proportional term dominates. $c$ describes the curvature of the graph, while $t$ determines the asymptote below which no signal is detected. Figure 4.12 shows an example of the application of this fit. As this function does not describe saturation of the ToT or the region below threshold, the fit is only applied on data points where the mean ToT is greater than 1 and less than 14.75.

4.3.2.3 Results

A known issue with the design of the CLICpix ASIC is the unwanted feedback capacitance between the discriminator output and amplifier input. This feedback leads to an additional fixed injected charge being measured for each recorded hit, due to the firing of the discriminator. The magnitude of this effect differs between even and odd columns across the CLICpix matrix due to slight differences in the physical layouts of alternating columns. By examining the distribution of the surrogate fit parameters, shown for SET 9 in figure 4.13, this effect can be seen.

The peaks at zero in the distribution of the $a$ and $b$ parameters, containing $\approx 150$ entries, correspond to noisy and dead pixels. These damaged pixels will be found in the device due to problems occurring in the manufacturing process. The majority of the $a$ and $b$ parameters are centred around a single value, which indicates a similar response in
the linear region of the surrogate function, however, the $c$ and $t$ parameters are centred around one of two values. When examining the distribution of these parameters as a function of position on the matrix, shown in figure 4.14 for a selected device, it can be seen that the structure is related to the column a given pixel is in. This feature is present in all devices considered and the underlying cause, the unwanted feedback capacitance, will be remedied in the next generation of the CLICpix ASIC.

The matrix-averaged surrogate function fit parameters for all devices can be found in tables 4.2 and 4.3, for the even and odd columns, respectively. The surrogate function for each device using these average parameters as input is shown in figure 4.15.

As figure 4.15 shows, the response of the CLICpix to the injected pulse height is largely uniform across all samples. For all devices the turn-on pulse height is $\approx 10$ mV and saturation, which occurs when the ToT output reaches the maximum value of 15, occurs at $\approx 150$ mV. The differing column structure exists due to the unwanted feedback capacitance between the discriminator output and amplifier input. This unwanted feedback leads to a sharper rise in ToT for even-numbered columns than for odd-numbered columns, because, in effect, the even-numbered columns are operating at a lower threshold.
column structure is present in all devices considered. The uniformity of the response of the CLICpix ASICs observed in this study make comparisons between the misaligned samples clearer. These results show that any performance differences observed between the misaligned devices do not originate from the intrinsic behaviour of the CLICpix ASIC.
Figure 4.14: The distribution of selected surrogate function parameters obtained when fitting the ToT as a function of injected pulse height for SET 9 as a function of matrix position. (a) and (b) show the distribution of the $c$ and $t$ parameters, respectively.

Figure 4.15: The average ToT response as a function of injected pulse height, which is represented using the surrogate function. Parameters for the surrogate function are obtained by fitting the ToT against pulse height curve for all pixels in the matrix. The results are divided into even and odd columns to account for the differing effective thresholds on alternate CLICpix columns.

4.4 Test Beam Analysis

Test beam measurements were used to characterise the behaviour of the prototype capacitively coupled pixel detectors. These measurements are particularly useful as they
The trajectory of any particles passing through the DUT was measured in the test beam setup using a telescope. The telescope consisted of several planes of pixel detectors mounted either side of the DUT. As low energy particles would be stopped by the telescope detector planes, telescopes can only be used to measure the trajectory of relatively high energy particles. This means that they cannot be used in lab based measurements, but can be used in test beams where high energy particles can be safely produced.
4.4.1 Test Beam Setup

Test beam experiments were carried out in August and September 2015 on the H6 beam line in the CERN SPS North Area. The beam consisted of positively charged hadrons of momenta 120 GeV/c. Mean particle rates of 500 kHz/cm$^2$ were observed during the 4.8 s spills at intervals of 25 s. The devices under test were mounted on an EUDET/AIDA telescope [68]. This telescope consisted of six planes of sensors, three on either side of the DUT, constructed of Mimosa pixel detectors. This telescope provided a resolution of 1.6 µm on the intercept position between tracks passing through the device and the DUT mounted on it.

4.4.2 Analysis

A number of cuts were applied to veto the effect of noisy pixels and tracks that underwent non-negligible multiple scattering [69]. These effects would lead to discrepancies in the reported efficiencies of the devices, which are not representative of the true device performance. Any pixels identified on the DUT that were deemed to be noisy were removed from the analysis. A pixel was deemed noisy if it responded at a mean rate greater than 5 σ in comparison to the average rate across the whole matrix. In addition to this, any tracks with an intercept on the DUT within half a pixel width of a noisy pixel were also rejected from the analysis. As tracks may undergo non-negligible multiple scattering, a $\chi^2$ cut was used to remove less precisely reconstructed tracks. Furthermore, all tracks occurring within 125 µm of each other were vetoed, in order to reduce the possibility of mis-association of clusters to tracks.

After the application of these cuts, the track position on the DUT was calculated using the measured particle trajectory through the telescope planes. This was followed by a search around the intercept position on the DUT to find an associated cluster. Clusters were associated to the track if they fell within 75 µm, or 3 pixels, about the intercept position. If multiple clusters were associated to a track the cluster position was calculated as the ToT-weighted centre-of-gravity.

Alignment of the telescope planes was essential for ensuring that the correct trajectory of the particles passing through the setup could be determined. Furthermore, alignment of the DUT with respect to the telescope planes was critical for ensuring the correct track intercept position was found. With that in mind the six telescope planes were aligned by minimising the total track $\chi^2$ with respect to the global alignment parameters.
The tracks that were created in the alignment procedure are referred to as "rough tracks" since they are produced using the hits from sensor planes that may not be ideally aligned. The alignment proceeded by one telescope plane at a time until all planes were accounted for. This procedure was iteratively repeated, updating the global alignment parameters to the optimal values found each time, until no further gains could be made. Once the telescope planes were aligned, the DUT was aligned in a similar manner, but this time minimising the root mean squared of the residual distribution. Here, residuals are defined as the distance between the track intercept and associated cluster centre-of-gravity on the DUT.

### 4.4.3 Results

The metric used for characterising the device performance in the test beam is the single hit efficiency, $\epsilon$. This is defined as the number of tracks with associated clusters recorded by the DUT, $n$, divided by the number of reconstructed tracks passing through the DUT recorded by the telescope, $m$. The errors shown on the efficiency measurements are given by $\sqrt{\epsilon(1 - \epsilon)/m}$, which follows from the variance of $n$ given binomial statistics with mean $\epsilon$. The single hit efficiency as a function of threshold for all devices is shown in figure 4.16. The threshold, in units of number of electrons, is the size of the signal that must be injected into the CLICpix ASIC to generate a hit.

**Figure 4.16:** The efficiency of the devices considered as a function of the threshold applied.
The data indicates that for all assemblies the single hit efficiency of the detector decreases when a higher amount of charge is required to generate a signal, which is to be expected. However, the efficiency of all samples, with the exception of the $\frac{1}{2}$-offset sample, is still above 99% up to a threshold of 2000 electrons. This is encouraging behaviour as the larger the threshold that can be applied, the lower the effects from noise will be. Reducing the effects from noise will aid tracking performance in the CLIC vertex detector, which is highly desirable. It is clear that the $\frac{1}{2}$ offset sample, SET 16, has a much lower efficiency as a function of threshold in comparison to the other samples. For the same deposited charge in the CCPDv3 the $\frac{1}{2}$-offset sample will, due to the reduced capacitance, produce a smaller signal in the CLICpix than the centred samples. This is the cause of the reduced efficiency as a function of threshold in this sample. More encouragingly is the behaviour of the $\frac{1}{4}$-offset sample, SET 14, which in terms of performance is comparable to the aligned samples. There is a degradation in efficiency of the $\frac{1}{4}$-offset sample with respect to the aligned sampled, which is to be expected given the reduced capacitance, however, it is relatively small. These results indicate that even with a relatively large misalignment between the CCPDv3 and CLICpix pads the device performance is not significantly affected, and therefore manufacturing tolerances of $\frac{1}{4}$ of a pixel width would not be problematic if this device were used for the CLIC vertex detector.

### 4.5 Conclusions

In summary, for the capacitively coupled pixel detectors considered in this analysis:

- The CCPDv3 sensor ASIC properties have been characterised and were found to be comparable across all devices.

- A calibration of the CLICpix readout ASIC was performed and their behaviour was found to be comparable across all devices.

- When combining the CCPDv3 and CLICpix characterisations, it becomes clear that any performance differences between the devices are due to the capacitive coupling gluing layer as opposed to the intrinsic behaviour of the ASICs.

- Test beam analysis of the devices found that device fabrication tolerances of up to $\frac{1}{4}$ of a pixel width would not harm the performance of these devices should they be used for the CLIC vertex detector.
Chapter 5

Energy Estimators

“He reveals the deep things of darkness and brings utter darkness into the light.”
— Job 12:22

5.1 Motivation

This section outlines a procedure for calibrating the Monte-Carlo (MC) response of the linear collider detector simulations with a focus on converting the detector response into accurate energy measurements, "energy estimators", for particles showering in the calorimeters. In the particle flow paradigm, all neutral particle energies are measured using the calorimeters, therefore, accurate energy reconstruction is crucial for determining detector performance. Additionally, comparisons of particle shower energy and charged particle track momenta govern the event reconstruction in PandoraPFA during the reclustering stage, which further emphasises the importance of reliable energy estimators.

The goal of a calorimeter is to measure the energy of particles that shower within it. Particle showers are a cascade of secondary particles that are produced when a high energy particle interacts with a dense material. The energy deposits produced by a showering particle in the calorimeter are referred to as hits. The number of hits created by a particle shower in a calorimeter depends upon the size and shape of the particle shower and the segmentation of the calorimeter. The energy of the showering particle, $E_{\text{Cluster}}$, is determined by grouping these energy deposits together into clusters and
summing their energy

\[ E_{\text{Cluster}} = \sum_{E\text{Cal hits}, i} E^i_{E\text{Cal}} + \sum_{H\text{Cal hits}, i} E^i_{H\text{Cal}}, \quad (5.1) \]

where \( E^i_{E\text{Cal}} \) is the energy of ECal hit \( i \) and \( E^i_{H\text{Cal}} \) is the energy HCal hit \( i \). In this example the energy deposits are assumed to be split across an ECal and a HCal, therefore, the sum runs over the hits in both calorimeters. This naive energy estimator will act as a starting point for the development of more sophisticated procedures aimed at improving the energy resolution.

The linear collider detector concepts employ highly-granular sampling calorimeters [6,34]. These calorimeters are comprised of alternating layers of active and absorber materials [32]. The absorber layers initiate particle showers and propagate their growth, while the active layers produce a signal that is proportional to the energy deposited within them. The signal produced in the active layers is measured by sampling calorimeters and used to estimate the energy deposited in the absorber layers. This estimation is made by assuming the energy deposited across a calorimeter hit, that is one active and one absorber layer, is uniform. Working under this assumption, the total calorimeter hit energy is proportional to the active layer hit energy. This estimation procedure is loosely referred to as digitisation and, in this way, the cluster energy estimator introduced above can be written as

\[ E_{\text{Cluster}} = \sum_{E\text{Cal hits}, i} \epsilon^i_{E\text{Cal}} \alpha_{E\text{Cal}} + \sum_{H\text{Cal hits}, i} \epsilon^i_{H\text{Cal}} \alpha_{H\text{Cal}}, \quad (5.2) \]

where \( \alpha_{E\text{Cal}} \) and \( \alpha_{H\text{Cal}} \) are digitisation constants for the ECal and HCal respectively, \( \epsilon^i_{E\text{Cal}} \) is the ECal active layer hit energy for hit \( i \) and \( \epsilon^i_{H\text{Cal}} \) is the HCal active layer hit energy for hit \( i \). The first stage of the calibration procedure presented in this chapter covers the determination of these digitisation constants, which convert the raw analogue-to-digital converter (ADC) response to a hit energy.

Once the basic energy estimator has been calibrated, it is possible to apply more advanced procedures designed to give a compensating calorimeter response [29]. A compensating calorimeter produces an identical response to a particle shower irrespective of whether the particle shower is electromagnetic or hadronic in nature. The primary cause of the difference in the response of a calorimeter to electromagnetic and hadronic showers is the undetectable energy component that is found in hadronic showers. These undetectable energy components are energy deposits produced from a showering particle.
that does not produce a signal in the calorimeters. Hadronic showers contain this undetectable component due to a combination of effects such as neutrons stopping within the calorimeter and nuclear binding energy losses. Typically, this leads to calorimeters having a weaker response to hadronic showers than to electromagnetic showers.

There are two distinct routes available for achieving a compensating response from a calorimeter: the first is hardware compensation [70], whereby calorimeters are constructed using materials that yield extra energy in response to hadronic showers; and the second is software compensation [4], whereby the uncompensated calorimetric energies for hadronic showers are modified at the software level.

A novel example of hardware compensation is the ZEUS calorimeter [70]. The ZEUS calorimeter was constructed using uranium as the absorber material. In response to neutral hadrons, the uranium undergoes fission producing extra energy that increases the hadronic response of the calorimeter. The amount of uranium was carefully chosen to achieve a fully compensating calorimeter response, i.e. identical calorimeter response to electromagnetic and hadronic showers. Although hardware compensation is possible for the linear collider’s calorimeters, it would require a large amount of radioactive material, which is highly undesirable.

The linear collider lends itself to software compensation as the fine segmentation of the calorimeters and precise reconstruction of individual particles makes identification of hadronic showers, and modification of their energies, feasible. A basic form of software compensation included in the linear collider reconstruction is the modification of the electromagnetic cluster energy estimator to

\[
E_{EM}^{EM} = \sum_{ECal \ hits, i} E_{ECal}^i \beta_{ECal}^{EM} + \sum_{HCal \ hits, i} E_{HCal}^i \beta_{HCal}^{EM}, \tag{5.3}
\]

and the hadronic cluster energy to

\[
E_{Had}^{Had} = \sum_{ECal \ hits, i} E_{ECal}^i \beta_{ECal}^{Had} + \sum_{HCal \ hits, i} E_{HCal}^i \beta_{HCal}^{Had}, \tag{5.4}
\]

where the \( \beta \)s are scaling factors that are applied to the energy of clusters of calorimeter hits associated with electromagnetic and hadronic clusters in the ECal and HCal. This simple scaling of energies achieves a compensating calorimeter response, which leads to better detector performance. Determination of these energy scale setting constants is the second stage of the calibration procedure that is presented in this chapter.
While this scaling of energies improves detector performance, it does not account for any changes to the $\beta$ scaling factors as a function of the total energy deposited. An energy dependence in the scaling factors is expected as the mechanisms governing the propagation of hadronic showers are sensitive to the shower energy [33]. To account for this, more sophisticated software techniques have been developed that vary the calorimeter cluster energy estimator as a function of energy in order to achieve a compensating response across a wider range of energies. These techniques make use of the fine segmentation of the linear collider’s calorimeters to identify hadronic showers. These techniques also address the problem of spuriously high energy calorimeter hits, which are caused by Landau fluctuations [71]. Landau fluctuations originate from high energy knock-on electrons appearing within particle showers [72] and can lead to overestimates of the particle shower energy if they occur in the active layers of a sampling calorimeter.

5.2 Calibration in the Particle Flow Paradigm

Calibration of the linear collider detector simulation is performed by two processors in the software framework; the digitiser, which performs the digitisation process for sampling calorimeters, and PandoraPFA. The input to the digitiser is the active layer calorimeter response (ADC values) and the output is the combined calorimeter hit energies i.e. the active and absorber layer energies. The hit energies are then used by PandoraPFA for event reconstruction. Calibration of the digitiser involves determining the digitisation constants ($\alpha_{ECal}$ and $\alpha_{HCal}$) and the minimum ionising particle (MIP) scale, which is the average energy response for a MIP on a per hit basis. Similarly, calibration of PandoraPFA requires setting the scaling factors, $\beta$, and the MIP response using the combined calorimeter hit energies.

The $\alpha$ and $\beta$ constants are determined by tuning the mean of reconstructed energy distributions. A number of cuts are applied when populating these reconstructed energy distributions that ensure the relevant reconstructed energy is being tuned. The application of these cuts means that linear scaling of the $\alpha$ and $\beta$ constants does not lead to a linear shift in the mean of the reconstructed energy distributions. Therefore, when calibrating the $\alpha$ and $\beta$ constants, an iterative approach is taken; the next iteration of the calibration constant is determined by repeating the reconstruction using the current iteration of the constant and adjusting the constant based on the mean of the reconstructed energy distribution.
Determining the MIP scale is included as part of the calibration procedure as it is used by PandoraPFA in the identification of muons and for applying energy thresholds designed to limit the impact of noise. This energy scale is also used by the digitiser when simulating electrical noise, saturation effects in scintillator readout technologies and for applying noise vetoing energy thresholds [73].

The non-zero peak in the distribution of calorimeter hit energies for 10 GeV $\mu^-$s entering the calorimeters at normal incidence is used to define the MIP scale in PandoraPFA, whereas in the digitiser ADC values are used [72]. In the linear collider detector simulation, several realistic effects are simulated by the digitiser including saturation effects, energy thresholds, timing cuts and electrical noise. Application of these effects at this point in the software chain means that the active layer hit energies are not subject to them, while the post digitisation combined calorimeter hit energies are. Consequently, the MIP scale in PandoraPFA cannot be obtained from the digitiser MIP scale, instead both have to be independently determined.

Although this overall procedure is referred to as calibration, strictly speaking this is not correct. Calibration sets the detector response to real data, while this procedure sets the simulated detector response to Monte-Carlo (MC) events. In a real detector, calibration would follow the setting of the simulated detector response to MC events so that simulations can be used to guide the calibration process.

### 5.2.1 Overview of the Calibration Procedure

The calibration procedure is split into four separate operations: determination of digitisation constants ($\alpha$s) in the digitiser; determination of scaling factor constants ($\beta$s) in PandoraPFA; MIP scale setting in the digitiser; and MIP scale setting in PandoraPFA. Calibration of the digitiser, digitisation constants and MIP scale, uses calorimetric energy measurements prior to any reconstruction, while calibration of PandoraPFA, scale factors and MIP scale, uses fully reconstructed particle flow objects (PFOs). As reconstructed PFOs are created using calorimetric energy measurements that have been digitised, it is wise to calibrate the digitiser before calibrating PandoraPFA, therefore, the calibration procedure is applied in the following order:

1. Setting the MIP response in the digitiser.
2. Setting the digitisation constants, $\alpha$s, in the digitiser.
3. Setting the MIP response in PandoraPFA.

4. Setting the scaling factors, $\beta$s, in PandoraPFA.

### 5.2.2 MIP Scale Determination in the Digitiser

The MIP scale in the digitiser was determined by simulating 10 GeV $\mu^-$ events and creating a distribution of active layer calorimeter hit energies for each calorimeter in the detector simulation. When populating these distributions, a direction correction factor of $\cos(\theta)$, where $\theta$ is the incident angle of the $\mu^-$ to the calorimeter cell, was applied to account for the path length of the MIP through the active medium of the calorimeter. This converts the individual response to a normal-incident equivalent response. No selection cuts were applied to the sample of muon events.

Figure 5.1 shows the distribution used to determine the MIP peak in the ECal, HCal barrel, HCal endcap and HCal ring. In the digitiser software only a single HCal MIP scale, taken as the HCal barrel, is implemented.

### 5.2.3 Digitisation Implementation

This section discusses how the digitisation constants, $\alpha$s, are determined. The digitisation constant for a given calorimeter depends upon several factors such as the material properties of the active and absorber layers, the magnetic field strength and energy losses occurring within the gaps in the detector. Therefore, each calorimeter in the ILD detector model has a distinct constant that must be determined independently.

#### 5.2.3.1 ECal Digitisation Implementation

The procedure for determining the digitisation constants in the ECal involves simulation of single photons at an energy $E_{MC} = 10$ GeV. Single photons at this energy are largely contained within the ECal, as shown in figure 5.2a. This makes them ideal for isolating the ECal digitisation calibration from that of the HCal digitisation calibration. Events are only used for calibrating the ECal digitisation if they are confined to the ECal. To that extent, cuts are applied ensuring that the sum of the reconstructed energy found outside the ECal is less than 1% of $E_{MC}$ and that $\cos(\theta) < 0.95$, where $\theta$ is the polar angle of the photon. Photons that convert are also vetoed in this event sample at MC.
The active layer calorimeter hit energy distributions for (a) the ECal, (b) the HCal barrel, (c) the HCal endcap and (d) the HCal ring for 10 GeV $\mu^-$ events. The hit energies were corrected to account for the path length of the muons through the active medium of the calorimeter. The vertical black dotted lines indicate the position of the peak in each of these distributions that is used for defining the MIP scale in the digitisation processor.

Figure 5.1: The active layer calorimeter hit energy distributions for (a) the ECal, (b) the HCal barrel, (c) the HCal endcap and (d) the HCal ring for 10 GeV $\mu^-$ events. The hit energies were corrected to account for the path length of the muons through the active medium of the calorimeter. The vertical black dotted lines indicate the position of the peak in each of these distributions that is used for defining the MIP scale in the digitisation processor.

The impact of these cuts on the sum of ECal hit energies for the $E_{MC} = 10$ GeV photons is shown in figure 5.2b.

The calibration of the digitisation in the ECal is an iterative procedure which begins with the simulation of single photons using a trial calibration, $\alpha_{ECal}^0$. Next the distribution of the sum of calorimeter hit energies within the ECal is produced for events passing the selection cuts, as shown in figure 5.2b. For an ideal calorimeter this distribution should be Gaussian, as described in chapter 6, therefore, a Gaussian fit is applied to this distribution and the mean, $E_{Fit}$, extracted. To remove the effect of any outliers in this
distribution, the fit is applied to the range of data with the smallest root mean square that contains at least 90% of the data. An example of such a fit is shown in figure 5.3. In the case of ideal calibration, the mean of this fit, $E_{\text{Fit}}$, would be equal $E_{\text{MC}}$. It is assumed that any difference between the two is due to the calibration, therefore, to correct this, the digitisation constant from the trial calibration, $\alpha^0_{\text{ECal}}$, is rescaled by the ratio of the $E_{\text{MC}}$ to $E_{\text{Fit}}$

$$\alpha^0_{\text{ECal}} \rightarrow \alpha_{\text{ECal}} = \alpha^0_{\text{ECal}} \times \frac{E_{\text{MC}}}{E_{\text{Fit}}}.$$ (5.5)

This procedure is then repeated until the $E_{\text{Fit}}$ falls within a specified tolerance of $E_{\text{MC}}$. The tolerance applied here was $|E_{\text{Fit}} - E_{\text{MC}}| < E_{\text{MC}} \times 5\%$. The binning used for the fitted histogram is chosen such that the bin width is equal to the desired tolerance on $E_{\text{Fit}}$ e.g. $E_{\text{MC}} \times 5\% = 0.5$ GeV. It should be emphasised that the PFO energies used for downstream analyses have the electromagnetic and hadronic energy scale corrections applied, which are calibrated to a much tighter accuracy.

### 5.2.3.2 HCal Digitisation Implementation

The calibration for the digitisation in the HCal proceeds in a similar manner to that described for the ECal with a few key differences. This calibration uses simulated MC long-lived neutral kaons ($K^0_L$s) at $E_{\text{MC}} = 20$ GeV. The higher energy, with respect to
the ECal digitisation, results in particle showers that sample deeper into the HCal. The $K_L^0$s must pass through the ECal, which contains one $\lambda_I$, before arriving at the HCal. Consequently, approximately 15% of these events begin showering in the ECal, as can be seen in figure 5.4a. Only events that deposit less than 5% of their energy in the ECal are used for calibrating the HCal digitisation constants. Furthermore, events that are not contained in the HCal are removed by requiring the last layer of the HCal where energy is deposited to be in the innermost 90% of the HCal. The impact of these cuts on the sum of HCal calorimeter hit energies for the $E_{MC} = 20 \text{ GeV}$ $K_L^0$ events is shown in figure 5.4b.

There are two HCal digitisation constants used in the detector simulation, one applied for the barrel and another for the endcap. The use of two digitisation constants accounts for differences in hadronic shower dynamics between the two, such as differing magnetic field configurations in the barrel and endcap. Both parameters are calibrated in the same manner, but have different cuts on $\theta$, the polar angle of the $K_L^0$. For the barrel region of the HCal events are selected if $0.2 < \cos(\theta) < 0.6$, while for the endcap events are selected if $0.8 < \cos(\theta) < 0.9$. These angular cuts account for the transverse profile of the hadronic showers and ensure that the showers are largely confined to the relevant sub-detector. The target reconstructed energy for these $K_L^0$s is the kinetic energy because many of the neutral hadrons appearing in jets are neutrons and their accessible energy is the kinetic energy [33] as opposed to the total energy.

After applying the above $K_L^0$ selection cuts, the calibration procedure for the digitisation of the HCal barrel and endcap proceeds in the same manner as was described
for the ECal. An example of the Gaussian fits applied to the sum of the calorimeter hit energies in the HCal barrel and endcap are shown in figure 5.5.

Figure 5.5: Gaussian fit to sum of the HCal calorimeter hit energies for 20 GeV $K_L^0$ events with selection cuts.

5.2.3.3 HCal Ring Digitisation Implementation

The HCal ring, as illustrated in figure 5.6, is a hadronic calorimeter that surrounds the ECal endcap and it is sandwiched between the HCal barrel and endcap. This calorimeter...
is required to ensure hermetic coverage of the hadronic calorimeter system across the barrel/endcap cross-over region [34].

The HCal ring has an independent digitisation constant to account for any difference in the hadronic shower development between the ring, barrel and endcap. Due to the thickness of the HCal ring, particle showers are never fully contained in it, so a different approach to calibration is required. To ensure that the HCal ring calibration is approximately correct, $\alpha_{\text{HCal ring}}$ is assumed to equal $\alpha_{\text{HCal endcap}}$ multiplied by several factors designed to account for differences in the active layer thickness, absorber layer thickness and the MIP response between the HCal endcap and ring. In detail

$$\alpha_{\text{HCal ring}} = \alpha_{\text{HCal endcap}} \times \frac{\langle \cos(\theta_{\text{endcap}}) \rangle}{\langle \cos(\theta_{\text{ring}}) \rangle} \times \frac{P_{\text{endcap}}}{P_{\text{ring}}} \times \frac{L_{\text{Active}}^{\text{endcap}}}{L_{\text{Active}}^{\text{ring}}} \times \frac{L_{\text{Absorber}}^{\text{endcap}}}{L_{\text{Absorber}}^{\text{ring}}},$$ \hspace{1cm} (5.6)

where $\theta$ is the incident angle of the incoming particle to the calorimeter determined using the 20 GeV $K_L^0$, $L^{\text{Active}}$ is the active layer thickness and $L^{\text{Absorber}}$ is the absorber layer thickness. $P$ is the position of the MIP peak in the distribution of active layer hit energies, which has been corrected so that the MIP appears to enter the calorimeter at normal incidence, and is determined using 10 GeV $\mu^-$ events.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5_6.png}
\caption{A PandoraPFA event display showing the nominal ILD calorimeters. (a) the ECal, (b) the full HCal and (c) the HCal ring, which covers the barrel/endcap cross-over region.}
\end{figure}

### 5.2.4 MIP Scale Determination in PandoraPFA

The MIP scale in PandoraPFA is set by simulating 10 GeV $\mu^-$ events and creating the distribution of combined calorimeter hit energies. The MIP scale in PandoraPFA must be determined for the calorimeters and, in contrast to the digitiser, the muon chamber. Consequently, an additional distribution showing the calorimeter hit energy for the muon
chamber must be constructed at this stage of the calibration. As was done for the digitiser, a direction correction factor was applied to the hit energies to account for the path length of the MIP through the active medium of the calorimeter and no selection cuts were applied.

Examples of the distributions used to set the MIP scale in PandoraPFA can be found in figure 5.7. Due to the energy thresholds applied in the digitiser, there are fewer populated bins with low hit energies. The double peak structure observed in the ECal calorimeter hit energy distribution is expected given the ECal absorber material thickness doubling in the back 10 layers of the ECal. The MIP peaks used for defining the MIP scale in PandoraPFA, figure 5.7, are broader than those used for determining MIP scale setting in the digitiser, 5.1, as the realistic effects applied by the digitiser are only present in the combined calorimeter hit energy distributions.

5.2.5 Electromagnetic Scale in PandoraPFA

Setting the electromagnetic scale in PandoraPFA is performed by examining the energies of particles reconstructed by PandoraPFA. The reconstruction is performed using the combined calorimeter hit energies that were set by the digitiser and having applied the noise vetoing MIP cuts.

The electromagnetic scale in the ECal, $\beta_{ECAl}^{EM}$, is determined using simulated photons at $E_{MC} = 10$ GeV. To ensure that the events used for this part of the calibration are largely confined to the ECal, a cut requiring less than 1% of the reconstructed energy to be found outside the ECal is applied. Furthermore, only events reconstructed as a single photon are used to veto conversions. The impact of the selection cuts on the electromagnetic energy measured in the ECal for 10 GeV photons is shown in figure 5.8a. The peak at zero electromagnetic energy in the ECal is due to events traveling down the beam pipe and photon conversions. In photon conversion events, the calorimetric energy deposits made by the $e^{\pm}$ are associated to charged particle tracks. In this case, the energy measured using the calorimeters will be reported as zero because the charged particle tracks are used to determine the reconstructed particle energies. The tail of events with low electromagnetic energy in the ECal occurs primarily due to pattern recognition failures in photon conversion events. In these events a small fraction of the calorimetric energy deposits made by the $e^{\pm}$ are not associated to charged particle tracks and instead are reconstructed as separate photons with a reconstructed energy much less than $E_{MC}$. 
Figure 5.7: The combined calorimeter hit energy distributions for (a) the ECal, (b) the HCal and (c) the muon chamber for 10 GeV $\mu^-$ events. These hit energies were corrected to account for the path length of the muons through the active medium of the calorimeter. The vertical black dotted lines indicate the position of the peak in each of these distributions that is used for defining the MIP scale in PandoraPFA.

The fitting procedure follows the procedure used for the ECal digitisation, described in section 5.2.3.1, whereby a trial calibration for the electromagnetic energy scale in the ECal, $\beta_{EM0}^{Ecal}$, is first assumed. The initial trial calibration is approximate and is iteratively updated until it converges to within a chosen tolerance. Using the trial calibration, the photons are reconstructed and the distribution of the electromagnetic energy in the ECal created. A Gaussian fit is then applied to this distribution in the range with the smallest root mean square containing at least 90% of the data. The mean of the fitted Gaussian,
\( E_{\text{Fit}} \), is then used to scale \( \beta_{ECal}^{EM0} \) in the following way

\[
\beta_{ECal}^{EM0} \rightarrow \beta_{ECal}^{EM} = \beta_{ECal}^{EM0} \times \frac{E_{MC}}{E_{\text{Fit}}}.
\] (5.7)

An example distribution and fit used in the calibration of the nominal ILD detector model can be found in figure 5.8b. This procedure is repeated using the updated \( \beta_{ECal}^{EM} \) until \( E_{\text{Fit}} \) falls within a specified tolerance. The tolerance applied here was \( |E_{\text{Fit}} - E_{MC}| < E_{MC} \times 0.5\% \). The binning for the fitted histogram is chosen such that the bin width is equal to the desired target tolerance on \( E_{\text{Fit}} \), e.g. \( E_{MC} \times 0.5\% = 0.05 \text{ GeV} \). This tolerance is tighter than was applied for the digitisation as it is these and only these energies that are used in downstream analyses.

![Histograms showing electromagnetic energy distribution with and without selection cuts.](image)

**Figure 5.8:** (a) The sum of the electromagnetic energy measured in the ECal for simulated 10 GeV photons with and without the selection cuts. (b) Gaussian fit to sum of the electromagnetic energy deposited in the ECal for simulated 10 GeV photons with selection cuts.

### 5.2.6 Hadronic Scale in PandoraPFA

The hadronic energy scale factors for the ECal, \( \beta_{ECal}^{Had} \), and HCal, \( \beta_{HCal}^{Had} \), are determined using simulated \( K^0_L \) events at \( E_{MC} = 20 \text{ GeV} \). As the ECal contains approximately one nuclear interaction length, a non-negligible amount of hadronic energy will be deposited in the ECal, which makes the hadronic scale in the ECal, \( \beta_{ECal}^{Had} \), important for detector performance. The hadronic scale in the ECal and HCal are simultaneously set as it is unfeasible to create a large sample of 20 GeV \( K^0_L \)s that are fully contained within the ECal.
For the reasons outlined in section 5.2.3.2, the target reconstructed energy for the sample of $K^0_L$s used for setting the hadronic energy scale is the kinetic energy, $E_K$, as opposed to the total energy. To ensure the events used are not affected by leakage of energy out of the back of the HCal, a cut is applied that vetoes events where energy is deposited in the outermost 10% of the HCal. In addition, a cut requiring a single neutral hadron to be reconstructed is applied to veto events with reconstruction failures and decays in the tracker. Finally, it is required that the total hadronic energy measured within the calorimeters falls within three $\sigma$ of the kinetic energy of the $K^0_L$, where $\sigma$ is taken to be $55\% \times \sqrt{E_K}$ GeV. This definition for $\sigma$ is approximately the energy resolution for neutral hadrons using the nominal ILD HCal [34]. This cut ensures that when fitting the two dimensional distribution of hadronic energy measured in the ECal and HCal, outliers do not skew the fit. The impact of these selection cuts can be seen in figure 5.9.

Determining the hadronic scale in PandoraPFA is an iterative process and begins by assuming trial values, $\beta^{Had}_{ECal}$ and $\beta^{Had}_{HCal}$, for the hadronic scale calibration factors $\beta^{Had}_{ECal}$ and $\beta^{Had}_{HCal}$. The $K^0_L$ events are first reconstructed using the trial scale factors. Then a linear fit is applied to the two dimensional distribution of the reconstructed hadronic energies measured in the ECal and HCal for events passing the selection cuts. The best fit is obtained by minimising $\chi^2$ with respect to variables describing a linear fit to the distribution. In this case, $\chi^2$ is defined as

$$\chi^2(\delta_{ECal}, \delta_{HCal}) = \sum_i \left( \frac{r_i}{\sigma_{r_i}} \right)^2,$$ (5.8)
where \( r_i \) is the perpendicular distance in the two dimensional plane of hadronic energies measured in the ECAL and HCAL from the point \((x_i, y_i)\) to a straight line passing through the points \((\delta^{\text{Had}}_{\text{ECAL}}, 0)\) and \((0, \delta^{\text{Had}}_{\text{HCAL}})\). In this definition, \( x_i \) and \( y_i \) are the hadronic energies measured in the ECAL and HCAL respectively for event \( i \). The variables \( \delta^{\text{Had}}_{\text{ECAL}} \) and \( \delta^{\text{Had}}_{\text{HCAL}} \) describe a linear fit to the hadronic energy distribution, which are to be varied when minimising \( \chi^2 \). The explicit definition of \( r_i \) is given in equation 5.9 and illustrated in figure 5.10. The uncertainty on \( r_i \) is given by \( \sigma_{r_i} \), which is explicitly defined in equation 5.10. This uncertainty is calculated by propagating the uncertainties on \( x_i \) and \( y_i \), which are assumed to be \( \sigma_{x_i/y_i} = 55\% \times \sqrt{x_i/y_i} \), into the expression for \( r_i \). The sum runs over all events, \( i \), passing the selection cuts.

\[
\begin{align*}
    r_i &= \frac{y_i \delta^{\text{Had}}_{\text{ECAL}} + x_i \delta^{\text{Had}}_{\text{HCAL}} - \delta^{\text{Had}}_{\text{ECAL}} \delta^{\text{Had}}_{\text{HCAL}}}{\sqrt{(\delta^{\text{Had}}_{\text{ECAL}})^2 + (\delta^{\text{Had}}_{\text{HCAL}})^2}}, \\
    \sigma_i &= \sqrt{\left(\sigma_{y_i} \delta^{\text{Had}}_{\text{ECAL}}\right)^2 + \left(\sigma_{x_i} \delta^{\text{Had}}_{\text{HCAL}}\right)^2}. 
\end{align*}
\]

The minimisation of \( \chi^2 \) is done by stepping over a range of \( \delta^{\text{Had}}_{\text{ECAL}} \) and \( \delta^{\text{Had}}_{\text{HCAL}} \) centred about 0.

**Figure 5.10:** An example showing the definition of \( r_i \). The blue dotted line corresponds to \( y_i = \delta^{\text{Had}}_{\text{HCAL}} - x_i \delta^{\text{Had}}_{\text{HCAL}}/\delta^{\text{Had}}_{\text{ECAL}} \).

The ideal value of \( E_K \) is found in search for the minimum \( \chi^2 \). Once the minima in \( \chi^2 \) is found the trial calibration factors \( \beta^{\text{Had}}_{\text{ECAL0}} \) and \( \beta^{\text{Had}}_{\text{HCAL0}} \) are rescaled to correct for any deviation from
the desired fit as follows

\[
\beta_{\text{Had}}^{E\text{Cal}} \rightarrow \beta_{\text{Had}}^{E\text{Cal}} = \beta_{\text{Had}}^{E\text{Cal}} \times \frac{E_K}{\Delta_{\text{Had}}^{E\text{Cal}}}, \quad (5.11)
\]

\[
\beta_{\text{Had}}^{H\text{Cal}} \rightarrow \beta_{\text{Had}}^{H\text{Cal}} = \beta_{\text{Had}}^{H\text{Cal}} \times \frac{E_K}{\Delta_{\text{Had}}^{H\text{Cal}}}, \quad (5.12)
\]

where \(\Delta_{\text{Had}}^{E\text{Cal}}\) and \(\Delta_{\text{Had}}^{H\text{Cal}}\) are the values of \(\delta_{\text{Had}}^{E\text{Cal}}\) and \(\delta_{\text{Had}}^{H\text{Cal}}\) giving the minimum \(\chi^2\). The step size used for minimising \(\chi^2\) with respect to \(\delta_{\text{Had}}^{E\text{Cal}}\) and \(\delta_{\text{Had}}^{H\text{Cal}}\) was chosen such that a single step would correspond to the final tolerance on \(\delta_{\text{Had}}^{H\text{Cal}}\), which in this case is \(\approx 0.1\) GeV. This procedure is then repeated using the updated hadronic scaling factors until \(\Delta_{\text{Had}}^{E\text{Cal}}\) and \(\Delta_{\text{Had}}^{H\text{Cal}}\) both fall within a specified final tolerance, which in this case is taken to be \(|\Delta_{E/H\text{Cal}}^{\text{Had}} - E_K| < E_K \times 0.5\% \approx 0.1\) GeV.

The electromagnetic scale in the HCal, \(\beta_{\text{HCal}}^{EM}\), is chosen to be equal to the hadronic scale in the HCal, \(\beta_{\text{HCal}}^{\text{Had}}\). For the ILC and CLIC, \(\beta_{\text{HCal}}^{EM}\) is not a critical parameter in the reconstruction as photons are largely contained within the ECal meaning little to no electromagnetic energy is measured in the HCal.

Setting the hadronic scale in PandoraPFA ensures that the energy estimators for neutral hadrons are accurate at 20 GeV, however, this is not true for all energies. The undetectable energy component of a hadronic shower varies as a function of particle shower energy [33]. This means the response of a calorimeter to neutral hadrons is non-linear with the hadron energy. This is an inherent limitation of this calibration procedure that will be addressed by the development of more sophisticated energy estimators in subsequent chapters.

5.2.7 Summary

The procedure for setting the MC response in the linear collider detector simulation has been outlined. Table 5.1 lists the calibration factors produced by this procedure when applied to the nominal ILD detector model. This procedure ensures that when modifying the detector simulation, the response of the detector will yield reliable energy estimators for particles showering in the calorimeter. For completion, after this calibration procedure has been applied, retraining of the likelihood data used by specific algorithms in PandoraPFA for the reconstruction of photons can be performed.
Table 5.1: The calibration factors determined by the procedure outlined in this section for the nominal ILD detector model. A 100 ns timing cut was applied to all calorimeter hits in these simulations.

<table>
<thead>
<tr>
<th>Calibration Factor</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>MIP Peak Active Layer Calorimeter Hit Energy (ECal)</td>
<td>0.000148 GeV</td>
</tr>
<tr>
<td>MIP Peak Active Layer Calorimeter Hit Energy (HCal)</td>
<td>0.000493 GeV</td>
</tr>
<tr>
<td>$\alpha_{ECal}$</td>
<td>43.0</td>
</tr>
<tr>
<td>$\alpha_{HCal,Barrel}$</td>
<td>49.3</td>
</tr>
<tr>
<td>$\alpha_{HCal,Endcap}$</td>
<td>55.3</td>
</tr>
<tr>
<td>$\alpha_{HCal,Ring}$</td>
<td>31.1</td>
</tr>
<tr>
<td>MIP Peak Combined Calorimeter Hit Energy (ECal)</td>
<td>0.00650 GeV</td>
</tr>
<tr>
<td>MIP Peak Combined Calorimeter Hit Energy (HCal)</td>
<td>0.0263 GeV</td>
</tr>
<tr>
<td>MIP Peak Combined Calorimeter Hit Energy (Muon Chamber)</td>
<td>0.0970 GeV</td>
</tr>
<tr>
<td>$\beta_{EM,ECal}$</td>
<td>1.00</td>
</tr>
<tr>
<td>$\beta_{EM,HCal}$</td>
<td>1.06</td>
</tr>
<tr>
<td>$\beta_{Had,ECal}$</td>
<td>1.16</td>
</tr>
<tr>
<td>$\beta_{Had,HCal}$</td>
<td>1.06</td>
</tr>
</tbody>
</table>

Setting the MC response for the linear collider detector simulation will play a vital role in the calibration of a real linear collider detector. While the procedure cannot be directly applied to a real detector, due to the lack of a source of single particle events of known energy and particle type, the simulated detector response, when combined with test beam measurements of prototype detector components, will be used to calibrate the detector in a first pass. From that point a previously measured energy scale, such as the Z boson peak, can be used to further refine the calibration.

### 5.3 Novel Energy Estimators

This section describes two novel energy estimators that are introduced with a view to improving the energy resolution for hadronic showers. Two techniques will be discussed: HCal hit energy truncation, which focuses on limiting the impact of Landau fluctuations; and software compensation, which focuses on obtaining a compensating calorimeter response. Both of these techniques are implemented by introducing weights, $\omega^i$, to calorimetric energy deposits made by showering particles in the HCal. The energy of a
showering particle, $E_{\text{Cluster}}$, is determined by grouping together clusters of calorimeter hits and summing their energies. When weights are applied to HCal hits this energy estimator becomes

$$E_{\text{Cluster}} = \sum_{E_{\text{Cal}} \text{ hits, } i} E^i_{\text{ECal}} + \sum_{H_{\text{Cal}} \text{ hits, } i} E^i_{\text{HCal}} \omega^i(\rho^i). \quad (5.13)$$

Weights are only applied to calorimeter hits in the HCal as these techniques modify the energy of hadronic showers, which are primarily contained within the HCal. The weights, $\omega^i$, vary a function of the energy density of the calorimeter hit, $\rho^i = E^i_{\text{HCal}} / V$ where $V$ is the physical volume of a calorimeter hit in the HCal. The physical volume of a calorimeter hit is defined as $V = c^2 t$, where $c$ is the calorimeter cell size and $t$ is the thickness of the calorimeter layer, which includes the active and absorber layer thicknesses as well as any instrumentation. Although the exact weights depend on the implementation of the technique, a general feature is that at large $E^i_{\text{HCal}}$ the weight is less than one. This limits the impact of spuriously high energy hits caused by Landau fluctuations. The energy loss probability distribution function for scintillator detectors, such as the ILD HCal, is given by a Landau function [71]. Energy deposits from the high energy tail of this distribution, which are known as Landau fluctuations, account for high energy knock-on electrons that appear within particle showers [72]. As Landau fluctuations deposit a disproportionately large amount of energy with respect to the bulk of the particle shower, they can lead to overestimates of the particle shower energy.

The energy loss probability distribution function for $n$ particles passing through a calorimeter hit is given by the convolution of $n$ Landau functions which, by the central limits theorem, will tend to a Gaussian as $n$ becomes large. Consequently, as the average number of particles passing through a calorimeter hit increases, the high energy tail in the energy loss probability distribution function for the hit becomes less pronounced and the impact of Landau fluctuations decreases. This means that the impact of Landau fluctuations on energy measurements is dictated by the density of particles within a particle shower and the transverse segmentation, or cell size, of the calorimeter in use. If the transverse segmentation, or cell size, of a calorimeter decreases, the average number of particles passing through each hit decreases and the impact of Landau fluctuations increases. Any technique used for minimising the impact of Landau fluctuations will be sensitive to the transverse segmentation of the calorimeters in use.
5.3.1 HCal Hit Energy Truncation

The first technique to be examined is a simple truncation of the hadronic energy, \( E \), recorded in any given HCal hit

\[
E \rightarrow E' = \begin{cases} 
E & \text{if } E < \kappa, \\
\kappa & \text{otherwise},
\end{cases}
\]

where \( \kappa \) is the value of the truncation. This improves the energy estimators for hadronic clusters by limiting the impact of Landau fluctuations. In terms of \( \omega \) introduced in equation 5.13 the truncation corresponds to

\[
\omega(\rho) = \begin{cases} 
1 & \text{if } \rho \times V < \kappa, \\
\frac{\kappa}{\rho \times V} & \text{otherwise}.
\end{cases}
\]

This weight as a function of hit energy density is shown in figure 5.11.

![Figure 5.11: The weights, \( \omega \), used in the HCal hit energy truncation as a function of the energy density of the HCal hit, \( \rho \). The truncation shown here corresponds to a 1 GeV truncation in the nominal ILD HCal.](image)

5.3.1.1 Legacy Energy Corrections

Alongside the HCal hit energy truncation, PandoraPFA also applied two other energy corrections designed at limiting the impact of Landau fluctuations. They are:

- **Clean Clusters.** This algorithm checks to see whether the energy measured within a calorimeter hit is anomalously high. Anomalously high energy hits are defined as
hits where the energy contained within the hit is greater than 10% of the energy of the cluster that the hit has been associated to. If a hit is deemed to have an anomalously high energy and if this energy is above a threshold (0.5 GeV) the hit energy used by PandoraPFA is modified. The updated hit energy is taken as the average hit energy in the calorimeter layers immediately before and after the layer containing the high energy hit.

- **Scale Hot Hadrons.** This algorithm calculates the average energy of the calorimeter hits in a given cluster in units of, normally incident, MIP equivalent particles. If this number is larger than a certain value, default 15 MIPs per hit, the cluster energy is rescaled to give a lower average number of MIPs per hit, default is 5 MIPs per hit.

In the reconstruction, these corrections are applied to each cluster of calorimeter hits, irrespective of the location of that cluster in the detector. These algorithms, with the HCal hit truncation, form the 'legacy' energy corrections that are used by PandoraPFA when performing the event reconstruction.

### 5.3.1.2 Impact on Single Particle Energy Resolution

Figure 5.12a shows the energy resolution for neutral hadrons as a function of the HCal hit energy truncation applied. The optimal truncation for the ILD detector model simulation was 1 GeV and, using this truncation, a neutral hadron energy resolution of $\sim 8.8\% = 62\%/\sqrt{E(\text{GeV})}$ was obtained for $E = 50 \text{ GeV} \ K^0_L$ events. In comparison, the neutral hadron energy resolution for $E = 50 \text{ GeV} \ K^0_L$ events obtained without a truncation was $\sim 10.4\% = 74\%/\sqrt{E(\text{GeV})}$. Smaller energy truncations begin to truncate the energy of calorimeter hits produced in typical hadronic shower development, while larger truncations allow for a larger impact from Landau fluctuations. Both of these effects result in worsening neutral hadron energy resolutions. Figure 5.13 shows the normalised distribution of hit energies for 50 GeV $K^0_L$ events. This distribution shows that a 1 GeV truncation affects less than 2% of the calorimeter hits in these events. For completeness the photon energy resolutions as a function of HCal hit energy truncation are shown in figure 5.12b. As expected the photon energy resolution is unaffected by the HCal hit energy truncation as the photons are largely contained within the ECal.
Figure 5.12: The energy resolution as a function of HCal cell truncation for (a) 50 GeV \(K^0_L\) events and (b) 100 GeV photons using the nominal ILD detector model.

Figure 5.13: The normalised hit energy distribution for 50 GeV \(K^0_L\) events in the nominal ILD detector.

5.3.1.3 Impact on Jet Energy Resolution

Figure 5.14 shows the jet energy resolution as a function of jet energy for selected values of the HCal hit energy truncation. The trends in this plot are complex as the optimal HCal hit energy truncation varies with the jet energy. For 45.5 GeV jets, the best jet energy resolution, \(\sim 3.6\%\), is obtained using a 0.5 GeV truncation, while for 180 GeV jets, the best jet energy resolution, \(\sim 2.9\%\), is obtained using a 1 GeV truncation. This is expected because at low jet energies the average number of particles passing through each calorimeter hit will be small. Therefore, the impact of Landau fluctuations is large and, to limit them, a low truncation energy is needed. As the jet energy increases,
more particles on average pass through each calorimeter hit and the impact of Landau fluctuations decreases.

![Image of a graph showing the jet energy resolution as a function of jet energy for various HCal hit energy truncations.](image)

**Figure 5.14:** The jet energy resolution as a function of jet energy for various HCal hit energy truncations. The results shown use the nominal ILD detector model, which contains $30 \times 30$ mm$^2$ square scintillator tiles in the HCal.

It is clear that a 1 GeV HCal hit energy truncation is beneficial for the performance of the nominal ILD detector model since the jet energy resolution is reduced by roughly $\sim 0.15\%$ across the jet energy range from 45.5 GeV to 250 GeV. As the HCal hit truncation technique offers significant performance gains, it is used for the calorimeter optimisation studies presented in chapter 6. These studies include optimisation of the HCal cell size. Increasing the HCal cell size will increase the average number of particles passing through each calorimeter hit, which in turn reduces the impact of Landau fluctuations and vice versa. For all detector models considered where the HCal cell size was varied, the HCal hit energy truncation was re-optimised to account for the changing impact of Landau fluctuations. For detector models with a HCal cell size of $10 \times 10$ mm$^2$, $20 \times 20$ mm$^2$, $30 \times 30$ mm$^2$, $40 \times 40$ mm$^2$, $50 \times 50$ mm$^2$ and $100 \times 100$ mm$^2$ the reoptimised truncation values were 0.5, 0.75, 1, 1.5, 2 and 5 GeV, respectively. Furthermore, the average particle density in a HCal hit will also be sensitive to the properties of the absorber material used in the calorimeters, therefore, the HCal hit energy truncation was also reoptimised in the HCal absorber material study. The optimal truncation energy cut for the $30 \times 30$ mm$^2$ cell size tungsten HCal option was 5 GeV, while for all other detector models considered
it was 1 GeV. The cause of increased truncation energy cut for tungsten is discussed in section 6.3.5.

Understanding the effect of the HCal hit energy truncation is crucial when performing optimisation studies. This can be seen in figure 5.15, which shows the results of the HCal cell size optimisation study when using a 1 GeV truncation and when optimising the truncation for each detector model. By applying a uniform HCal hit energy truncation the importance of the HCal cell size to particle flow calorimetry is vastly overinflated. For example, if the HCal cell size is increased from 10 mm to 100 mm the jet energy resolution for 250 GeV jets goes from $\sim 2.8\%$ to $\sim 4.5\%$ for the flat 1 GeV truncation, but only $\sim 3.5\%$ when using an optimised truncation. As the jet energy and HCal cell size increase, the flat 1 GeV truncation throws away a larger fraction of typical hadronic shower energy measurements, which causes the jet energy resolution to degrade rapidly.

![Graph showing jet energy resolution as a function of HCal cell size](image)

**Figure 5.15:** The jet energy resolution as a function of HCal cell size in the ILD detector model using a HCal hit energy truncation that is (a) optimised and (b) fixed at 1 GeV.

### 5.3.2 Software Compensation

Particle showers that are produced when a hadron interacts with a calorimeter contain two components [33]; an electromagnetic shower core (which originates from the production and decay of $\pi^0$s and $\eta$s) and a hadronic shower component originating from other interacting and decaying particles. By identifying each of these components in the reconstruction, it is possible to modify their energies to give a compensating calorimeter response. This technique is known as software compensation.
Software compensation achieves a compensating calorimeter response by applying weights, as introduced in equation 5.13, that modify the energy of calorimeter hits in the HCal. These weights increase the energy found in the hadronic hits to compensate for the undetectable energy component found in hadronic showers. Additionally, these weights reduce the energy of spuriously high energy hits to minimise the impact of Landau fluctuations. The weights vary as a function of the calorimeter hit energy density, $\rho_i$, and the uncompensated energy of the particle shower, $E_{\text{Raw}}$, where

$$E_{\text{Raw}} = \sum_{\text{ECal hits}, i} E_{\text{ECal}, i} + \sum_{\text{HCal hits}, i} E_{\text{HCal}, i}. \quad (5.16)$$

The electromagnetic and hadronic components of a hadronic particle shower are treated differently in this approach by applying weights that are sensitive to the energy density of the calorimeter hits. Hits with large energy densities are likely to be part of the electromagnetic core, while low energy density hits are likely to be part of satellite hadronic hits around the electromagnetic shower core [3]. By tailoring the weights as a function of the energy density, a compensating calorimeter response can be obtained. Figure 5.16 shows the electromagnetic and hadronic shower components, determined by the energy density of the calorimeter hits, for a hadronic shower in a 500 GeV $Z \rightarrow \text{uds}$ di-jet event.

Figure 5.16: An event display for a 500 GeV $Z \rightarrow \text{uds}$ di-jet event reconstructed using the nominal ILD detector. (a) The full event environment. (b) A single hadronic cluster from the same event where shading indicates the energy density in the HCal. High energy density cells are coloured red, while lower energy density cells are coloured blue. All ECal hits are shaded black. The high energy density electromagnetic core of the selected hadronic cluster is clearly visible.
The software compensation weights also depend on $E_{\text{Raw}}$, the total raw cluster energy, to account for the sensitivity of the hit energy density distribution on the total particle shower energy. For hadronic showers, the fraction of the total energy carried in the electromagnetic core increases as the total shower energy increases [33], therefore a dependancy of the weights on $E_{\text{Raw}}$ is needed to obtain a compensating calorimeter response across a wide range of energies.

The precise form of the weights used in this technique are

$$\omega(E_{\text{Raw}}, \rho) = p_1(E_{\text{Raw}}) \times \exp(p_2(E_{\text{Raw}}) \times \rho) + p_3(E_{\text{Raw}})$$

with

$$p_1(E_{\text{Raw}}) = p_{11} + p_{12} \times E_{\text{Raw}} + p_{13} \times E_{\text{Raw}}^2$$
$$p_2(E_{\text{Raw}}) = p_{21} + p_{22} \times E_{\text{Raw}} + p_{23} \times E_{\text{Raw}}^2$$
$$p_3(E_{\text{Raw}}) = \frac{p_{31}}{p_{32} + \exp(p_{33} \times E_{\text{Raw}})}$$

where $p_{\alpha \beta}$ are constants and

$$E_{\text{Raw}} = \sum_{\text{ECal hits}, i} E_{\text{ECal}}^i + \sum_{\text{HCal hits}, i} E_{\text{HCal}}^i$$

The functional form of these weights was developed by the CALICE collaboration [3] and originates from an empirical fit to test beam measurements. In this study, the parameters $p_{\alpha \beta}$ were determined by minimising $\chi^2(p_{\alpha \beta})$ where

$$\chi^2(p_{\alpha \beta}) = \sum_{\text{Events}} \left( \frac{(E_{\text{Cluster}}(p_{\alpha \beta}) - E_{\text{MC}})}{0.5 \times \sqrt{E_{\text{MC}}}} \right)^2$$

where the sum runs over single $K_L^0$ events that ranged in energy from 10 to 100 GeV in steps of 10 GeV. At each energy the same number of events was used to avoid biasing to particular energies. In each event, $E_{\text{Cluster}}$ is the software compensated energy estimator for the reconstructed event and $E_{\text{MC}}$ is the MC energy of the $K_L^0$. Normalising the deviation of $E_{\text{Cluster}}$ from $E_{\text{MC}}$ by the stochastic term in the HCal energy resolution, $\sim 50% \times \sqrt{E}$, made sure events of different MC energy contributed the same weight to $\chi^2$. 

Figure 5.18 shows $\omega$ as a function of $\rho$ for selected values of $E_{\text{Raw}}$ and figure 5.17 shows $p_1$, $p_2$ and $p_3$ as a function of $E_{\text{Raw}}$. These weights shown in figures 5.18 and 5.17 were obtained by training the software compensation technique on samples simulated using the nominal ILD detector model. Figure 5.18 shows that the high energy density hits are being reduced in energy to compensate for the effects of Landau fluctuations, while the low energy density hits are being increased in weight to compensate for the undetectable energy component found in hadronic showers. Furthermore, the weights vary as a function of the raw hadronic shower energy to account for the changing energy density topology of hadronic showers with increasing shower energy.

\[
\begin{align*}
\text{(a)} & \quad p_1 \quad \text{vs} \quad E_{\text{Raw}} \quad [\text{GeV}] \\
\text{(b)} & \quad p_2 \quad \text{vs} \quad E_{\text{Raw}} \quad [\text{GeV}] \\
\text{(c)} & \quad p_3 \quad \text{vs} \quad E_{\text{Raw}} \quad [\text{GeV}]
\end{align*}
\]

**Figure 5.17:** The software compensation parameters (a) $p_1$, (b) $p_2$ and (c) $p_3$ as a function of $E_{\text{Raw}}$, the total raw cluster energy. These weights were obtained by training the software compensation technique on samples simulated using the nominal ILD detector model.

The software compensation technique is applied in the PandoraPFA framework in the form of an energy correction function, which means whenever the energy of a cluster of hits is considered by PandoraPFA the software compensated energy is used. Applying software compensation in this way benefits the detector energy resolution in two ways; firstly, the intrinsic energy resolution of the detector improves and secondly, the confusion contribution to the energy resolution is reduced.

As software compensation only modifies the energy of HCal hits there is freedom to apply further energy corrections to the ECal hits. Applying the 'Clean Clusters' logic, described in section 5.3.1.1, to the ECal hits alongside software compensation was found to be beneficial to the jet energy resolution. Therefore, the application of software compensation within PandoraPFA implicitly involves the application of the 'Clean Clusters’ logic to the ECal hits.

Software compensation was tuned using a maximum $K^0_L$ energy of 100 GeV, therefore, it is only applied to clusters where $E_{\text{Raw}} < 100$ GeV; sensible behaviour outside this
Figure 5.18: The software compensation weight applied to a calorimeter hit as a function of calorimeter hit energy density for various cluster energies.

range cannot be ensured. While it would be possible to modify the energy range of the training sample to go to higher energies, hadronic clusters with energy greater than 100 GeV will be rare at the ILC-like energies, i.e. $\sqrt{s} \leq 500$ GeV, considered here.

5.3.2.1 Impact on Single Particle Energy Resolution

Figure 5.19 shows the energy resolution as a function of MC energy for single $K^0_L$ events obtained using the various energy correction configurations in PandoraPFA. When comparing the energy resolution given by software compensation to that obtained using no energy corrections, it can be seen that software compensation offers an improvement in the energy resolution of $\sim 15\%$ across the energy range considered. The uniformity of this improvement is encouraging, indicating that software compensation is achieving a compensating calorimeter response across this wide range of energies.

Comparing the performance of software compensation to the legacy corrections, described in section 5.3.1.1, it can be seen that software compensation gives a better energy resolution across almost the entire range of energies considered. The only exception to this is around $E_{K^0_L} \sim 50$ GeV where the performance of software compensation and the legacy corrections are comparable. By removing the hit truncation from the legacy options it is clear that the changes in energy resolution when using the legacy options are
being driven by the hit truncation. This makes the trend in energy resolution observed using the legacy corrections clear as, at low $K^0_L$ energies, very few hits are affected by the truncation so the performance is comparable to not using any energy corrections. At high $K^0_L$ energies, the truncation is too aggressive and removes energy from hits that are not spuriously high leading to a worsening energy resolution. Between these two extremes, $E_{K^0_L} \sim 50$ GeV, the truncation works ideally and the improvement in energy resolution when using the legacy corrections is the largest.

![Figure 5.19](image)

**Figure 5.19:** The energy resolution as a function of the MC energy for single $K^0_L$ events using various energy correction settings. The black line represents no energy corrections, the blue line represents software compensation, the magenta line represents the legacy energy corrections and the red line represents the legacy corrections without the HCal hit energy truncation. The nominal ILD detector model was used in these simulations.

### 5.3.2.2 Impact on Jet Energy Resolution

The improvements in the intrinsic energy resolution of the detector observed when using software compensation will propagate into the reconstruction of jets. Figure 5.20 shows the jet energy resolution as a function of jet energy when using selected energy correction configurations in PandoraPFA. It can be seen that software compensation improves the jet energy resolution by $\sim 15\%$ across the energy range considered in comparison to using no energy corrections. Furthermore, software compensation offers an improvement in the jet energy resolution of the order of $5\%$ for jet energies $\gtrsim 100$ GeV in comparison
to the legacy corrections, which prior to the development of software compensation had
given the best jet energy resolutions.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.20.png}
\caption{The jet energy resolution as a function of the jet energy for a variety of different
energy correction options. The black line represents no energy corrections, the
blue line represents software compensation, the magenta line represents the
legacy energy corrections and the red line represents the legacy corrections
without the HCal hit energy truncation. The nominal ILD detector model was
used in these simulations.}
\end{figure}

Figure 5.21 shows the intrinsic energy resolution and confusion contributions to the
jet energy resolution as a function of jet energy when using selected energy correction
configurations in PandoraPFA. The intrinsic energy resolution contribution shows that
software compensation is significantly better than all other energy corrections options,
which is to be expected from the energy resolution studies presented in section 5.3.2.1.
When compared to the legacy energy corrections, software compensation improves the
intrinsic energy resolution by up to 12% across the energy range considered, with the
largest improvement occurring for 100 GeV jets. As jets contain a broad spectrum of
hadronic cluster energies, there is no jet energy for which the intrinsic energy resolution
of the detector is comparable between the legacy corrections and software compensation.
The confusion contributions to the jet energy resolution when using software compensation
and the legacy corrections are almost identical. This indicates that the improvement
seen in the jet energy resolution when comparing software compensation to the legacy
corrections, shown in figure 5.20, is being driven by the intrinsic energy resolution.
The "Clean Clusters" and "Scale Hot Hadrons" energy corrections, i.e. the legacy corrections without the HCal hit energy truncation, benefits the pattern recognition by reducing the confusion contribution. The confusion contribution is reduced by $\sim 18\%$ for 45.5 GeV jets using these energy corrections, however, as the jet energy increases, the magnitude of this improvement decreases, such that at 250 GeV jets no improvement is seen. These corrections do not significantly affect the intrinsic energy resolution of the detector. As these corrections benefit pattern recognition, selected aspects of their logic is applied to ECal hits in the software compensation energy correction as previously discussed.

![Figure 5.21](image)

**Figure 5.21:** The contributions to the jet energy resolution as a function of the jet energy for a variety of different energy correction options. The jet energy resolution contributions presented are (a) the intrinsic energy resolution of the detector and (b) the total confusion contribution. The jet energy resolution obtained using the standard reconstruction is given by the quadrature sum of the intrinsic energy resolution and total confusion contributions. The black line represents no energy corrections, the blue line represents software compensation, the magenta line represents the legacy energy corrections and the red line represents the legacy corrections without the HCal hit energy truncation. The nominal ILD detector model was used in these simulations.

### 5.3.3 Summary

The effects on single particle and jet energy resolution of the HCal hit energy truncation and software compensation have been examined. Although relatively simplistic, the HCal hit energy truncation was found to be beneficial for detector performance by limiting the impact of Landau fluctuations. The more sophisticated software compensation procedure was found to be highly effective at producing a compensating calorimeter response across
a wide range of energies, which translated into excellent performance in terms of jet energy resolution.

5.4 Timing Cuts

The linear collider will operate using trigger-less readout whereby the recorded data for each sub-detector is read out between collisions of the $e^+$ and $e^-$ bunches. The bunch train structure for ILC and CLIC is compared in table 5.2.

Event selection at the linear collider involves the identification of any hard interactions, prior to full event reconstruction, and only putting data into the event reconstruction if it is measured within a chosen time window about these interactions. The recorded time of a calorimeter hit, which is cut on to determine whether the hit will proceed to full reconstruction, is corrected for straight time-of-flight to the IP. This ensures that the amount of time particle showers have to develop in the calorimeters is independent of their position in the detector. The energy resolution of a calorimeter is sensitive to the choice of time window applied because energy measurements made outside the time window are rejected. Therefore, the overall detector performance will be sensitive to the choice of time window used.

The small bunch separation time at CLIC, 0.5 ns, means the integration time of the calorimeters will necessarily span several bunch crossings. When this is combined with the intense beam-induced backgrounds, identification of energy deposits produced from a hard interaction of interest becomes difficult. By placing tight timing constraints on the energy deposits made in the CLIC calorimeters, it is possible to minimise the impact of the beam-induced backgrounds. As well as minimising the impact of the backgrounds, these tight timing requirements will also change how particle showers from the hard interaction of interest are sampled. Understanding the impact of these timing requirements on physics performance is vital to the success of the CLIC experiment. Application of the time constrains applied to measurements made at the ILC is less challenging than at CLIC because the bunch separation is much larger, meaning the calorimeters could be read out between bunches, and the beam-induced backgrounds are much smaller.

For all choices of time window considered in this study the calibration procedure described in section 5.2 was reapplied. This ensures that the mean of the reconstructed
energy distributions will not depend on the calorimeter timing window because the calibration will compensate for energy losses incurred by rejecting energy measurements made outside the time window.

For the results presented in this chapter and the optimisation studies found in chapter 6, a 100 ns timing window was applied to all detector models considered. This value was chosen as it reflects particle shower development time [6] and could be reasonably achieved using readout technology options presently available [74].

### 5.4.1 Impact on Single Particle Energy Resolution

Figure 5.22 shows the energy resolution of the nominal ILD detector for 100 GeV photons and 50 GeV $K_L^0$s as a function of the timing window applied to the calorimeter hits. The timing cut makes little difference to the energy resolution of photons, however, the energy resolution for neutral hadrons gets significantly worse as the time window is reduced. The neutral hadron energy resolution becomes worse by almost 20% when the time window is reduced from $10^6$ ns to 10 ns. These trends are to be expected because electromagnetic showers develop far more rapidly than their hadronic counterparts [33]. This can be seen from figure 5.23, which shows the distribution of the measurement time of calorimeter hits, corrected for time-of-flight, for selected shower components for 91 GeV $Z\rightarrow uds$ events. Hadronic showers develop more slowly as they often involve intermediate states that must decay to continue the propagation of the shower.

If a narrow calorimeter timing window is used, energy measurements from the hadronic shower will be lost and the energy resolution will degrade, which is what is observed. On the other hand, electromagnetic showers develop so rapidly that even the 10 ns time window does not reject many energy measurements.
Figure 5.22: The energy resolution as a function of calorimeter timing window for (a) 100 GeV photons and (b) 50 GeV $K^0_L$ events using the nominal ILD detector model.

Figure 5.23: The normalised distribution of the time of the electromagnetic and hadronic shower calorimeter hits, corrected for time of flight to the impact point, for 91 GeV $Z\rightarrow uds$ events. Electromagnetic shower energy deposits are deposited very rapidly, while hadronic shower energy deposits are deposited over a much longer time period.

5.4.2 Impact on Jet Energy Resolution

Figure 5.24 shows the jet energy resolution as a function of the jet energy for selected calorimeter time windows. As expected, the jet energy resolution becomes worse when the calorimeter timing window is reduced.
The jet energy resolution as a function of jet energy for various calorimeter timing cuts. The nominal ILD detector model was used in these simulations.

The time window applied to the calorimeter hits affects both the neutral hadron and jet energy resolutions with a larger timing window leading to better resolutions. It can be seen that by applying an aggressive choice of time window, such as 10 ns, the jet energy resolution would be degraded because many of the hadronic showers are not fully sampled. However, even using a 10 ns timing cut, the jet energy resolutions are still sufficiently low to give excellent detector performance. Both the single particle and jet energy resolutions indicate that the majority of hadronic showers at the energies considered will be fully sampled using a 100 ns time window and that there is little to be gained by increasing this time window further.

5.4.3 Summary

Simulations were performed to study the impact of the calorimeter hit time window used at the linear collider experiments. The energy resolution for electromagnetic showers did not change significantly when varying the size of time window, whereas the neutral hadron energy resolution becomes worse as the size of the time window is reduced. The jet energy resolution is also sensitive to the size of the time window used, however, the trend was far weaker than that seen for neutral hadrons because only 10% of the jet energy is carried in the form of neutral hadrons. Increasing the time window beyond
100 ns did not have any significant benefit, indicating that the majority of hadronic showers are fully sampled in this time.
Chapter 6

Calorimeter Optimisation Studies

“The simple believes everything, but the prudent gives thought to his steps.”
— Proverbs 14:15

6.1 Introduction

This chapter describes the optimisation of the calorimeters used at the linear collider, with focus placed on obtaining the best energy resolution for jets. Parameters such as the number of layers, cell size and material choices for the calorimeters are investigated. Several global detector parameters such as the magnetic field strength and the inner radius of the ECal are also studied. These parameters are not calorimeter specific, but affect the jet energy resolution obtained from particle flow.

6.2 Electromagnetic Calorimeter Optimisation

The purpose of an electromagnetic calorimeter (ECal) is to measure the energy deposits from electromagnetic showers. The nominal ILD ECal, summarised in table 6.1, is a silicon-tungsten sampling calorimeter. It contains 29 readout layers and 24 radiation lengths ($X_0$), which is sufficient to contain all but the highest energy electromagnetic showers. The absorber thickness of the last nine layers is twice that of the first 20 layers to reduce the number of readout channels and cost of the calorimeter. The high
longitudinal sampling frequency is crucial for the pattern recognition aspect of particle flow calorimetry, especially in the region where particle showers start developing.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Default Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell Size</td>
<td>$5 \times 5 \text{ mm}^2$ square cells</td>
</tr>
<tr>
<td>Number of Layers</td>
<td>29 readout layers</td>
</tr>
<tr>
<td>Active Material Choice</td>
<td>Silicon or Scintillator</td>
</tr>
<tr>
<td>Active Material Thickness</td>
<td>0.5 mm (Silicon) or 2 mm (Scintillator)</td>
</tr>
<tr>
<td>Absorber Material Choice</td>
<td>Tungsten</td>
</tr>
<tr>
<td>Absorber Material Thickness</td>
<td>20 layers of 2.1 mm followed by 9 layers of 4.2 mm</td>
</tr>
</tbody>
</table>

Table 6.1: The configuration of the silicon and scintillator ECal options for the ILD detector model [34].

The calorimeter performance was simulated for a number of detector models where the following detector parameters were varied:

- **Cell size**: This is a vital aspect of the detector in the particle flow paradigm as smaller cell sizes leads to better separation between nearby showering particles, which helps to minimise the effect of confusion. Modifying the cell size should have little effect on the intrinsic energy resolution of the detector.

- **Longitudinal sampling frequency**: The longitudinal sampling frequency in the ECal was varied by changing the number of layers in the ECal while simultaneously changing the thicknesses of the layers such that the total depth, in radiation lengths, was held constant. Increasing the number of layers in a sampling calorimeter means any particles showering within it are sampled more, which leads to a reduction in the stochastic contribution to the energy resolution. Therefore, varying the number of layers is expected to change in intrinsic energy resolution of the calorimeter.

- **Active material choice**: The options under consideration for the active sensor material are silicon or plastic scintillator. As well as providing different intrinsic energy resolutions the readout mechanics of these two options are significantly different. There is no clear prior knowledge as to which should provide better performance.
6.2.1 ECal Cell Size

Different detector models were considered where the cell size in the ECal was varied about the nominal value of $5 \times 5$ mm$^2$ square cells. The granularities considered were $3 \times 3$ mm$^2$, $5 \times 5$ mm$^2$, $7 \times 7$ mm$^2$, $10 \times 10$ mm$^2$, $15 \times 15$ mm$^2$ and $20 \times 20$ mm$^2$ square cells for both the silicon and scintillator active material options.

The energy resolution, using 100 GeV photons, as a function of the ECal cell size is shown in figure 6.1a for the silicon option and in figure 6.1b for the scintillator option. At this energy, the photons will be largely contained within the ECal and the reported energy resolution reflects solely the performance of the ECal. For both the silicon and scintillator ECal options the energy resolution does not depend strongly on the ECal cell size. This is to be expected as there is no change in the number of layers, which is the main factor in determining the energy resolution of a sampling calorimeter.

The only statistically significant variation observed occurs for the scintillator ECal option. A degradation in the energy resolution of $\sim 10\%$ is observed when reducing the ECal cell size from $5 \times 5$ mm$^2$ to $3 \times 3$ mm$^2$. The most likely cause is the "dead" region in the active material, which represents the readout multi pixel photon counter (MPPC) [35]. The MPPC occupies a fixed area of the cell, irrespective of cell size, and so the dead region of the cell fractionally increases as cell size is reduced. The larger this dead region, the worse the sampling of the electromagnetic showers in the ECal and the worse the resolution. While this effect will be present in all scintillator ECal options, it will only be significant for the small cell sizes when the dead region is fractionally the largest.

The ability to separate nearby electromagnetic particle showers within a calorimeter is limited by the Moliére radius of the absorber material and the cell size. The Moliére radius controls the width of the electromagnetic shower, while the cell size controls how the transverse shower profile is sampled. By reducing the cell size, it becomes easier to resolve nearby electromagnetic showers, which in turn reduces the effect of confusion. Therefore, it is expected that the jet energy resolution will be sensitive to the ECal cell size, even though the intrinsic energy resolution is not. The jet energy resolution as a function of ECal cell size is shown in figure 6.2a for the silicon option and figure 6.2b for the scintillator option. There is a strong dependance on the ECal cell size, with smaller cell sizes leading to lower values of the jet energy resolution; the jet energy resolution for 250 GeV jets for both ECal options goes from $\sim 3.0\%$ to $\sim 4.3\%$ when the ECal cell size goes from $3 \times 3$ mm$^2$ to $20 \times 20$ mm$^2$. The origin of this trend is best illustrated by
considering the intrinsic energy resolution and confusion contributions to the jet energy resolution. These contributions are shown as a function of ECal cell size for 45 and 250 GeV jets in figure 6.3. It is clear from these contributions that the intrinsic energy resolution of the detector does not change when varying the cell size, which agrees with both prior expectations of calorimeter behaviour and the single particle energy resolution study. As expected, it can be seen that the trend in jet energy resolution as a function of the ECal cell size is being driven purely by changes to the confusion contribution and, in particular, the confusion caused by the reconstruction of photons.

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**Figure 6.1:** The energy resolution as a function of ECal cell size for 100 GeV photons using the nominal ILD detector model with (a) the silicon and (b) the scintillator ECal option.

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**Figure 6.2:** The fractional jet energy resolution as a function of ECal cell size for various jet energies using the nominal ILD detector model with (a) the silicon and (b) the scintillator ECal option.
It is clear that the ECal cell size is extremely important for jet energy measurements, although it has little bearing on the intrinsic energy resolution of the ECal. Separation of the hadronic decays of the W and Z bosons, i.e. $\sigma_E/E \lesssim 3.8\%$ [29], can be achieved across the jet energy range considered here using a maximum ECal cell size of $15 \times 15 \text{ mm}^2$. However, as reducing the ECal cell size further continues to benefit the jet energy resolution, minimising the ECal cell size is desirable.
6.2.2 ECAL Longitudinal Sampling Frequency

The detector performance was simulated where the number of layers in the ECAL was varied, while keeping the total material budget ($X_0$) approximately constant. This study was performed for both the silicon and scintillator active material options. In all cases tungsten was used for the ECAL absorber material and the active layer thicknesses were not changed from those used in the nominal ILD ECAL summarised in table 6.1. The different ECAL layouts considered are summarised in table 6.2.

<table>
<thead>
<tr>
<th>Total Number of Layers</th>
<th>$N_{Layers}$ Region 1</th>
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<th>Absorber Thickness Region 2 [mm]</th>
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</tr>
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<td>4.00</td>
<td>5</td>
<td>8.00</td>
<td>22.31</td>
</tr>
</tbody>
</table>

Table 6.2: The longitudinal structure of the ECAL models considered in the optimisation study. The radiation length of tungsten absorber is 3.504 mm [22]. Note that a presampler layer contributes one extra layer to the cumulative number of layers.

The energy resolution, for 100 GeV photons, as a function of the number of layers in the ECAL is shown in figure 6.4a for the silicon option and in figure 6.4b for the scintillator option. When the number of layers is increased $\sigma_E/E$ decreases, which is expected because the energy resolution for a sampling calorimeter is $\propto 1/\sqrt{E \times N_{Layers}}$, where $E$ is the reconstructed energy and $N_{Layers}$ is the number of layers in the calorimeter.

When the number of layers in the ECAL is increased, the intrinsic energy resolution benefits; the intrinsic energy resolution of the ECAL improves by $\sim 25\%$ in both ECAL options when increasing the number of layers from 16 to 30. This has the knock-on effect of reducing the confusion contribution to the jet energy resolution, which can be seen in figures 6.5a and 6.5b for the silicon and scintillator ECAL options, respectively. In both cases, the jet energy resolution was found to improve when the number of layers in the ECAL was increased; the jet energy resolution goes from $\sim 4.4$ to $\sim 3.6\%$ for the silicon option and from $\sim 4.1$ to $\sim 3.6\%$ for the scintillator option when increasing the number of layers from 16 to 30. The magnitude of the change in jet energy resolution is dependent upon the jet energy, with a stronger dependancy being observed for low
energy jets. This is expected from the stochastic contribution to the energy resolution for a sampling calorimeter. For high jet energies, changing the number of layers in the ECal does not significantly affect the jet energy resolution because the jet energy resolution is dominated by confusion. For low jet energies, the stochastic contribution to the energy resolution is bigger making it possible to resolve the changes to it when varying the number of layers in the ECal.

The decomposition of the jet energy resolution into the intrinsic energy resolution and confusion contributions for 45 and 250 GeV jets are shown, for both the silicon and scintillator ECal options, in figure 6.6. As expected, the improvement to the intrinsic energy resolution seen when increasing the number of layers in the ECal leads to the knock-on effect of lowering the confusion. However, significantly the magnitude of the change to the intrinsic energy resolution and confusion contributions to the jet energy resolution when varying the number of layers in the ECal are comparable in size. This shows that pattern recognition is as important for detector performance in the particle flow paradigm as intrinsic energy resolution.

6.2.3 ECal Active Material

In sections 6.2.1 and 6.2.2 the performance of the ECal was reported for both the silicon and scintillator options and to a large extent the performance of the two options was similar, but not identical:
• The intrinsic energy resolution of the silicon ECal option is better than that of the scintillator option at very high energies. For 500 GeV photons the intrinsic energy resolution is $\sim 25\%$ better for the silicon option. Section 3.7 contains a comparison between the photon energy resolution for the two ECal options, which clearly illustrates this. The most likely origin of the differing energy resolutions is the implementation of Birks’ law [47] for scintillator active materials, which states

$$d\mathcal{L}/dx \propto \frac{dE/dx}{1 + k_B dE/dx},$$

where $d\mathcal{L}/dx$ is the scintillation light yield per unit path length, $dE/dx$ is the energy deposited per unit path length and $k_B$ is a material property constant. For large energy deposits per unit length, such as those found in high energy photons, the light yield saturates causing a degradation in the energy resolution. When comparing the photon energy resolution for the silicon and scintillator ILD ECal options, which can be found in section 3.7, the saturation effect starts to degrade the energy resolution for the scintillator option for photons of energy $\gtrsim 50$ GeV.

• The 'dead' region due to the presence of the MPPC in the simulation of the scintillator ECal option degrades performance of the detector for small transverse granularities, see figure 6.1.

In summary, the performance of the two options, in terms of energy and jet energy resolution, at ILC-like energies is comparable. However, the silicon option is preferred.
Figure 6.6: Contributions to the jet energy resolution shown as function of number of layers in the ECal using the nominal ILD detector model for (a) the silicon ECal option and 45 GeV jets, (b) the scintillator ECal option and 45 GeV jets, (c) the silicon ECal option and 250 GeV jets and (d) the scintillator ECal option and 250 GeV jets. The black curves correspond to the standard reconstruction, the blue curves to the intrinsic energy resolution contribution to the jet energy resolution, the red curves to the confusion contribution to the jet energy resolution and the magenta curves to the confusion contribution to the jet energy resolution related solely to photon reconstruction.

when manufacture and implementation of the two models is compared. While constructing silicon wafers to fit a 5 × 5 mm² square cell size is achievable, this would be extremely challenging for scintillator tiles. To resolve this in actuality, the scintillator ECal option would have to use 5 × 45 mm² scintillator strips that are arranged in alternating directions in each ECal layer [34]. By combining information from neighbouring layers it becomes possible to approach an effective 5 × 5 mm² square cell size.
6.3 Hadronic Calorimeter Optimisation

The purpose of a hadronic calorimeter (HCal) is to measure the energy deposits from hadronic showers. The HCal in the default ILD detector model, summarised in table 6.3, is approximately 6 nuclear interaction lengths ($\lambda_I$) deep. The ECal contributes approximately one $\lambda_I$ giving a total of $\approx 7\lambda_I$, which is sufficient to contain jets at ILC like energies. The longitudinal structure of this model consists of 48 readout layers each containing a 3 mm active layer of scintillator and a 20 mm absorber layer of iron.

<table>
<thead>
<tr>
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</tr>
</thead>
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</tr>
<tr>
<td>Number of Layers</td>
<td>48 readout layers</td>
</tr>
<tr>
<td>Active Material Choice</td>
<td>Scintillator</td>
</tr>
<tr>
<td>Active Material Thickness</td>
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</tr>
<tr>
<td>Absorber Material Choice</td>
<td>Steel</td>
</tr>
<tr>
<td>Absorber Material Thickness</td>
<td>20 mm</td>
</tr>
</tbody>
</table>

Table 6.3: The configuration of the HCal in the nominal ILD detector model [34].

There are several readout approaches under consideration for the HCal including fully analogue, fully digital and semi-digital. Analogue readout reports the energy within each HCal cell using a continuous variable, while digital readout only produces a response if the energy deposited within a calorimeter cell is above a given threshold. The semi-digital approach mirrors that of the digital approach, but has three responses each with a different energy threshold. While the energy resolution for digital calorimeters is not as good as that of analogue calorimeters, it is possible to construct smaller cell sizes using a digital readout. In traditional calorimetry, a digital calorimeter would give a worse jet energy resolution than the analogue equivalent, however, that is not necessarily the case in particle flow calorimetry. If a digital calorimeter could be realised with a much smaller cell size than the analogue equivalent, then the effect of confusion in the digital calorimeter may be reduced such that it compensates for any loss to intrinsic energy resolution. In the following studies only the optimisation of the analogue HCal is presented as this is the readout approach used in the nominal ILD detector model.

A number of options were simulated where the following parameters in the HCal were varied:
• Cell size: This is crucial for successful application of particle flow calorimetry for making associations between clusters of calorimeter hits and charged particle tracks. It is expected that the intrinsic energy resolution be invariant to changes in the HCal cell size.

• Number of readout layers: The number of layers in the HCal are varied, however, the thickness of those layers match those of the nominal ILD HCal design. This means the total depth of the HCal in $\lambda_I$ is changing. It is expected that this study will determine the effect of leakage of energy out of the back of the HCal.

• Longitudinal sampling frequency: This involves changing the number of readout layers in the HCal while simultaneously changing the thicknesses of the active and absorber layers to keep the total number of $\lambda_I$ in the HCal constant. As this modifies the sampling of particle showers in the HCal, it will affect the intrinsic energy resolution of the HCal.

• Sampling fraction: This is the ratio of the active medium thickness to the absorber medium thickness. This controls how particle showers within the calorimeter are sampled. In this study the total depth of the HCal in $\lambda_I$ is held constant between detector models.

• Absorber material choice: Two options have been considered: steel and tungsten. This choice affects the growth and propagation of hadronic showers.

### 6.3.1 HCal Cell Size

The HCal cell size is an important detector parameter in the application of particle flow calorimetry. Smaller HCal cell sizes will lead to a finer spatial resolution that can be used to better separate charged and neutral particle calorimetric energy deposits. On the other hand, this will also lead to an increase in the number of readout channels that will raise the cost of the calorimeter. Therefore, it is highly desirable to achieve the optimal physics performance using the largest cell size possible. The nominal ILD HCal has a 30 mm square cell size and in this study the following cell sizes were considered; $10 \times 10$ mm$^2$, $20 \times 20$ mm$^2$, $30 \times 30$ mm$^2$, $40 \times 40$ mm$^2$, $50 \times 50$ mm$^2$ and $100 \times 100$ mm$^2$.

In the nominal ILD detector, 50 GeV long-lived neutral kaons ($K_L^0$s) will deposit $\sim 65\%$ of their energy in the HCal and $\sim 35\%$ in the ECal. As 50 GeV $K_L^0$s deposit the bulk of their energy in the HCal, they are appropriate to use when determining
the performance of the HCal. However, it should be emphasised that the \( \mathcal{K}_L^0 \) energy resolutions represent the intrinsic energy resolution of the whole ILD detector and not purely that of the HCal.

Figure 6.7 shows the energy resolution for 50 GeV \( \mathcal{K}_L^0 \)s as a function of cell size. As expected, the hadronic energy resolution does not strongly depend on the HCal cell size. The only statistically significant variation in energy resolution is observed for the \( 100 \times 100 \text{ mm}^2 \) HCal cell size. For this model the energy resolution gets worse by \( \sim 8\% \) in comparison to the other models considered. The most likely cause of this is a reduction in the effectiveness of the HCal hit energy truncation, which is described in section 5.3.1. The reduced effectiveness is expected because the precision used when obtaining the optimal energy truncation becomes worse as HCal cell size diverges from the nominal value.

![Figure 6.7: The energy resolution as a function of HCal cell size for 50 GeV \( \mathcal{K}_L^0 \) events using the nominal ILD detector model.](image)

A smaller HCal cell size will lead to better separation of charged and neutral hadron calorimetric energy deposits, therefore, it is expected that the confusion contribution to the jet energy resolution will be reduced by using smaller HCal cell sizes. Figure 6.8 shows the jet energy resolution as a function of cell size in the HCal. At low jet energies there is no strong dependency of the jet energy resolution on the HCal cell size, which is as expected from the \( \mathcal{K}_L^0 \) energy resolution study. For high energy jets there is a clear dependence, with lower HCal cell sizes leading to better jet energy resolutions; the jet energy resolution for 250 GeV jets goes from \( \sim 2.7\% \) to \( \sim 3.5\% \) when the HCal cell size is increased from \( 10 \times 10 \text{ mm}^2 \) to \( 100 \times 100 \text{ mm}^2 \). Examining the different contributions to the jet energy resolution, shown in figure 6.9 it can be seen that the intrinsic energy resolution contribution does not depend on the HCal cell size; it is the confusion contribution that drives the overall trend in the jet energy resolution.
This is particularly clear at high jet energies where the confusion contribution to the jet energy resolution dominates that of the intrinsic energy resolution contribution. At high jet energies smaller HCal cell sizes leads to a reduction in the effect of confusion; the confusion contribution to the jet energy resolution is reduced by $\sim 25\%$ when reducing the HCal cell size from $100 \times 100 \text{ mm}^2$ to $10 \times 10 \text{ mm}^2$. At low jet energies the trend is less clear, as the confusion contribution is less dominant. Nevertheless, a reduction in the effect of confusion with decreasing cell size is still visible for all but the smallest HCal cell size. The most likely cause of the increase in confusion for the smallest HCal cell size at low energies is the tuning of the PandoraPFA algorithms to the nominal ILD HCal cell size. For both the 45 and 250 GeV jets, the photon confusion does not depend on the HCal cell size. This indicates that changes to the confusion term seen when varying the HCal cell size are related solely to the reconstruction of hadrons.

![Figure 6.8](image)

**Figure 6.8:** The jet energy resolution as a function of HCal cell size for various jet energies using the nominal ILD detector model.

A comparison of the results from the ECal and HCal cell size optimisation studies shows that the jet energy resolution has a stronger dependency on the ECal cell size than on the HCal cell size; increasing the nominal ECal cell size by a factor of three makes the jet energy resolution for 250 GeV jets worse by $\sim 20\%$, while increasing the nominal HCal cell size by the same factor makes the jet energy resolution worse by $\sim 12\%$. This is to be expected as in the particle flow paradigm $\approx 30\%$ of jet energy is recorded in the ECal, while only $\approx 10\%$ is recorded in the HCal. Consequently, the potential effect of double counting and omitting energy deposits, i.e. confusion, is greater in the ECal than the HCal. Therefore, minimising confusion in the ECal is expected to be more crucial for the overall jet energy resolution, which is what is observed. Furthermore, as PandoraPFA groups calorimeter hits together using a cone clustering approach, identifying the start of a particle shower is key for determining how calorimeter hits are grouped together deeper
132 Calorimeter Optimisation Studies

Figure 6.9: Contributions to the jet energy resolution shown as function of HCal cell size using the nominal ILD detector model for (a) 45 GeV jets and (b) 250 GeV jets. The black curves correspond to the standard reconstruction, the blue curves to the intrinsic energy resolution contribution to the jet energy resolution, the red curves to the confusion contribution to the jet energy resolution and the magenta curves to the confusion contribution to the jet energy resolution related solely to photon reconstruction.

into the calorimeters. In effect, this means the grouping of calorimeter hits in the HCal depends upon information gathered in the ECal. Therefore, if the ECal performance is sufficiently good, even with coarse HCal cell sizes, excellent performance can be achieved.

In summary, the confusion contribution to the jet energy resolution falls as the HCal cell size is reduced, while the intrinsic energy resolution of the detector is largely unaffected. As this dependancy is relatively weak, even the use of 100 × 100 mm$^2$ HCal cell sizes would be enough to allow for separation of the hadronic decays of W and Z bosons, i.e. $\sigma_E/E \lesssim 3.8\%$ [29], at ILC like energies. However, there are benefits to having smaller HCal cell size; the jet energy resolution is reduced from $\sim 3.5\%$ to $\sim 2.8\%$ for 250 GeV jets when decreasing the HCal cell size from 100 × 100 mm$^2$ to 10 × 10 mm$^2$.

6.3.2 HCal Number of Layers

In this study, the total number of layers in the HCal was varied. In contrast to the longitudinal sampling frequency study, the active and absorber layer thicknesses in the HCal were not altered. Changing the number of layers in this way leads to a change in the total thickness of the calorimeter. This study is sensitive to the effects, if any, of leakage of energy out of the back of the calorimeters. The manufacturing cost of the HCal
is proportional to the number of readout channels and layers. Therefore, minimising
the number of layers, while retaining excellent physics performance is important. Here
detector models were simulated with a HCal containing 36, 42, 48 (nominal), 54 and 60
layers.

It is expected that the energy resolution of the detector will improve when the number
of layers in the HCal is increased since fewer events should suffer from the effects of
leakage. Any improvements seen by increasing the number of layers in the HCal is
expected only up to the point where the majority of hadronic showers are fully contained
by the calorimeters. The energy resolution as a function of number of layers in the HCal
for 50 GeV $K^0_L$ is shown in figure 6.10. The energy resolution becomes worse as the
number of layers in the HCal is reduced below 48 layers, while above this point additional
layers do not change the energy resolution. This indicates that the majority of hadronic
showers at this energy are fully contained by a 48 layer HCal. As reducing the number of
HCal layers to 36 only causes a small degradation, $\sim 10\%$, in the neutral hadron energy
resolution, it is feasible to consider reducing the number of layers in the ILD HCal.

![Figure 6.10](image)

**Figure 6.10:** The energy resolution as a function of number of layers in the HCal for 50 GeV
$K^0_L$ events using the nominal ILD detector model.

Figure 6.11 shows the jet energy resolution as a function of the number of layers in
the HCal. For low energy jets, where intrinsic energy resolution dominates, the jet energy
resolution does not depend on the number of layers in the HCal. At high jet energies,
where confusion dominates, increasing the number of layers in the HCal improves the jet
energy resolution; the jet energy resolution is goes from $\sim 3.4\%$ to $\sim 3.0\%$ for 250 GeV
jets when increasing the number of HCal layers from 36 to 48. The origin of these trends
is leakage of energy out of the back of the calorimeters, which becomes more problematic
as the number of layers in the HCal is reduced and the jet energy increases.
Figure 6.12 shows the jet energy resolution contributions as a function of the number of layers in the HCal. These results appear somewhat counterintuitive in that the intrinsic energy resolution of the detector does not seem to depend on the number of layers in the HCal even for high energy jets. However, this is expected given only 10% of jet energy is carried in the form of neutral hadrons and the neutral hadron energy resolution, for 50 GeV hadrons, is only weakly dependent on the number of HCal layers. Leakage does have an effect on the intrinsic energy resolution, however, the use of \( \text{RMS}_{90} \) obscures part of this by excluding events where leakage is significant. The fractional decrease in \( \text{RMS}_{90} \) for the intrinsic energy distribution when increasing the number of HCal layers from 36 to 60 is \( \sim 4\% \), however, the change in the full RMS is \( \sim 23\% \). Figure 6.12 also shows that the confusion contribution is far more sensitive to the number of layers in the HCal than the intrinsic energy resolution. This sensitivity originates from the reclustering stage of the reconstruction in events where leakage has occurred. In these events, when PandoraPFA compares the momentum of a charged particle track to the cluster of calorimeter hits that it produces, there will be a disparity. To resolve the disparity, PandoraPFA will associate other calorimeter energy deposits that were not produced by the charged particle to the track to compensate for the leaked energy, which produces confusion. As photons are largely contained within the ECal at these energies, the photon confusion contribution to the jet energy resolution has no dependence on the number of layers in the HCal.

In summary, even if the number of layers in the HCal were reduced by 25%, the jet energy resolution would be sufficient for separating the hadronic decays of the W and Z bosons at ILC energies, i.e. \( \sigma_E/E \lesssim 3.8\% \) [29]. Although, the effects of leakage do make the jet energy resolution worse for ILC like energies, once the number of layers in the
Figure 6.12: Contributions to the jet energy resolution shown as function of number of layers in the HCal using the nominal ILD detector model for (a) 45 GeV jets and (b) 250 GeV jets. The black curves correspond to the standard reconstruction, the blue curves to the intrinsic energy resolution contribution to the jet energy resolution, the red curves to the confusion contribution to the jet energy resolution and the magenta curves to the confusion contribution to the jet energy resolution related solely to photon reconstruction.

HCal is reduced from 48 layers, therefore, it is desirable to have a minimum of 48 layers in the ILD HCal.

6.3.3 HCal Longitudinal Sampling Frequency

Several detector models were simulated where the longitudinal sampling frequency in the HCal was modified. The longitudinal sampling frequency was altered by changing the number of layers in the HCal, while simultaneously changing the active and absorber layer thicknesses, to maintain the total number of nuclear interaction lengths. For each model considered, the absorber material was steel, containing a total of 5.72 $\lambda_I$, and the active material was scintillator, containing a total of 0.19 $\lambda_I$. The ratio of the active to absorber layers thicknesses (the sampling fraction) in these models is the same as in the nominal ILD HCal. A summary of the detector models considered is given in table 6.4.

Figure 6.13 shows the energy resolution for 50 GeV $K_L^0$ as a function of number of layers in the HCal. As the number of layers in the HCal is increased, the energy resolution improves. This is because increasing the number of layers in a sampling calorimeter, while leaving the total material budget unchanged, will lead to greater sampling of particles
### Table 6.4: Longitudinal configuration of the HCal in the detector models considered.

<table>
<thead>
<tr>
<th>Number</th>
<th>$N_{\text{Layers HCal}}$</th>
<th>Absorber Thickness [mm]</th>
<th>Active Thickness [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>16.00</td>
<td>2.40</td>
<td></td>
</tr>
<tr>
<td>54</td>
<td>17.78</td>
<td>2.67</td>
<td></td>
</tr>
<tr>
<td>48</td>
<td>20.00</td>
<td>3.00</td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>22.86</td>
<td>3.43</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>26.67</td>
<td>4.00</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>32.00</td>
<td>4.80</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>40.00</td>
<td>6.00</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>53.33</td>
<td>8.00</td>
<td></td>
</tr>
</tbody>
</table>

The energy resolution is less pronounced than the naive expectation of $1/N_{\text{HCal}}$, where $N_{\text{HCal}}$ is the number of layers in the HCal, because this relationship only holds for the energy resolution of a single sampling calorimeter and these results are for the full ILD detector, including the $\approx 1\lambda_I$ in the ECal. Furthermore, the $1/N_{\text{HCal}}$ functional form neglects a number of effects, such as instrumentation defects and electrical noise, that should be included when parameterising the energy resolution [32].

**Figure 6.13:** The energy resolution as a function of the longitudinal sampling frequency in the HCal for 50 GeV $K^0_L$ events using the nominal ILD detector model.

Figure 6.14 shows the jet energy resolution as a function of the longitudinal sampling frequency in the HCal. Increasing the number of layers in the HCal leads to an improvement in the HCal; when the number of layers in the HCal is increased from 18 to 60 the jet energy resolution for 250 GeV jets improves by $\sim 17\%$. 
Figure 6.15 shows that both the intrinsic energy resolution and confusion improve with increasing longitudinal sampling frequency. For 250 GeV jets, when increasing the number of HCal layers from 18 to 60 the intrinsic energy resolution contribution goes from $\sim 1.9\%$ to $\sim 1.6\%$ and the confusion contribution goes from $\sim 3.0\%$ to $\sim 2.4\%$. The twofold improvement is expected because increasing the longitudinal sampling frequency improves the intrinsic energy resolution of a sampling calorimeter, which has the knock-on effect of lowering the confusion. The resulting reduction in confusion is due to the improved precision obtained when comparing the momenta of charged particle tracks and the energy of clusters of calorimeter hits. These comparisons are used to guide event reconstruction in PandoraPFA, therefore, if the precision of these comparisons is improved, the confusion is reduced as described in section 3.6.2.

![Figure 6.14: The jet energy resolution as a function of longitudinal sampling frequency in the HCal for various jet energies using the nominal ILD detector model.](image)

It is clear that a larger number of layers in the HCal benefits both the intrinsic energy resolution of the ILD detector as well as reducing the confusion contribution to the jet energy resolution. As there are few physics analyses that rely on the identification and categorisation of individual neutral hadrons, but there are many that rely on identification and categorisation of photons, the intrinsic energy resolution of the HCal is less crucial from a physics perspective than that of the ECal. However, these studies show the HCal has a crucial role to play in jet reconstruction in the particle flow paradigm. To achieve a jet energy resolution of $\sigma_E/E \lesssim 3.8\%$ [29], which is required to separate the W and Z hadronic decays, the ILD detector will require a minimum of 42 layers in the HCal. This longitudinal sampling frequency is required particularly for low energy jets where the energy resolution is dominated by the intrinsic energy resolution of the detector.
**6.3.4 HCal Sampling Fraction**

The performance of the ILD detector was studied for different ratios of active to absorber layer thicknesses in the HCal. In the nominal detector model, the active scintillator layer thickness is 3 mm, while the absorber layer thickness is 20 mm giving a sampling fraction of 0.15. HCal models were simulated where this ratio was changed from 0.05 to 0.25 in steps of 0.05, while retaining the same number of interaction lengths.

No performance changes in the energy resolution for 50 GeV \(K^0_L\)s or the jet energy resolution for 91, 200, 360 and 500 GeV \(Z\rightarrowuds\) di-jet events were observed when varying the ratio of active to absorber layer thicknesses. Based on these simulations, there is no suggestion that varying this ratio has any statistically significant effect on the physics performance. Although this study indicates that thinning the active layer thickness would not change performance, hardware effects must also be considered to determine whether these conclusions hold true in a real detector. A study into the effects of the readout electronics is required before changing the active layer thicknesses to determine whether a MIP signal can be clearly distinguished when changing the sampling fraction.
6.3.5 HCal Absorber Material

The nominal choice of HCal absorber material is steel with tungsten providing a feasible alternative [41]. Although tungsten is more expensive than steel, it contains a larger number of nuclear interaction lengths per unit length. Therefore, using tungsten as the absorber material would allow for a reduction in the size of the HCal, while retaining the same number of nuclear interaction lengths. Reducing the depth of the calorimeter would decrease the size of the solenoid required, which would offset some of the additional cost of tungsten.

Table 6.5 shows the configuration for the steel and tungsten HCal options that were used in the full ILD simulation. To isolate the effects of changing the absorber material, the total depth, in nuclear interaction lengths, was kept constant when comparing the two options. Furthermore, the sampling fraction was also held constant. A number of different physics lists exist within GEANT4 for the modelling of hadronic showers. The default model for high energy physics calorimetry is the QGSP_BERT physics list. This uses the quark-gluon string model [75] with the precompound model of nuclear evaporation [76] (QGSP) for high energy interactions and the Bertini (BERT) cascade model [77] for intermediate energy interactions. For the study of absorber materials both the QGSP_BERT and the QGSP_BERT_HP physics lists were used. The QGSP_BERT_HP list uses the high precision neutron package (NeutronHP) to deal with the transportation of neutrons from below 20 MeV to thermal energies. This added detail is necessary for accurate modelling of hadronic showers in tungsten [74].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Steel HCal Option</th>
<th>Tungsten HCal Option</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell Size</td>
<td>$30 \times 30 \text{ mm}^2$ square cells</td>
<td>$30 \times 30 \text{ mm}^2$ square cells</td>
</tr>
<tr>
<td>Number of Layers</td>
<td>48 readout layers</td>
<td>48 readout layers</td>
</tr>
<tr>
<td>Absorber Material Thickness [mm]</td>
<td>20.0</td>
<td>12.0</td>
</tr>
<tr>
<td>Active Material Choice</td>
<td>Scintillator</td>
<td>Scintillator</td>
</tr>
<tr>
<td>Active Material Thickness [mm]</td>
<td>3.0</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Table 6.5: The configuration of the steel and tungsten HCal options [34].

One of the dominant processes governing the energy deposition of hadronic showers in calorimeters is spallation [33]. Spallation begins with the collision of a high energy incident particle with nucleons in the calorimeter absorber material. This collision creates a cascade of high energy hadronic particles, e.g. protons, neutrons and pions, within the
nucleus. If these energies are large enough, some of these particles may escape the nucleus and form secondary particles in the hadronic shower. After this initial collision, the nuclei of the absorbing material are left in an excited state. Assuming the excited nuclei are sufficiently stable that they will not undergo fission, they will return to a stable state by ejecting energy in the form of particles in a process called evaporation. Evaporation of neutrons, which is the dominant form of evaporation, significantly delays the growth of hadronic showers as after the evaporation process some of these neutrons participate in neutron capture [74]. Neutron capture involves an absorber nuclei capturing a neutron and then emitting a photon as it returns to a stable state. The time taken for the neutron capture mechanism to proceed is limited by the lifetime of the unstable nuclei [78], which typically makes neutron capture one of the slowest mechanisms by which hadronic showers can propagate. The number of evaporation neutrons released in a hadronic shower increases with the atomic number, Z, of the absorber material of a calorimeter increases. This is because of the increase in neutron content of the absorber material nuclei [74]. As the number of evaporation neutrons increases, more neutron capture processes are initiated, which results in a longer hadronic shower development time. Figure 6.16 shows the shower development times for hadronic showers in the tungsten (Z=74) and steel (iron, Z=26) HCal options and, as expected, the shower development time is greater for tungsten.

![Figure 6.16](image.png)

**Figure 6.16:** The fraction of the total calorimetric energy deposited in the HCal as a function of time for 25 GeV $K^0_L$ events using the steel and tungsten HCal options. Results are shown for both the QGSP_BERT and QGSP_BERT_HP physics lists. The calorimeter hit times have been corrected for straight line time of flight to the impact point.

Table 6.6 shows the energy resolution for 50 GeV $K^0_L$'s obtained using the nominal ILD detector model with various HCal absorber materials and GEANT4 physics lists. In comparison to steel, tungsten option offers an $\sim 8\%$ improvement in the energy resolution.
for 50 GeV neutral hadrons (using the QGSP_BERT_HP physics list). This can be attributed to differences in the nuclear structure of the two materials, which will lead to different developments of the hadronic showers within them. For example, the energy losses to nuclear binding energies are smaller in tungsten than steel, as the target nucleons are less stable than in iron, therefore, less energy is needed to liberate them. This will lead to a larger signal for tungsten and a reduction in the energy resolution in comparison to steel. The results of table 6.6 also indicate that the addition of the high precision neutron package was not important for this study.

<table>
<thead>
<tr>
<th>HCal Option</th>
<th>Energy Resolution [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel, QGSP_BERT</td>
<td>8.8 ± 0.2</td>
</tr>
<tr>
<td>Steel, QGSP_BERT_HP</td>
<td>9.0 ± 0.3</td>
</tr>
<tr>
<td>Tungsten, QGSP_BERT</td>
<td>8.3 ± 0.2</td>
</tr>
<tr>
<td>Tungsten, QGSP_BERT_HP</td>
<td>8.3 ± 0.2</td>
</tr>
</tbody>
</table>

Table 6.6: The energy resolution for 50 GeV K⁰s obtained using the nominal ILD detector with various HCal absorber materials and GEANT4 physics lists. A 100 ns timing cut was applied to the steel and tungsten HCal options in these simulations.

It should be emphasised that the HCal hit energy truncation, as described in chapter 5, used for the tungsten and steel HCal options differs because tungsten contains a larger number of radiation lengths per nuclear interaction length than steel does. As the HCal primarily measures hadronic showers, one may naively expect the number of radiation lengths in the HCal to be irrelevant, given both options have the same number of nuclear interaction lengths. However, this is not the case because all hadronic showers have an electromagnetic component generated by the decays of hadrons to photons, e.g. \( \pi^0 \rightarrow \gamma\gamma \) and \( \eta \rightarrow \gamma\gamma \). This leads to hadronic showers depositing more energy per calorimeter hit in tungsten than in steel and makes retuning the HCal hit energy truncation a necessity. As expected, the truncation used for tungsten, 5 GeV, is larger than for steel, 1 GeV, because of the increased average hit energy.

Table 6.7 shows the jet energy resolutions for selected jet energies obtained using the nominal ILD detector with various HCal absorber materials and GEANT4 physics lists. These results indicate that steel outperforms tungsten as the HCal absorber material. The magnitude of the improvement offered using steel grows as the jet energy increases; the jet energy resolution is \( \sim 3\% \) better for the steel option for 45 GeV jets, while for 250 GeV jets the improvement is \( \sim 11\% \). The intrinsic energy resolution and confusion
contributions to the jet energy resolution for 45 and 250 GeV jets are shown in table 6.8. The intrinsic energy resolution contribution to the jet energy resolution is almost identical for the two HCal options, which is expected because the $K_L^0$ energy resolution was only slightly better for the tungsten option. The tungsten option is unlikely to give a significantly better intrinsic energy resolution because only the small fraction of jet energy associated with neutral hadrons is measured in the HCal. The confusion contribution to the jet energy resolution is larger for tungsten than for steel; for 250 GeV jets the confusion contribution is $\sim 3.4\%$ in tungsten and only $\sim 3.0\%$ in steel. The larger confusion contribution is expected for the tungsten option because hadronic showers are generally wider in tungsten. The transverse profile of hadronic showers in the two HCal options is illustrated in figure 6.17, which shows the normalised distribution of the energy weighted transverse distance from the shower axis to the calorimeter hits for 50 GeV hadronic showers for both the steel and tungsten HCal options. Increasing the average hadronic shower width makes resolving individual particle showers in a dense jet environment more challenging, which means more calorimetric energy deposits will be incorrectly clustered together. This in turn results in incorrect associations being made between calorimetric energy deposits and charged particle tracks i.e. an increased confusion contribution. Again, the use of the QGSP_BERT_HP physics list, as opposed to QGSP_BERT, made a minimal impact on these results.

<table>
<thead>
<tr>
<th>HCal Option</th>
<th>Jet Energy Resolution [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>45 GeV</td>
</tr>
<tr>
<td>Steel, QGSP_BERT</td>
<td>3.65 ± 0.05</td>
</tr>
<tr>
<td>Steel, QGSP_BERT_HP</td>
<td>3.67 ± 0.05</td>
</tr>
<tr>
<td>Tungsten, QGSP_BERT</td>
<td>3.78 ± 0.05</td>
</tr>
<tr>
<td>Tungsten, QGSP_BERT_HP</td>
<td>3.80 ± 0.05</td>
</tr>
</tbody>
</table>

Table 6.7: The jet energy resolution for selected jet energies obtained using the nominal ILD detector with various HCal absorber materials and GEANT4 physics lists. A 100 ns timing cut was applied to the steel and tungsten HCal options in these simulations.

The impact of the choice of HCal absorber material is small on both the neutral hadron energy resolution and intrinsic energy resolution, however, the steel HCal option outperforms the tungsten option in terms of pattern recognition confusion. When examining the mechanical properties of steel and tungsten, it is clear that steel has a significant advantage over tungsten in terms of rigidity [6]. This means that fewer
<table>
<thead>
<tr>
<th>HCal Option</th>
<th>Jet Energy Resolution [%]</th>
<th>45 GeV</th>
<th>250 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Intrinsic</td>
<td>Confusion</td>
</tr>
<tr>
<td>Steel, QGSP_BERT</td>
<td></td>
<td>2.93 ± 0.04</td>
<td>2.16 ± 0.06</td>
</tr>
<tr>
<td>Steel, QGSP_BERT_HP</td>
<td></td>
<td>2.98 ± 0.04</td>
<td>2.15 ± 0.06</td>
</tr>
<tr>
<td>Tungsten, QGSP_BERT</td>
<td></td>
<td>2.97 ± 0.04</td>
<td>2.34 ± 0.06</td>
</tr>
<tr>
<td>Tungsten, QGSP_BERT_HP</td>
<td></td>
<td>2.92 ± 0.04</td>
<td>2.42 ± 0.06</td>
</tr>
</tbody>
</table>

Table 6.8: The contributions to the jet energy resolution obtained using the nominal ILD detector with various HCal absorber materials and GEANT4 physics lists. A 100 ns timing cut was applied to the steel and tungsten HCal options in these simulations.

Figure 6.17: The normalised distribution of the energy weighted transverse distance of the calorimeter hits from a 50 GeV hadronic shower to the shower axis. The blue and red lines show the energy weighted transverse distance obtained using a steel and tungsten HCal absorber material in the ILD detector respectively. The simulations used the QGSP_BERT_HP physics list.

support structures would be required for the calorimeter leading to less dead material and better performance, which makes steel the preferred option.

6.4 Global Detector Parameters

The overall detector size and the magnetic field strength are major cost drivers for the ILD detector. Both will affect the jet energy resolution and studies showing their impact on detector performance are presented here.
6.4.1 The Magnetic Field Strength

In the particle flow paradigm the momentum of charged particles is obtained through the curvature of their trajectory as they bend in the magnetic field. Therefore, the magnetic field is an integral element for the successful application of particle flow calorimetry. Furthermore, the magnetic field deflects charged particles away from neutral particles in jets. The stronger the magnetic field, the larger the average separation between the calorimetric energy deposits made by charged and neutral particles in jets, which reduces the effect of confusion. Therefore, it is expected that a stronger magnetic field will lead to better jet energy resolutions through a reduction of the confusion contribution to the jet energy resolution.

Detector models were simulated where the magnetic field was varied from 1.0 to 5.0 T in steps of 0.5 T and the resulting jet energy resolutions are shown in figure 6.18. The larger the magnetic field strength, the better the jet energy resolution. Increasing the magnetic field strength from 1.0 to 5.0 T improves the jet energy resolution for 250 GeV jets by \( \sim 25\% \). The higher the jet energy, the stronger the dependence of the jet energy resolution on the magnetic field strength.

![Figure 6.18: The jet energy resolution using the nominal ILD detector as a function of the magnetic field strength for various jet energies.](image)

Figure 6.19 shows the breakdown of the jet energy resolution into the various contributions. As expected, there is a reduction in the confusion contribution with increasing magnetic field strength. Furthermore, there is a reduction in intrinsic energy resolution with increasing magnetic field strength for low energy jets. This is most likely due to particles being directed into the forward region of the detector. When a charged particle passes through a magnetic field it will, assuming no energy losses, traverse a helix. The
radius of curvature, \( R \), of that helix is given by

\[
R = \frac{p_T}{qB},
\]

(6.2)

where \( p_T \) is the transverse momentum of the charged particle with respect to the magnetic field, \( q \) is the electric charge of the particle and \( B \) is the magnetic field strength. When the magnetic field strength increases, the radius of curvature for charged particles will decrease and more low charged particles will be directed toward the forward regions of the detector. As the tracking coverage in the forward region of the detector is worse in the central region [34], increasing the magnetic field strength leads to fewer charged particles being reconstructed, which is illustrated in figure 6.20, and a degradation in the energy resolution. For high jet energies, low transverse momentum charged particles will still get directed to the forward regions of the detector, however, these contribute fractionally less energy to the total reconstructed energy. Therefore, the trend of worsening intrinsic energy resolution with increasing magnetic field strength is less pronounced as the jet energy grows.

At high jet energies, reducing the magnetic field strength appears to degrade the intrinsic energy resolution; the intrinsic energy resolution for 250 GeV jets goes from \( \sim 1.8\% \) to \( \sim 2.1\% \) when reducing the magnetic field strength from 5.0 T to 1.0 T. This trend is due to an artefact in the definition of the intrinsic energy resolution for jets meaning it is not a genuine effect. The intrinsic energy resolution is highly non-trivial to determine; Monte-Carlo (MC) information is used to make all associations between charged particle tracks and clusters of calorimeter hits as follows:

1. Each calorimeter hit is associated to the MC particle that deposits the largest amount of energy in that hit;
2. Clusters of calorimeter hits formed by the same MC particle are clustered together;
3. Each charged particle track is associated to the MC particle that produced it;
4. Clusters of calorimeter hits are associated to charged particle tracks if they are made by the same MC particle.

This procedure assumes that only one MC particle deposits significant energy per calorimeter hit. If multiple MC particles deposit significant energy in the same calorimeter hit, this assumption breaks down and errors are made when associating charged particle tracks to calorimetric energy deposits. These errors cause the same double counting and
omission of energy deposits as confusion does, however, they have a smaller effect because multiple MC particles deposit significant energy in the same calorimeter hit is rare in finely segmented calorimeters. As the overlap of particle showers within the calorimeter grows, as it does for high energy jets when reducing the magnetic field, the intrinsic energy resolution appears to get worse because of this confusion-like effect. Because this effect is small in comparison to changes in the confusion contribution, the overall dependence of the detector performance on the magnetic field strength can be confidently quantified.

![Graphs showing contributions to jet energy resolution](image)

**Figure 6.19:** Contributions to the jet energy resolution shown as function of the magnetic field strength using the nominal ILD detector model for (a) 45 GeV jets and (b) 250 GeV jets. The black curves correspond to the standard reconstruction, the blue curves to the intrinsic energy resolution contribution to the jet energy resolution, the red curves to the confusion contribution to the jet energy resolution and the magenta curves to the confusion contribution to the jet energy resolution related solely to photon reconstruction.

In summary, increasing the magnetic field strength is beneficial to the jet energy resolution because it reduces confusion from associating tracks to calorimetric energy deposits from charged particles. The intrinsic energy resolution is also dependent upon the magnetic field strength, however, the effect is small in comparison to the confusion. Although a magnetic field of 1.5 T would give good enough performance to be able to separate the hadronic decays of W and Z bosons, i.e. \( \sigma_E/E \lesssim 3.8\% \) [29], at the energies considered, increasing the field strength further significantly improves the detector performance.
Figure 6.20: The mean number of reconstructed charged particles as a function of the magnetic field strength for 91 GeV $Z \rightarrow uds$ di-jet events. The nominal ILD detector model was used and the pattern recognition has been fully cheated using the MC information.

6.4.2 Inner ECAL Radius

The impact on the jet energy resolution of the overall size of the detector was studied by simulating detector models where the ECAL inner radius was altered. The ECAL inner radii considered were 1208, 1408, 1608, 1808 (nominal) and 2008 mm.

Figure 6.21 shows the dependence of the jet energy resolution on the ECAL inner radius. Increasing the ECAL inner radius increases the separation between particles as they enter the calorimeters, which reduced the effect of confusion and improves the jet energy resolution. As confusion is more dominant at higher energies, the benefits to using a larger ECAL radius grow with increasing jet energy; increasing the ECAL inner radius from 1208 mm to 2008 mm improves the jet energy resolution by $\sim 9\%$ for 45 GeV jets, but by $\sim 25\%$ for 250 GeV jets. Figure 6.22 shows the decomposition of the jet energy resolution into its different components. These results explicitly show a reduction in confusion with increasing ECAL inner radius; the confusion contribution goes from $\sim 3.4\%$ to $\sim 2.4\%$ when increasing the ECAL inner radius from 1208 mm to 2008 mm. The intrinsic energy resolution of the detectors shows no strong dependence on the inner ECAL radius. The apparent degradation in intrinsic energy resolution at low ECAL inner radii is an artefact of the association of a single MC particle per calorimeter cell when running the cheated pattern recognition as explained in section 6.4.1. The dominant effect driving the jet energy resolution is, as expected, the confusion.
Figure 6.21: The jet energy resolution using the nominal ILD detector as a function of the ECal inner radius for various jet energies.

Figure 6.22: Contributions to the jet energy resolution shown as function of the ECal inner radius using the nominal ILD detector model for (a) 45 GeV jets and (b) 250 GeV jets. The black curves correspond to the standard reconstruction, the blue curves to the intrinsic energy resolution contribution to the jet energy resolution, the red curves to the confusion contribution to the jet energy resolution and the magenta curves to the confusion contribution to the jet energy resolution related solely to photon reconstruction.

In conclusion, increasing the ECal inner radius benefits the jet energy resolution because it increases the separation between particles as they enter the calorimeter, which reduces confusion.
6.5 Summary

The effect of varying the configuration of the calorimeters, the magnetic field strength and the overall detector size on the single particle and jet energy resolutions were presented in this chapter. For both the ECal and the HCal, the dominant factor determining the intrinsic energy resolution was the longitudinal sampling frequency. However, the jet energy resolution had the strongest sensitivity to the ECal cell size, which shows that spatial recognition is more important when using particle flow calorimetry than intrinsic energy resolution. The HCal cell size was found to be less significant than the ECal cell size for determining the jet energy resolution because separation of nearby particle showers in the HCal uses the spatial information gathered in the ECal. In the particle flow paradigm, fine segmentation in the ECal can compensate for the coarser HCal granularities. The jet energy resolution also showed a strong dependence on the magnetic field strength and the overall detector size. Increasing both the magnetic field and overall detector size leads to greater separation of nearby particle showers in the calorimeters, which reduces the effect of confusion.

Optimal performance for the ILD detector in terms of the jet energy resolution will be achieved using the smallest possible cell sizes in the calorimeters. However, reducing the cell size in either the ECal or HCal will significantly increase their manufacturing costs. Reducing the cell size by a factor of $r$ increases the number of readout channels in a calorimeter by a factor of $r^2$. Therefore, in order to reduce the ECal cell size in the ILD detector from $5 \times 5$ mm$^2$ to $3 \times 3$ mm$^2$, the cost of manufacturing the ECal will rise by a factor of nearly three. As the ECal is the most expensive ILD sub-detector in terms of manufacturing costs [34], significantly increasing the cost is highly undesirable. The manufacturing costs of the ILD HCal are approximately a third that of the ECal, which means that reducing the cell size would also add significant cost. Therefore, while there is margin for improving the jet energy resolution by reducing the calorimeter cell sizes, the additional cost is likely to make this unfeasible.

Similarly, optimal performance for the ILD detector in terms of intrinsic energy resolution will be achieved by maximising the number of layers in the calorimeters. However, increasing the number of layers will linearly increase the number of readout channels and the cost of the calorimeters. As the dependence of the intrinsic energy resolution on the number of layers is weak, the physics benefits obtained from increasing the number of layers is unlikely to merit the significant cost increase that would be incurred.
Chapter 7

The Sensitivity of CLIC to Anomalous Gauge Couplings through Vector Boson Scattering

“Does not wisdom call out? Does not understanding raise her voice?”
— Proverbs 8:1

7.1 Motivation

Vector boson scattering is the interaction of the form $VV \rightarrow VV$ where $V$ is any of the electroweak gauge bosons $W^+, W^-, Z$ or $\gamma$. This is an interesting process to study because it provides understanding of how the Standard Model Higgs is able to unitarise the otherwise unbounded cross-section for longitudinal massive gauge boson scattering. Vector boson scattering also provides insights into beyond Standard Model physics that impacts the electroweak sector by probing potential anomalous triple and quartic gauge couplings.

Triple and quartic gauge couplings lead to interactions of the form $V \rightarrow VV$ and $VV \rightarrow VV$, respectively. In the Standard Model there are five allowed vertices, shown in figure 7.1, which arise from the kinematic term $L_{kin} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W_{\mu\nu} W^{\mu\nu}$ in the Standard Model Lagrangian.
The Sensitivity of CLIC to Anomalous Gauge Couplings through Vector Boson Scattering

![Diagram of vector boson scattering](image)

**Figure 7.1:** Triple and quartic gauge boson vertices in the Standard Model.

Anomalous triple and quartic gauge couplings are introduced as parameters in effective field theories (EFTs). These couplings either modify the Standard Model triple and quartic gauge boson vertices or introduce new triple and quartic vertices that were previously forbidden. EFTs are a mathematical construct designed to introduce new physics in a manner that builds upon the Standard Model. They work under the assumption that new physics exists at an energy scale, $\Lambda$, that is much higher than the energy scales currently accessible to modern day particle physics experiments. In the limit $\Lambda \rightarrow \infty$, the Standard Model is reproduced as the new physics becomes kinematically inaccessible. Such theories are model independent, giving them a wide span in the search for new physics. A classic example of an EFT theory is the Fermi theory for beta decay [79]. At energies much below the mass of the W boson, the weak interaction occurring when a neutron decays into a proton, electron and anti-neutrino can be treated as a four-point vertex with quartic coupling strength $G_F$, the Fermi Coupling constant as shown in figure 7.2.

The study presented in this chapter examines the anomalous quartic gauge couplings $\alpha_4$ and $\alpha_5$ through vector boson scattering process. The anomalous gauge couplings that are to be examined are introduced as part of an EFT that is described in chapter 2. The anomalous gauge couplings $\alpha_4$ and $\alpha_5$ appear in the Lagrangian through the following
The Sensitivity of CLIC to Anomalous Gauge Couplings through Vector Boson Scattering

\[ d \rightarrow G_F \rightarrow u \]
\[ e^- \]
\[ \bar{\nu}_e \]

Figure 7.2: Four-point vertex proposed for explanation of beta decay by Fermi.

terms

\[ \alpha_4 [\text{Tr}(V^{\mu}V_{\mu})]^2 \quad \text{and} \quad \alpha_5 \text{Tr}(V^{\mu}V_{\nu})\text{Tr}(V^{\nu}V_{\mu})] \],

(7.1)

where \( V_{\mu} \) corresponds, in a carefully chosen gauge, to a linear combination of the massive gauge bosons \( W^+, W^- \) and \( Z \). These terms modify the Standard Model vertices \( W^+W^- \rightarrow W^+W^- \) and \( W^+W^- \rightarrow ZZ \) as well as introducing the new vertex \( ZZ \rightarrow ZZ \). The anomalous gauge couplings \( \alpha_4 \) and \( \alpha_5 \) can be studied in vector boson scattering processes such as those shown in figure 7.3.

CLIC is designed for precision measurements in \( e^+e^- \) collisions at high energies and it is ideal for a study of vector boson scattering. The application of Particle Flow Calorimetry with fine granularity calorimeters gives CLIC excellent jet energy resolution, which allows it to clearly characterise multi-jet final states and final states containing missing energy in the form of neutrinos. The excellent jet energy resolution also allows for accurate separation of \( W \) and \( Z \) bosons through di-jet invariant mass, which will be invaluable for event selection.

The cross-sections for vector boson scattering processes are sufficiently large at the proposed running energies for CLIC to give large signal sample size. A study of anomalous gauge boson couplings at CLIC has the potential to give results several orders of magnitude better than the complementary studies performed at the LHC because of the reduction in hadronic backgrounds and increased cross-section for vector boson scattering processes. To date, the tightest limits on the anomalous gauge couplings \( \alpha_4 \)
The Sensitivity of CLIC to Anomalous Gauge Couplings through Vector Boson Scattering

Figure 7.3: Example of vector boson scattering Feynman diagrams showing sensitivity to quartic gauge boson self-interaction vertices. The processes shown are relevant for CLIC. In these diagrams q represents the u, d, s, c and b quarks; l represents $e^-$, $\mu^-$ and $\tau^-$ leptons; and $\nu$ represents the $\nu_e$, $\nu_\mu$ and $\nu_\tau$ neutrinos.

and $\alpha_5$ are provided by the ATLAS collaboration [80]. That analysis examined $W^\pm V jj$ events, where V is either $W^\pm$ or Z, using $\sim 20$ fb$^{-1}$ of data at $\sqrt{s} = 8$ TeV and found 95% confidence limits on the anomalous gauge couplings of:

$$-0.024 < \alpha_4 < 0.030, \quad (7.2)$$

$$-0.028 < \alpha_5 < 0.033. \quad (7.3)$$

As CLIC can offer a higher sensitivity to vector boson scattering processes than the LHC, there is a strong case for performing an anomalous gauge coupling analysis via vector boson scattering at CLIC.
The branching fractions for the hadronic decays of both the $W^\pm$ and $Z$ bosons is of the order of 70% [13], therefore, the signal final states for the analysis presented in this chapter are vector boson scattering processes where the outgoing bosons decay purely hadronically: $\nu\nuqqqq$, $\nu lqqqq$ and $llqqqq$.

7.2 Event Generation, Simulation and Reconstruction

Events were generated using Whizard [81, 82] version 1.95. Due to the presence of beamstrahlung photons in the CLIC beam, events were generated for collisions of $e^+e^-$, $e^+\gamma$, $\gamma e^-$ and $\gamma\gamma$. The energy spectra used for all particles involved in these collisions took into account the effects of radiation in the form of beamstrahlung photons and the intrinsic energy spread of the CLIC beam. Furthermore, events involving the interaction between the electromagnetic field of the beam particles involving quasi-real photon mediators with low momenta, described by the Weizsacker-Williams approximation [83, 84] or the Equivalent Photon Approximation (EPA), were generated using Whizard and included in this analysis. Fragmentation and hadronisation was implemented using PYTHIA 6.4 [53] that was tuned for OPAL $e^+e^-$ collision data recorded at LEP [54]. The decays of tau leptons was simulated using TAUOLA [55]. The full list of events used in this analysis, along with their Standard Model cross-section at $\sqrt{s} = 1.4$ TeV can be found in table 7.1. The samples comprise all final states that are relevant, either as signal or background processes, for an analysis involving the purely hadronic decay channels of the vector boson scattering process:

- Final states from the purely hadronic decay channels of the vector boson scattering process. These states are expected to show sensitivity to the anomalous couplings $\alpha_4$ and $\alpha_5$: $e^+e^- \rightarrow \nu\nuqqqq$, $e^+e^- \rightarrow \nu lqqqq$ and $e^+e^- \rightarrow llqqqq$

- Final states with four primary quarks arising from $e^+e^-$ interactions: $e^+e^- \rightarrow qqqq$.

- Final states with two primary quarks arising from $e^+e^-$ interactions: $e^+e^- \rightarrow \nu\nuqq$, $e^+e^- \rightarrow \nu lqq$, $e^+e^- \rightarrow llqq$ and $e^+e^- \rightarrow qq$.

- Final states with four primary quarks arising from the interactions of either $e^+$ or $e^-$ with a beamstrahlung photon: $e^-\gamma_{BS} \rightarrow e^-qqqq$, $e^+\gamma_{BS} \rightarrow e^+qqqq$, $e^-\gamma_{BS} \rightarrow \nu_eeqqq$ and $e^+\gamma_{BS} \rightarrow \nu_eeqqq$. 
• Final states with four primary quarks arising from the interactions of either $e^+$ or $e^-$ with the electromagnetic field of the opposing beam particle. These cross-sections are calculated using the EPA approximation, which represents the electromagnetic field of the opposing beam particle as a series of photons, so the final states appear as interactions of $e^+$ or $e^-$ with photons: $e^-\gamma_{\text{EPA}} \rightarrow e^-qqqq$, $e^+\gamma_{\text{EPA}} \rightarrow e^+qqqq$, $e^-\gamma_{\text{EPA}} \rightarrow \nu_eqqqq$ and $e^+\gamma_{\text{EPA}} \rightarrow \bar{\nu}_eqqqq$.

• Final states with four primary quarks arising from the interaction of the electromagnetic fields of opposing beam particles using the EPA approximation: $\gamma_{\text{EPA}}\gamma_{\text{EPA}} \rightarrow qqqq$.

• Final states with four primary quarks arising from the interaction of the electromagnetic field of either $e^+$ or $e^-$ using the EPA approximation with a beamstrahlung photon: $\gamma_{\text{EPA}}\gamma_{\text{BS}} \rightarrow qqqq$ or $\gamma_{\text{BS}}\gamma_{\text{EPA}} \rightarrow qqqq$.

• Final states with four primary quarks arising from the interaction of two beamstrahlung photons: $\gamma_{\text{BS}}\gamma_{\text{BS}} \rightarrow qqqq$.

In the above list $q$ represents $u$, $\bar{u}$, $d$, $\bar{d}$, $s$, $\bar{s}$, $c$, $\bar{c}$, $b$ or $\bar{b}$; $l$ represents $e^\pm$, $\mu^\pm$ or $\tau^\pm$; and $\nu$ represents $\nu_e$, $\bar{\nu}_e$, $\nu_\mu$, $\bar{\nu}_\mu$, $\nu_\tau$ and $\bar{\nu}_\tau$.

Monte-Carlo (MC) samples were simulated using the CLID_ILD detector model [35]. Further details of this detector model can be found in chapter 3. The simulation was performed in MOKKA [44], which is a GEANT4 [45] wrapper providing detailed geometric descriptions of detector concepts for the linear collider. Events were reconstructed using the MARLIN [51] c++ framework, designed for reconstruction at the linear collider. PandoraPFA [29,31] was used to apply Particle Flow Calorimetry in the reconstruction, the full details of which can be found in chapter 3. To ensure consistency with the pre-existing background samples provided by the CLIC collaboration, all reconstructed events in this analysis used a fixed calibration as opposed to the procedure discussed in chapter 5. For the same reason, neither software compensation or the HCal hit energy truncation was applied in this analysis. The sensitivity of CLIC to the anomalous gauge couplings reported in this analysis would benefit from the application of both the calibration procedure and software compensation.

The effect of the $\gamma\gamma \rightarrow \text{hadrons}$ backgrounds, discussed in section 1.1.3.2, were incorporated in the analysis by overlaying $\gamma\gamma \rightarrow \text{hadrons}$ events onto the signal and background event samples. The overlaid backgrounds were added prior to reconstruction so that their impact on the reconstruction was fully accounted for. For each physics
Final State | Cross-section [fb]
---|---
e^+e^- → ννqqqq | 24.7
e^+e^- → νlqqqq | 110.4
e^+e^- → llqqqq | 62.1
e^+e^- → qqqq | 1245.1
e^+e^- → ννqq | 787.7
e^+e^- → νlqq | 4309.7
e^+e^- → llqq | 2725.8
e^+e^- → qq | 4009.5
e^-γEPA → e^-qqqq | 287.1
e^-γBS → e^-qqqq | 1160.7
e^+γEPA → e^+qqqq | 286.9
e^+γBS → e^+qqqq | 1156.3
e^-γEPA → νeqqqq | 32.6
e^-γBS → νeqqqq | 136.9
e^+γEPA → νeqqqq | 32.6
e^+γBS → νeqqqq | 136.4
γEPAγEPA → qqqq | 753.0
γEPAγBS → qqqq | 4034.8
γBSγEPA → qqqq | 4018.7
γBSγBS → qqqq | 21406.2

Table 7.1: Cross-sections of signal and background processes at $\sqrt{s} = 1.4$ TeV. In the above table q represents u, ū, d, d̄, s, s̄, c, c̄, b or b̄; l represents e±, μ± or τ±; and ν represents νe, νe, νμ, νμ, ντ and ντ. The EPA and BS subscript on the incoming photon indicates whether the photon is generated from the equivalent photon approximation or beamstrahlung.

event of interest, $γγ → hadrons$ background events equivalent to 60 bunch crossings (BXs) are included. As readout time windows are applied in detector readout, 60 BXs is sufficient for accounting for the $γγ → hadrons$ backgrounds. These backgrounds occur in a time window of −5 ns to 25 ns around the physics event and the BXs are separated by 0.5 ns, to mimic the CLIC bunch train structure. The number of background events overlaid per BX is drawn from a Poisson distribution with a mean of 1.3 (3.2) events per bunch crossing at $\sqrt{s} = 1.4$ (3) TeV [6].
Detector readout is simulated using a readout time window of 10 ns on all detectors apart from the TPC and HCal barrel. In the TPC, all hits are retained and in the HCal barrel a 100 ns time window is used to account for the additional time it takes hadronic showers to develop in tungsten [6]. All readout times are corrected for straight time-of-flight to the impact point (IP). Any hits that have are measured outside of these windows are not used in the reconstruction.

7.3 Modelling of Anomalous Gauge Couplings

The samples that were sensitive to the anomalous gauge couplings $\alpha_4$ and $\alpha_5$ were generated using Whizard version 1.97, instead of the previously quoted version 1.95. This change was required as version 1.97 contained a unitarisation scheme that ensured cross-sections for processes involving longitudinal gauge boson scattering did not violate unitarity at the energies considered here.

Two alternative methods exist for modelling the sensitivity of the vector boson scattering process to the anomalous gauge couplings $\alpha_4$ and $\alpha_5$. The first is to generate multiple samples with different values of $\alpha_4$ and $\alpha_5$ and the second is to generate a single sample with $\alpha_4 = 0$ and $\alpha_5 = 0$ and reweight that sample. The latter approach was taken in this analysis as the former approach is impractical when considering a fine sampling of the $\alpha_4$ and $\alpha_5$ space.

Event weights, $w$, are calculated according to the ratio of the matrix elements, $M$, for the particular event configuration [85]

$$w(\alpha_4, \alpha_5) = \frac{|M(event, \alpha_4, \alpha_5)|^2}{|M(event, 0, 0)|^2}.$$  \hfill (7.4)

Figure 7.4 shows the dependence of the event weights on $\alpha_4$ and $\alpha_5$ for four individual $\nu\nuqqqq$ final state events, generated at $\sqrt{s} = 1.4$ TeV.

Only final states involving contributions from massive gauge boson quartic vertices require reweighting. Whizard was used to evaluate the cross-sections for all final states shown in table 7.1 with $\alpha_4 = \alpha_5 = 0$ and with $\alpha_4 = \alpha_5 = 0.05$. Only the three final states shown in table 7.2 were found to have a dependency on $\alpha_4$ and $\alpha_5$. 
To maximise the sensitivity to the anomalous gauge couplings, the $\nu\nuqqqq$ final state is used to define signal in this analysis. The $\nu qqqq$ and $llqqqq$ final states are treated as backgrounds that are invariant to changes in $\alpha_4$ and $\alpha_5$ because they have a much
reduced sensitivity to the anomalous gauge couplings in comparison to the $\nu\nu qq qq$ final state. Furthermore, the $\nu lqqq$ and $llqqq$ final states can be easily vetoed during event selection because of the presence of the primary lepton. This means the sensitivity of the $\nu lqqq$ and $llqqq$ final states to the anomalous gauge couplings will have a negligible effect on the results from this study.

Use of the unitarisation scheme in Whizard 1.97, which is needed to ensure cross-sections do not violate unitarity when studying anomalous gauge couplings at CLIC like energies, requires a unit CKM matrix [85]. The impact of this requirement was examined by comparing several reconstructed and MC distributions for $\nu\nu qq qq$ final state events generated with Whizard using a Standard Model and unit CKM matrix. No significant differences were observed, which indicates that enforcing a unit CKM matrix when generating the $\nu\nu qq qq$ final state samples did not significantly affect this analysis.

7.4 Data Analysis

The following section contains a description of how the variables used throughout the anomalous gauge coupling sensitivity study were determined.

7.4.1 Limiting Beam Related Backgrounds

During the reconstruction, after the inner detector tracks have been reconstructed, the CLICTrackSelection processor [31] is applied, which vetoes poorly reconstructed and fake tracks by applying simple quality cuts to the number of hits in the tracking sub-detectors. The CLICTrackSelection processors also reject tracks where the time of arrival at the calorimeter differs by more than 50 ns between a straight line of flight and a helix fit to the track. Applying this cut ensures that associations made between charged particles tracks and calorimetric energy deposits are consistent.

Following the reconstruction, the CLICPfoSelector processor [31] is applied to remove reconstructed particle flow objects (PFOs) that originate from beam related backgrounds. This processor applies cuts on the $p_T$ and timing information of the PFOs, which vary as a function of position in the detector and the PFO type to target regions of the detector where backgrounds are more prominent, e.g. low $p_T$ for $\gamma\gamma \rightarrow$ hadrons events. Three configurations of the CLICPfoSelector have been developed for the CLIC environment and
were considered in this analysis. They are, in order of increasing background rejection, the Loose, Default and Tight selections [31].

### 7.4.2 Jet Finding

After the application of the CLICPfoSelector, the MarlinFastJet processor, a wrapper for the FastJet [86] processor, was used to cluster each event into four jets. These jets are then paired up to form two candidate bosons working under the assumption that the correct pairing is achieved when the difference between the invariant masses of the candidate bosons is a minimum. In the case of the signal final state, $\nu\nuqqqq$, it is assumed that the four jets and two candidate bosons map onto the four primary quarks and two outgoing bosons in the vector boson scattering process. The jet clustering was performed using the longitudinally invariant $k_t$ jet algorithm [87, 88] in exclusive mode. The longitudinally invariant $k_t$ algorithm proceeds as follows:

1. Determine the $k_t$ distance, $d_{ij}$, for each pair of particles, $i$ and $j$, and the beam, $d_{iB}$, distance for each particle, $i$. These distances are defined as

   $d_{ij} = \min(p_{ti}^2, p_{tj}^2) \Delta R_{ij}^2 / R^2$, \hspace{1cm} (7.5)

   $d_{iB} = p_{ti}^2$, \hspace{1cm} (7.6)

   where $\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$, $p_{ti}$ is the transverse momentum of particle $i$, $y_i$ is the rapidity of particle $i$, $\phi_i$ is the azimuthal angle of the direction of travel of particle $i$ and $R$ is a configurable parameter that typically is of the order of 1.

2. Find the minimum distance, $d_{\text{min}}$, of all the $k_t$ and beam distances. If the minimum occurs for a $k_t$ distance, particles $i$ and $j$ are merged, summing their 4-momenta. If the beam distance is the minima, particle $i$ was declared to be part of the "beam" jet and the particle is removed from the list of particles and not included in the final jet output.

3. Repeat until the desired number of jets is created. Alternatively, in inclusive mode this would be repeated until no particles are left in the event.

Two other clustering algorithms were considered, however, they were found to be inappropriate for the experimental conditions at CLIC. These alternative algorithm choices are applied in the same manner as the longitudinally invariant $k_t$ algorithm, however, they differ in the definition of $d_{ij}$ and $d_{iB}$. Figure 7.5 shows the distribution of the invariant...
The Sensitivity of CLIC to Anomalous Gauge Couplings through Vector Boson Scattering

The mass of the candidate bosons at $\sqrt{s} = 1.4$ TeV $\nu\nu q\bar{q}q$ final state events for each of the jet algorithms considered. The candidate boson masses are determined by forcing the events into 4 jets and then pairing the jet pairs to form candidate bosons. The jet pairing configuration is determined by pairing jets such that the mass differences between the two candidate bosons is a minimum.

![Invariant Mass of Candidate Boson](image)

**Figure 7.5:** The reconstructed masses for different choices of jet algorithm at $\sqrt{s} = 1.4$ TeV $\nu\nu q\bar{q}q$ final state events. These samples should be dominated by vector boson scattering involving pairs of outgoing $W$ bosons and so it is expected that a peak at the $W$ boson mass, $m_W = 80.385 \pm 0.015$ GeV [13], should be observed. In the case of the $k_t$ algorithm and the $e^+e^-k_t$ algorithm an R parameter of 0.7 was used. All distributions show raw number of events.

The first alternative jet algorithm considered was the $k_t$ algorithm for $e^+e^-$ colliders [89], the $e^+e^-k_t$ or Durham algorithm. In this algorithm $d_{iB}$ is not used and

$$d_{ij} = 2\min(E_i^2, E_j^2)(1 - \cos\theta_{ij}) \quad (7.7)$$

where $\theta_{ij}$ is the opening angle of particles $i$ and $j$ and $E_i$ is the energy of particle $i$. In the collinear limit $d_{ij}$ corresponds to the relative transverse momenta of the particles. The major failure of this algorithm when applied to CLIC is the absence of $d_{iB}$, which leads to large numbers of beam related background particles being associated to jets. As figure 7.5 shows, the invariant mass of the paired jets, which should peak around the $W$ and $Z$ boson masses, is much larger than expected, due to the presence of these backgrounds. Also this algorithm is not invariant to boosts along the beam direction.
meaning that it is inappropriate for use at CLIC given the beam induced backgrounds modify the nominal collision kinematics.

The second alternative jet algorithm considered was the Cambridge-Aachen jet algorithm \cite{90} where

\[
d_{ij} = \Delta R_{ij}^2 / R^2, \quad (7.8)
\]

\[
d_{iB} = 1. \quad (7.9)
\]

This algorithm performs poorly as it does not account for the transverse momentum or the energy of the particles being clustered. In essence, this is a cone clustering algorithm with a cone radius defined through $\Delta R_{ij} = R$, which even for large R was found to discard too much energy in the event to be useful for this analysis. This can be seen in figure 7.5 where the invariant mass of the paired jets is much lower than expected. This algorithm is appropriate for events that contain highly boosted jets, however, at CLIC the jets are too disperse for this algorithm to be successful.

### 7.4.2.1 Optimal Jet Finding Algorithm

Optimisation of the jet finding procedure was performed on both the PFO selection and the value of the R parameter used in the longitudinally invariant $k_t$ algorithm. The optimisation procedure involved performing the sensitivity study, described in section 7.6, using solely the $\nu\nuqqqq$ signal final state. This methodology ensures that the optimisation was done with respect to the physics of interest without having to perform the jet reconstruction for the large number of background events for each jet algorithm configuration considered.

Table 7.3 shows the 68% confidence limits on the measurement of $\alpha_4$ and $\alpha_5$ obtained using the $\nu\nuqqqq$ signal final state only at $\sqrt{s} = 1.4$ TeV for different jet algorithm configurations. These confidence limits represent the idealised sensitivity of the CLIC experiment to the anomalous gauge couplings. Once the effects of backgrounds and event selection are included in the analysis, these confidence limits will increase in size.

The configuration for the jet algorithm for the $\sqrt{s} = 1.4$ TeV analysis was chosen as selected PFOs with an R parameter of 0.9. While the loose PFO selection gives a marginally better performance, the selected PFO selection was preferred to minimise the effect of the $\gamma\gamma \rightarrow hadrons$ background. Figure 7.6a shows confidence contours,
The Sensitivity of CLIC to Anomalous Gauge Couplings through Vector Boson Scattering

Table 7.3: The 68% confidence limits on the measurement of $\alpha_4$ and $\alpha_5$ obtained using the $\nu\nuqqqq$ signal final state only at $\sqrt{s} = 1.4$ TeV for different jet algorithm configurations.

<table>
<thead>
<tr>
<th>R Parameter</th>
<th>Tight Selected PFOs</th>
<th>Selected PFOs</th>
<th>Loose Selected PFOs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>$-0.0039 &lt; \alpha_4 &lt; 0.0051$</td>
<td>$-0.0035 &lt; \alpha_4 &lt; 0.0047$</td>
<td>$-0.0037 &lt; \alpha_4 &lt; 0.0047$</td>
</tr>
<tr>
<td></td>
<td>$-0.0027 &lt; \alpha_5 &lt; 0.0031$</td>
<td>$-0.0025 &lt; \alpha_5 &lt; 0.0031$</td>
<td>$-0.0024 &lt; \alpha_5 &lt; 0.0028$</td>
</tr>
<tr>
<td>0.9</td>
<td>$-0.0036 &lt; \alpha_4 &lt; 0.0047$</td>
<td>$-0.0035 &lt; \alpha_4 &lt; 0.0045$</td>
<td>$-0.0035 &lt; \alpha_4 &lt; 0.0045$</td>
</tr>
<tr>
<td></td>
<td>$-0.0026 &lt; \alpha_5 &lt; 0.0031$</td>
<td>$-0.0023 &lt; \alpha_5 &lt; 0.0027$</td>
<td>$-0.0022 &lt; \alpha_5 &lt; 0.0027$</td>
</tr>
<tr>
<td>1.1</td>
<td>$-0.0036 &lt; \alpha_4 &lt; 0.0047$</td>
<td>$-0.0036 &lt; \alpha_4 &lt; 0.0048$</td>
<td>$-0.0026 &lt; \alpha_4 &lt; 0.0046$</td>
</tr>
<tr>
<td></td>
<td>$-0.0026 &lt; \alpha_5 &lt; 0.0031$</td>
<td>$-0.0025 &lt; \alpha_5 &lt; 0.0029$</td>
<td>$-0.0024 &lt; \alpha_5 &lt; 0.0028$</td>
</tr>
</tbody>
</table>

given a null hypothesis of $\alpha_4 = \alpha_5 = 0$, for the selected PFO and R parameter of 0.9 jet algorithm configuration at $\sqrt{s} = 1.4$ TeV. Figures 7.6b and 7.6c show the one dimensional $\chi^2$ distribution for $\alpha_4$ and $\alpha_5$, assuming $\alpha_5 = 0$ and $\alpha_4 = 0$, respectively, for the same configuration.

### 7.4.3 Lepton Finding

An isolated lepton finder [91] was included in the analysis chain to reject background final states containing primary leptons. Leptons produced via hadronisation are unlikely to be flagged as isolated because all hadronisation products are boosted along the direction of the parent quark. This means isolated leptons are likely to correspond to primary leptons, which makes the number of isolated leptons a powerful discriminating variable to use in event selection.

The isolated lepton finder determines whether a PFO is an electron or muon by first checking that the PFO has a single charged particle track associated to it. If that is the case, the calorimetric energy deposits of the PFO are examined to see if they are consistent with what is expected for an electron or muon. If they are consistent with expectations, the properties of the charged particle track are examined to determine whether the track originates from the IP. If the PFO is deemed to have originated from the IP and has an energy greater than 15 GeV then it is counted as an isolated lepton. Table 7.4 shows the fraction of events containing at least one isolated lepton for selected final states at $\sqrt{s} = 1.4$ TeV.
Figure 7.6: $\chi^2$ sensitivity distributions from a fit to $M_{VV}$ for the signal $\nu\nuqqqq$ final state only at $\sqrt{s} = 1.4$ TeV. These results use the optimal jet algorithm configuration of selected PFOs and an R parameter of 0.9 in the $k_t$ algorithm. (a) $\chi^2$ sensitivity contours in $\alpha_4$ and $\alpha_5$ space. (b) $\chi^2$ as a function of $\alpha_4$ assuming $\alpha_5 = 0$. (c) $\chi^2$ as a function of $\alpha_5$ assuming $\alpha_4 = 0$. All distributions are normalised to an integrated luminosity of $L_{int} = 1.5$ ab$^{-1}$. 
Table 7.4: The fraction of events containing at least one isolated lepton for the $\nu\nuqqqq$ and $\nu lqqqq$ final states at $\sqrt{s} = 1.4$ TeV.

7.4.4 Discriminant Variables

The next stage of the analysis involved the calculation of a number of event-based variables that were found to be useful for this analysis. The variables that were calculated are as follows

- **Particle level** variables:
  - number of PFOs in each jet;
  - energy of the highest energy PFO;
  - energy of the highest energy electron that was identified by PandoraPFA;
  - cosine of the polar angle of the highest energy track;
  - the number of isolated leptons found using the isolated lepton finder.

- **Candidate boson** variables:
  - energy of the candidate bosons;
  - invariant mass of the candidate bosons;
  - acolinearity of the candidate boson pair, which is defined as 180 degrees minus the opening angle of the pair of bosons in the rest frame of the detector.

- **Event based** variables:
  - the invariant mass of the visible system, $M_{VV}$;
  - the vector sum of the transverse momentum of all PFOs in the event;
  - sphericity, defined through the sphericity tensor $S^{ab}$,
  \[
  S^{ab} = \frac{\sum_{i} p_i^a p_i^b}{\sum_{i,a=x,y,z} |p_i^a|^2},
  \]
where \( p_i \) are the components of the momenta of the \( i^{th} \) PFO in the rest frame of the detector and the sum \( \Sigma \) runs over all particles in the event. Sphericity is defined as \( S = (3/2)(\lambda_2 + \lambda_3) \), where \( \lambda_i \) are the eigenvalues of the sphericity tensor defined such \( \lambda_1 \geq \lambda_2 \geq \lambda_3 \). This provides a measure of how spherical the reconstructed event topology is with isotropic events having \( S \approx 1 \), while two jet events have \( S \approx 0 \).

- **Jet clustering parameters** variables:
  - the \( y_{ij} \) variables where \( i = 3, 4 \) and \( j = i + 1 \). These are the smallest \( k_t \) distance found when combining \( j \) jets into \( i \) jets.

### 7.4.5 Jet Energy Resolution at CLIC

The importance of the jet energy resolution, which is extensively discussed in chapters 5 and 6, should be emphasised at this point. Many of the discriminant variables that are calculated for this analysis are dependant upon the jet energy resolution. In particular, all variables related to the candidate bosons, which are formed from pairing up jets, are dependent upon the measurement of jet energies.

Figure 7.7 shows the jet energy resolution as a function of the MC jet energy for the \( \nu \nu qqqq \) event sample used in the \( \sqrt{s} = 1.4 \) TeV analysis. The MC jet energy was obtained by pairing up quarks appearing in the final state to the reconstructed jets. The events were then binned in terms of their MC jet energy and the jet energy resolution calculated for each bin. When calculating the jet energy resolution, a narrower range of jet energies was used in compared to previous studies, 60% of the data with narrowest RMS as opposed to 90%, to minimise the effects of jet finding and beam-induced backgrounds. The jet energy resolutions reported here are marginally worse than those quoted in earlier chapters, however, this is to be expected given that the calibration procedure and software compensation techniques used in earlier chapters are not applied here.

### 7.5 Event Selection

This section discusses the event selection procedure. The goal of this procedure is to isolate the \( \nu \nu qqqq \) final state from the background final states, i.e. those containing two and four primary quarks. The procedure consists of a set of preselection cuts...
followed by the application of a multivariate analysis (MVA). All event numbers have been normalised, prior to event selection, to an integrated luminosity of $L_{\text{int}} = 1.5 \text{ ab}^{-1}$ for the $\sqrt{s} = 1.4 \text{ TeV}$ analysis and $L_{\text{int}} = 2 \text{ ab}^{-1}$ for the $\sqrt{s} = 3 \text{ TeV}$ analysis. These luminosities correspond to CLIC operating for approximately four to five years at each energy stage.

### 7.5.1 Preselection

A refined selection of the $\nu qqqq$ signal final state is achieved using a MVA, however, to ensure efficiency in the training and application of that MVA a number of simple preselection cuts were developed to veto obvious background final states prior to the application of the MVA. Preselection cuts were applied to the transverse momentum of the system and the number of isolated leptons found in the event. The raw distributions of these variables is shown in figure 7.8 and based on these distributions the following cuts were applied

- Transverse momentum of system $> 100 \text{ GeV}$. This cut is effective due to the presence of missing energy in the form of neutrinos in the signal final state.
- Number of isolated leptons in system = 0. This cut is effective as the signal final state does not contain leptons, while numerous background final states do.

The impact of these preselection cuts can be found in table 7.6, which can be found on page 176.

**Figure 7.8:** Distribution of the preselection cut variables at $\sqrt{s} = 1.4$ TeV: (a) the transverse momentum of the visible system; and (b) the number of isolated leptons in the system. All distributions are normalised to an integrated luminosity of $L_{int} = 1.5$ ab$^{-1}$.

### 7.5.2 Multivariate analysis

Having established the preselection cuts, a MVA was applied using the TMVA toolkit [92], to refine the event selection. The signal and background final state samples were separated into two equally sized samples; one sample was used to independently train the MVA and the other sample was used in the subsequent analysis.
The performance of several MVA classifiers was examined to determine the optimal classifier for this analysis. The MVA classifiers considered were [92]:

- **Boosted Decision Tree (BDT)**. Decision trees are formed by the sequential application of cuts that split the data into multiple classes. After the application of the final cut, the remaining classes are used to classify whether the input event corresponds to signal or background. Boosting a decision tree involves the use of several decision trees. A single classifier output is obtained from a weighted average of the individual decision trees. The cuts applied in the decision tree are determined using the training sample.

- **$k$-Nearest Neighbour (KNN)**. For a given input event, the $k$ closest neighbours from the training sample are found. The classifier for that input event is determined as the fraction of those $k$ events that belong to the signal sample. Distances in this classifier are defined as the Euclidean distance between events in the $n$-dimensional space of the variables used for training the classifier. Weights are applied when calculating the distances to account for the differing widths of the input variable distributions. The value of $k$ used in this analysis was 20.

- **Multilayer Perceptron (MLP)**. This is an example of a neural network. Neural networks consist of an interconnected series of neurons each with a different response to a set of input signals. The signal for the first layer of neurons are the event variables used to train the MVA. The input signal proceeds to travel through several layers of neurons. The number of neurons in a given layer is reduced as the number of layers passed through increases until two neurons are left, one corresponding to signal and the other background. The neuron giving the larger response in the final layer determines the event classifier. The training sample is used to determine the response of each neurons in the network.

- **Fisher and H-Matrix Discriminants**. These procedures involve the calculation of a hyperplane in $n$-dimensional space that maximally separates signal and background events in the training sample. The location of an input event in that $n$-dimensional space with respect to that hyperplane determines the classifier for the event. The hyperplane is determined by maximising the differences between the means of the input event variables normalised by a measure of their spread. Both the Fisher and H-Matrix discriminants search for the hyperplane in $n$-dimensional space, however, the Fisher discriminant begins this procedure by transforming the input variables into a variable space with no linear correlations.
• **Likelihood.** The likelihood is determined using the probability density function (PDF) for each of the input variables. PDFs are determined using the training sample for both signal and background events. For a given event, the likelihood is given by the product of the probability of obtaining each of the input variables for that event. The signal and background likelihoods are calculated using the signal and background PDFs, respectively, and the ratio of the signal likelihood to the sum of the signal and background likelihoods gives the event classifier.

The input variables used for these MVA classifiers were:

- number of PFOs in each jet;
- energy of the highest energy PFO;
- energy of the highest energy electron;
- cosine of the polar angle of the highest energy track;
- energy of the candidate bosons;
- invariant mass of the candidate bosons;
- acolinearity of the candidate boson pair;
- the vector sum of the transverse momentum of all PFOs in the event;
- the sphericity of the event;
- and the derived jet clustering parameter variables $-\log_{10}(y_{ij})$ where $y_{ij}$ are jet clustering parameters, $i = 3, 4$ and $j = i + 1$.

Figure 7.9 shows the background rejection, which is equivalent to one minus the background efficiency, as a function of signal efficiency for various MVA classifiers. Efficiency is defined as the fraction of events classified as signal by the MVA. The efficiencies reported by TMVA are calculated after the application of the preselection cuts, which are described in section 7.5.1.

The classifier giving the optimal performance in terms of signal efficiency and background rejection was the BDT. The performance of the BDT was optimised further by varying the number of trees used and the depth of the trees. An optimal significance, $S/\sqrt{(S + B)}$, where S and B are the number of signal and background events passing the preselection, respectively, of 52.7 was obtained using the BDT. Table 7.5 shows the
The Sensitivity of CLIC to Anomalous Gauge Couplings through Vector Boson Scattering

Figure 7.9: Background rejection as a function of signal efficiency for a variety of MVA options at \( \sqrt{s} = 1.4 \) TeV. The solid black circle shows the signal efficiency and background rejection obtained when maximising the significance for the BDT method.

The event selection is summarised using the distribution of the invariant mass of the candidate bosons, which for the signal final state should peak around the W mass. This distribution is shown in figure 7.11 with: a) no event selection; b) with the preselection cuts applied; and c) with both preselections cuts and MVA applied. The event selection efficiencies are also summarised in table 7.6.

As expected the dominant background processes after the MVA is applied are those that have the same topology as the signal process, i.e. four primary quarks with missing energy. Two smaller sources of background are also present: two jet events with missing energy that are confused with four jet events with missing energy and events where a lepton is not properly reconstructed causing the event to look like four jets and missing energy.
Table 7.5: Ranking of the MVA variables used by the BDT.

### 7.6 Anomalous Coupling Fitting Methodology

This section describes the procedure used for constructing the $\chi^2$ surface and the subsequent confidence contours used to determine the sensitivity of CLIC to the anomalous gauge couplings $\alpha_4$ and $\alpha_5$.

#### 7.6.1 Sensitive Distribution

The sensitivity of CLIC to the anomalous gauge couplings will be determined through the use of a $\chi^2$ fit. Three variables showing sensitivity to the anomalous gauge couplings were considered for use in the $\chi^2$ fit:

- $M_{VV}$. The invariant mass of the visible system;
- $\cos \theta^*_Bosons$. The angle between the boost direction and the back-to-back candidate bosons in the rest frame of the visible system;
The Sensitivity of CLIC to Anomalous Gauge Couplings through Vector Boson Scattering

Figure 7.10: Distributions of the highest ranked variables used by the BDT: (a) the acolinearity of the candidate boson pair; (b) the invariant mass of the highest energy candidate boson; and (c) the number of PFOs in the highest energy jet. All distributions include effect of preselection and are normalised to an integrated luminosity of \( L_{\text{int}} = 1.5 \text{ ab}^{-1} \).

- \( \cos \theta_{\text{jets}} \). The angle between the boost direction and the back-to-back jets in the rest frame of the candidate bosons. As each event contains two candidate bosons, there are two \( \cos \theta_{\text{jets}} \) variables per event.

Figure 7.12 shows the distribution of these variables for the \( \nu \nu qqqq \) final state for selected values of the anomalous gauge couplings \( \alpha_4 \) and \( \alpha_5 \). A \( \chi^2 \) fit to each of these variables was applied to obtain confidence limits on the sensitivity of CLIC to the anomalous gauge couplings, as described in section 7.6.2. The distributions used for the \( \chi^2 \) fit contained signal and background events that passed the event selection. Table 7.7 shows the 68% confidence limits on the measurement of \( \alpha_4 \) and \( \alpha_5 \) obtained using each of the variables considered. The \( M_{VV} \) distribution shows the greatest sensitivity to
Figure 7.11: Impact of preselection and MVA on the reconstructed invariant mass of the candidate bosons at $\sqrt{s} = 1.4$ TeV: (a) no cuts; (b) after preselection; and (c) after preselection and MVA. All distributions correspond to an integrated luminosity of $L_{\text{int}} = 1.5 \text{ ab}^{-1}$. 
The Sensitivity of CLIC to Anomalous Gauge Couplings through Vector Boson Scattering

<table>
<thead>
<tr>
<th>Final State</th>
<th>$\epsilon_{\text{presel}}$</th>
<th>$\epsilon_{\text{BDT}}$</th>
<th>$N_{\text{BDT}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+e^- \rightarrow \nu\nuqqqq$</td>
<td>64.1%</td>
<td>44.5%</td>
<td>16,470</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow \nu\ellqqqq$</td>
<td>26.1%</td>
<td>5.2%</td>
<td>8,582</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow \ellqqqq$</td>
<td>0.8%</td>
<td>0.1%</td>
<td>100</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow qqqq$</td>
<td>0.3%</td>
<td>0.1%</td>
<td>1,698</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow \nu\ellqq$</td>
<td>43.4%</td>
<td>0.5%</td>
<td>5,351</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow \ellqq$</td>
<td>0.1%</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow \nu\ellqq$</td>
<td>19.1%</td>
<td>0.1%</td>
<td>9,319</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow \nuqq$</td>
<td>0.6%</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$e^-\gamma_{\text{EPA}} \rightarrow e^-qqqq$</td>
<td>0.2%</td>
<td>-</td>
<td>48</td>
</tr>
<tr>
<td>$e^-\gamma_{\text{BS}} \rightarrow e^-qqqq$</td>
<td>0.1%</td>
<td>-</td>
<td>42</td>
</tr>
<tr>
<td>$e^+\gamma_{\text{EPA}} \rightarrow e^+qqqq$</td>
<td>0.3%</td>
<td>-</td>
<td>19</td>
</tr>
<tr>
<td>$e^+\gamma_{\text{BS}} \rightarrow e^+qqqq$</td>
<td>-</td>
<td>-</td>
<td>65</td>
</tr>
<tr>
<td>$e^-\gamma_{\text{EPA}} \rightarrow \nu_eqqqq$</td>
<td>26.0%</td>
<td>9.0%</td>
<td>4,421</td>
</tr>
<tr>
<td>$e^-\gamma_{\text{BS}} \rightarrow \nu_eqqqq$</td>
<td>36.1%</td>
<td>15.0%</td>
<td>23,150</td>
</tr>
<tr>
<td>$e^+\gamma_{\text{EPA}} \rightarrow \nu_eqqqq$</td>
<td>25.9%</td>
<td>9.2%</td>
<td>4,495</td>
</tr>
<tr>
<td>$e^+\gamma_{\text{BS}} \rightarrow \nu_eqqqq$</td>
<td>36.4%</td>
<td>15.3%</td>
<td>23,410</td>
</tr>
<tr>
<td>$\gamma_{\text{EPA}}\gamma_{\text{EPA}} \rightarrow \nuqq$</td>
<td>0.2%</td>
<td>-</td>
<td>81</td>
</tr>
<tr>
<td>$\gamma_{\text{EPA}}\gamma_{\text{BS}} \rightarrow \nuqq$</td>
<td>0.1%</td>
<td>-</td>
<td>55</td>
</tr>
<tr>
<td>$\gamma_{\text{BS}}\gamma_{\text{EPA}} \rightarrow \nuqq$</td>
<td>-</td>
<td>-</td>
<td>53</td>
</tr>
<tr>
<td>$\gamma_{\text{BS}}\gamma_{\text{BS}} \rightarrow \nuqq$</td>
<td>-</td>
<td>-</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 7.6:** Event selection efficiencies at $\sqrt{s} = 1.4$ TeV. In the above table, $\epsilon_{\text{presel}}$ denotes the number of events passing the preselection as a fraction of the total number of events, while $\epsilon_{\text{BDT}}$ denotes the number of events passing both the preselection and the BDT as a fraction of the total number of events. The EPA and BS subscript on the incoming photon indicates whether the photon is generated from the equivalent photon approximation or beamstrahlung. Entries with a dash indicate an efficiency of less than 0.1%. The event numbers correspond to an integrated luminosity of $\mathcal{L}_{\text{int}} = 1.5 \text{ ab}^{-1}$.

The anomalous gauge couplings; therefore, it will be used by all subsequent $\chi^2$ fits when reporting sensitivities. This distribution shows the greatest sensitivity of the variables considered because the couplings primarily affect events with large values of $M_{VV}$ and there are relatively few of these events.
Figure 7.12: The distributions of (a) $M_{VV}$, (b) $\cos^2 \theta_{J_{\text{Jets}}}$ and (c) $\cos^2 \theta_{\text{Bosons}}$ for selected values of the anomalous gauge couplings $\alpha_4$ and $\alpha_5$ for the $\nu\nuqqqq$ final state at $\sqrt{s} = 1.4$ TeV. The jet algorithm used was the longitudinally invariant $k_t$ algorithm with an R parameter of 0.9 and Selected PFOs. All distributions are normalised to an integrated luminosity of $L_{\text{int}} = 1.5 \text{ ab}^{-1}$. 
### Table 7.7: Sensitive Variable 68% Confidence Limit

<table>
<thead>
<tr>
<th>Sensitive Variable</th>
<th>68% Confidence Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{VV}$</td>
<td>$-0.0082 &lt; \alpha_4 &lt; 0.0116$</td>
</tr>
<tr>
<td></td>
<td>$-0.0055 &lt; \alpha_5 &lt; 0.0078$</td>
</tr>
<tr>
<td>$\cos\theta^*_\text{Bosons}$</td>
<td>$-0.0111 &lt; \alpha_4 &lt; 0.0155$</td>
</tr>
<tr>
<td></td>
<td>$-0.0082 &lt; \alpha_5 &lt; 0.0110$</td>
</tr>
<tr>
<td>$\cos\theta^*_\text{Jets}$</td>
<td>$-0.0100 &lt; \alpha_4 &lt; 0.0142$</td>
</tr>
<tr>
<td></td>
<td>$-0.0070 &lt; \alpha_5 &lt; 0.0098$</td>
</tr>
</tbody>
</table>

7.6.2 $\chi^2$ Surface and Confidence Limit Definition

A $\chi^2$ surface was used to determine confidence limits on the anomalous gauge couplings given the null hypothesis that $\alpha_4 = \alpha_5 = 0$. This surface is defined as

$$\chi^2 = \sum_i \frac{(O_i - E_i)^2}{E_i},$$

(7.11)

where $O_i$ is the observed, $\alpha_4 = \alpha_5 = 0$, and $E_i$ the expected, $\alpha_4 \neq 0$ and $\alpha_5 \neq 0$, bin content for bin $i$ in the distribution of interest. The summation $\Sigma_i$ runs over bins in the distribution of interest.

When applying the $\chi^2$ fit to the $M_{VV}$ distribution, the distribution was binned using 13 bins as shown in figure 7.13. The first bin spanned the invariant mass range between 0 GeV and 200 GeV, this was followed by 11 bins of width 100 GeV ranging from 200 GeV to 1300 GeV and finally the last bin contained all invariant masses above 1300 GeV. The expanded bin widths at the tails of the distribution were chosen to ensure the bin contents were sufficiently large to give a reliable estimate of the likelihood function using the $\chi^2$ parameter. This choice of bin width also ensured the bin contents were sufficiently large to minimise fluctuations arising from individual events with large weights. When applying the $\chi^2$ fit to distributions of the $\cos\theta^*_\text{Bosons}$ and $\cos\theta^*_\text{Jets}$ variables, the distributions were binned using 10 bins ranging from zero to one. As there are two $\cos\theta^*_\text{Jets}$ variables per event, the $\chi^2$ fit was applied to a two dimensional distribution of $\cos\theta^*_\text{Jets}$, where a distinction between the two $\cos\theta^*_\text{Jets}$ variables was made based on the energy of the candidate bosons. The use of a two dimensional distribution in the $\chi^2$ fit was needed to account for any correlation between the two $\cos\theta^*_\text{Jets}$ variables.
Confidence limits describing the sensitivity of the CLIC experiment to the anomalous gauge couplings were found by examining the $\chi^2$ surface in the space of $\alpha_4$ and $\alpha_5$. Deviations from the minima of this surface, which by construction occurs at $\alpha_4 = \alpha_5 = 0$, yield confidence limits that indicate the probability of observing a particular value of $\alpha_4$ and $\alpha_5$ given the null hypothesis that $\alpha_4 = \alpha_5 = 0$. The confidence limits reported in subsequent sections, 68%, 90% and 99%, are defined using fixed deviations from the minima of $\chi^2$ surface ($\Delta \chi^2$) of 2.28, 4.61 and 9.21, respectively.

Confidence limits on the individual parameters $\alpha_4$ and $\alpha_5$ were determined by setting the corresponding coupling term to zero and examining the remaining one dimensional $\chi^2$ distribution. A fourth order polynomial was fitted to the minima of this distribution and the 68% confidence limit defined using $\Delta \chi^2 = 1$. The value of $\Delta \chi^2$ corresponding to a 68% confidence limit is sensitive to the number of degrees of freedom in the fit, therefore, it differs when examining the one and two dimensional $\chi^2$ distributions.

**Figure 7.13:** The distribution of the invariant mass of the system, $M_{VV}$, for both signal and background finals states that are used in the $\chi^2$ fit at $\sqrt{s} = 1.4$ TeV. The distribution includes effect of event selection and corresponds to an integrated luminosity of $L_{int} = 1.5$ ab$^{-1}$.

### 7.6.3 Event Weight Interpolation Scheme

In order to obtain a smooth $\chi^2$ surface a fine sampling of the event weights in the $\alpha_4$ and $\alpha_5$ space is required, however, it is unfeasible to generate a finely sampled grid of event weights on an event by event basis because event generation is highly CPU intensive. To resolve this issue, an interpolation scheme was applied to determine the event weights within a sampled region of the $\alpha_4$ and $\alpha_5$ space. This allows for an infinite sampling...
of the event weights in the space of $\alpha_4$ and $\alpha_5$ without having to call the generator an infinite number of times.

A bicubic interpolation scheme, cubic interpolation along the two dimensions, was applied to the event weights produced by the generator. This procedure is best illustrated by figure 7.14, which shows the interpolated event weight surface superimposed with the raw event weights from the generator for four $\nu\nuqqqq$ events at $\sqrt{s} = 1.4$ TeV. This interpolation scheme produces a smooth and continuous surface that can be used for generating a smooth $\chi^2$ surface.

![Figure 7.14](image-url)

*Figure 7.14:* The event weight, $w$, as a function of the anomalous couplings $\alpha_4$ and $\alpha_5$ for a selection of $\sqrt{s} = 1.4$ TeV $\nu\nuqqqq$ final state events. The black circles show the event weight produced from the generator and the blue surface is determined using bicubic interpolation between these points.
7.7 Results

The sensitivity of the CLIC experiment to the anomalous gauge couplings $\alpha_4$ and $\alpha_5$ at $\sqrt{s} = 1.4$ TeV is shown in figure 7.15a. This result shows the sensitivity after the application of preselection and MVA purposed to remove the included background channels. These contours give a 68% confidence limit for CLIC operating at $\sqrt{s} = 1.4$ TeV of

$$-0.0082 < \alpha_4 < 0.0116, \quad (7.12)$$
$$-0.0055 < \alpha_5 < 0.0078. \quad (7.13)$$

7.7.1 Systematic Uncertainties

A source of systematic error in this experiment is the uncertainty on the cross-sections for the signal and background processes. Based on the event selection summary shown in table 7.6, the dominant source of background in this analysis comes from the $e^\pm \gamma_{\text{BS}} \rightarrow \nu_{\text{e}}qqqq$ processes. Therefore, uncertainties on the cross-section for these processes, as well as the signal process $e^+e^- \rightarrow \nu\nuqqqq$, will be considered.

The uncertainty on the cross-section for a given process is included in the $\chi^2$ definition through the use of a nuisance parameter. This procedure allows the cross-section for a process to fluctuate, however, the magnitude of the fluctuation, $r$, is moderated by an additional penalty term in the $\chi^2$ as follows

$$\chi^2(r) = \sum_i \left( \frac{O_i - E_i(r)}{E_i(r)} \right)^2 + \frac{(r - 1)^2}{\sigma_r^2}, \quad (7.14)$$

where $O_i$ is the observed, $\alpha_4 = \alpha_5 = 0$, bin content for bin $i$ in the distribution of $M_{VV}$ with no background fluctuations and $E_i(r)$ is the expected, $\alpha_4 \neq 0$ and $\alpha_5 \neq 0$, bin content for bin $i$ in the distribution of $M_{VV}$ where the cross-section for the process of interest has been scaled by a factor of $r$. The sum $\sum_i$ runs over the bins in the $M_{VV}$ distribution. The $\sigma_r$ variable is the width of the distribution of $r$, which indicates the uncertainty on the measurement of the cross-section of interest. A $\chi^2$ surface is constructed in the space of $\alpha_4$ and $\alpha_5$ by minimising $\chi^2(r)$ at each point.
The Sensitivity of CLIC to Anomalous Gauge Couplings through Vector Boson Scattering

Figure 7.15: $\chi^2$ sensitivity distributions from a fit to $M_{VV}$ at $\sqrt{s} = 1.4$ TeV. Results include the effect of backgrounds after the application of a series of preselection cuts and MVA. (a) $\chi^2$ sensitivity contours in $\alpha_4$ and $\alpha_5$ space. (b) $\chi^2$ as a function of $\alpha_4$ assuming $\alpha_5 = 0$. (c) $\chi^2$ as a function of $\alpha_5$ assuming $\alpha_4 = 0$.

The 68% confidence region is shown with the inclusion of a nuisance parameter for the signal process $e^+e^- \rightarrow \nu\nuqqqq$ and the dominant background processes $e^+\gamma_{BS} \rightarrow \nuqqqq$ in figures 7.16a and 7.16b, respectively. Minimal changes in sensitivity are observed when allowing the signal and dominant backgrounds to fluctuate. This can be understood by considering the shape of the $M_{VV}$ distribution for the signal and dominant background processes.
processes, which is shown in figure 7.17. These distribution shows that anomalous couplings primarily affect events with large invariant masses, while both the signal and dominant backgrounds peak at low invariant masses. Therefore, by fluctuating the cross-section for the signal and dominant background processes, it is not possible to gain a significantly better match between the observed and expected bin contents in the $M_{VV}$ distribution. This is encouraging as despite the $e^\pm \gamma_{BS} \to \nu_q qqq$ backgrounds dominating the $\chi^2$ fit that determines the sensitivity of CLIC to the anomalous gauge couplings, precise knowledge of their cross-section is not crucial. As the uncertainty on these cross-sections does not significantly affect the confidence regions, no cross-section uncertainties are accounted for when reporting the sensitivity of CLIC to the anomalous gauge couplings elsewhere in this analysis.

![Confidence Region](image)

Figure 7.16: The 68% confidence region including the effect of uncertainties in the cross-section for (a) the signal process $e^+ e^- \to \nu_q qqqq$ and (b) the dominant background processes $e^\pm \gamma_{BS} \to \nu_q qqqq$.

### 7.8 Sensitivity at $\sqrt{s} = 3$ TeV

The anomalous gauge coupling sensitivity study described in this chapter was repeated for CLIC operating at $\sqrt{s} = 3$ TeV. As this analysis largely mirrors that of the $\sqrt{s} = 1.4$ TeV analysis, this section focuses on the differences between the two analyses.
The signal and background final states for the \( \sqrt{s} = 3 \) TeV analysis were identical to those used for the \( \sqrt{s} = 1.4 \) TeV analysis. Cross-sections for these processes at \( \sqrt{s} = 3 \) TeV are given in table 7.8. The data analysis and event selection procedures used for the \( \sqrt{s} = 3 \) TeV analysis mirrored those used for the \( \sqrt{s} = 1.4 \) TeV analysis.

Jet finding was performed using the longitudinally invariant \( k_t \) algorithm as described in section 7.4.2. The jet algorithm configuration was optimised using the sensitivity of CLIC to the anomalous gauge couplings using pure signal only, as described in section 7.4.2.1. The optimal jet algorithm configuration for the \( \sqrt{s} = 3 \) TeV analysis used tight selected PFOs and an \( R \) parameter of 1.1. As the cross-section for the \( \gamma\gamma \to \text{hadrons} \) increases with energy, the effect of these background is more problematic at \( \sqrt{s} = 3 \) TeV than at \( \sqrt{s} = 1.4 \) TeV [31]. Therefore, the optimal PFO selection at \( \sqrt{s} = 3 \) TeV should be more aggressive at vetoing these backgrounds than the optimal PFO selection at \( \sqrt{s} = 1.4 \) TeV, which is what is observed.

As opposed to training the MVA using 50% of the signal and background events, as was done for the \( \sqrt{s} = 1.4 \) TeV analysis, the \( \sqrt{s} = 3 \) TeV analysis trained the MVA using 10% of the signal and background events. This modification prevented those events with very large event weights from dominating the \( \chi^2 \) fit and producing exaggerated
Table 7.8: Cross-sections of signal and background processes at $\sqrt{s} = 3$ TeV. In the above table q represents $u, \bar{u}, d, \bar{d}, s, \bar{s}, c, \bar{c}, b$ or $\bar{b}$; l represents $e^{\pm}$, $\mu^{\pm}$ or $\tau^{\pm}$; and $\nu$ represents $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau$ and $\bar{\nu}_\tau$. The EPA and BS subscript on the incoming photon indicates whether the photon is generated from the equivalent photon approximation or beamstrahlung.

sensitivities. The sensitivity to the anomalous gauge couplings grows with increasing centre of mass energy, therefore, at $\sqrt{s} = 1.4$ TeV very large event weights were not problematic. The sample sizes for all signal and background processes was sufficiently large that training on 10% of the total sample was sufficient to achieve good MVA performance. Event selection for the $\sqrt{s} = 3$ TeV analysis is summarised in table 7.9.

Due to the increased sensitivity of the signal sample, event weights were sampled with greater frequency in the space of $\alpha_4$ and $\alpha_5$ at $\sqrt{s} = 3$ TeV than at $\sqrt{s} = 1.4$ TeV analysis. Bicubic interpolation was again used to make a continuous surface for the event
The Sensitivity of CLIC to Anomalous Gauge Couplings through Vector Boson Scattering

<table>
<thead>
<tr>
<th>Final State</th>
<th>$\epsilon_{\text{presel}}$</th>
<th>$\epsilon_{\text{BDT}}$</th>
<th>$N_{\text{BDT}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+e^- \rightarrow \nu\nuqqqq$</td>
<td>74.4%</td>
<td>46.0%</td>
<td>65,740</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow \nu lqqqq$</td>
<td>40.0%</td>
<td>12.0%</td>
<td>25,660</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow llqqqq$</td>
<td>7.5%</td>
<td>1.1%</td>
<td>3,570</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow qqqq$</td>
<td>3.7%</td>
<td>0.3%</td>
<td>3,224</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow \nu\nuqq$</td>
<td>50.5%</td>
<td>1.2%</td>
<td>30,510</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow \nu lqq$</td>
<td>32.0%</td>
<td>0.4%</td>
<td>48,320</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow llqq$</td>
<td>1.4%</td>
<td>-</td>
<td>1,028</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow qqq$</td>
<td>1.4%</td>
<td>0.1%</td>
<td>3,268</td>
</tr>
<tr>
<td>$e^-\gamma_{\text{EPA}} \rightarrow e^-qqqq$</td>
<td>6.6%</td>
<td>0.8%</td>
<td>4,736</td>
</tr>
<tr>
<td>$e^-\gamma_{\text{BS}} \rightarrow e^-qqqq$</td>
<td>4.6%</td>
<td>0.7%</td>
<td>13,660</td>
</tr>
<tr>
<td>$e^+\gamma_{\text{EPA}} \rightarrow e^+qqqq$</td>
<td>6.5%</td>
<td>0.8%</td>
<td>4,686</td>
</tr>
<tr>
<td>$e^+\gamma_{\text{BS}} \rightarrow e^+qqqq$</td>
<td>4.7%</td>
<td>0.7%</td>
<td>13,310</td>
</tr>
<tr>
<td>$e^-\gamma_{\text{EPA}} \rightarrow \nu eqqqq$</td>
<td>45.6%</td>
<td>17.2%</td>
<td>18,610</td>
</tr>
<tr>
<td>$e^-\gamma_{\text{BS}} \rightarrow \nu eqqqq$</td>
<td>55.9%</td>
<td>26.7%</td>
<td>110,900</td>
</tr>
<tr>
<td>$e^+\gamma_{\text{EPA}} \rightarrow \nu eqqqq$</td>
<td>45.9%</td>
<td>17.3%</td>
<td>18,750</td>
</tr>
<tr>
<td>$e^+\gamma_{\text{BS}} \rightarrow \nu eqqqq$</td>
<td>56.5%</td>
<td>27.4%</td>
<td>113,700</td>
</tr>
<tr>
<td>$\gamma_{\text{EPA}}\gamma_{\text{EPA}} \rightarrow qqqq$</td>
<td>5.3%</td>
<td>0.7%</td>
<td>5,531</td>
</tr>
<tr>
<td>$\gamma_{\text{EPA}}\gamma_{\text{BS}} \rightarrow qqqq$</td>
<td>3.5%</td>
<td>0.4%</td>
<td>16,640</td>
</tr>
<tr>
<td>$\gamma_{\text{BS}}\gamma_{\text{EPA}} \rightarrow qqqq$</td>
<td>3.5%</td>
<td>0.4%</td>
<td>15,900</td>
</tr>
<tr>
<td>$\gamma_{\text{BS}}\gamma_{\text{BS}} \rightarrow qqqq$</td>
<td>0.6%</td>
<td>-</td>
<td>4,124</td>
</tr>
</tbody>
</table>

Table 7.9: Event selection efficiencies at $\sqrt{s} = 3$ TeV. In the above table, $\epsilon_{\text{presel}}$ denotes the number of events passing the preselection as a fraction of the total number of events, while $\epsilon_{\text{BDT}}$ denotes the number of events passing both the preselection and the BDT as a fraction of the total number of events. The EPA and BS subscript on the incoming photon indicates whether the photon is generated from the equivalent photon approximation or beamstrahlung. Entries with a dash indicate an efficiency of less than 0.1%. The event numbers correspond to an integrated luminosity of $L_{\text{int}} = 2 \text{ ab}^{-1}$.

weights. These event weight surfaces were then used to construct the $M_{VV}$ distribution and the $\chi^2$ surface used to determine the reported sensitivities. Figure 7.18 shows an example of the event weights extracted from the generator and the interpolated surface used to define the $\chi^2$ surface as a function of $\alpha_4$ and $\alpha_5$ for a selected $\nu\nuqqqq$ event at $\sqrt{s} = 3$ TeV.
A $\chi^2$ was applied to the distribution of $M_{VV}$ to determine the sensitivity of CLIC to the anomalous gauge couplings $\alpha_4$ and $\alpha_5$ at $\sqrt{s} = 3$ TeV. The $M_{VV}$ distribution used for the fit had an increased number of bins with respect to the $\sqrt{s} = 1.4$ TeV analysis; the first bin spanned the invariant mass range between 0 GeV and 200 GeV, this was followed by 27 bins of width 100 GeV ranging from 200 GeV to 1300 GeV and finally the last bin contained all invariant masses above 2900 GeV. Figure 7.19 shows the $M_{VV}$ distribution for signal and background processes at $\sqrt{s} = 3$ TeV that was used in the $\chi^2$ fit.

Figure 7.18: The event weights, $w$, as a function of the anomalous couplings $\alpha_4$ and $\alpha_5$ for a selected $\nu\nuqqqq$ final state events at $\sqrt{s} = 3$ TeV. These weights are calculated using (a) the generator and (b) bicubic interpolation.

Figure 7.19: The distribution of the invariant mass of the system for both signal and background finals states that is used in the $\chi^2$ fit at $\sqrt{s} = 3$ TeV. The distribution includes effect of event selection and corresponds to an integrated luminosity of $L_{int} = 2 \text{ ab}^{-1}$.
The sensitivity of the CLIC experiment to the anomalous gauge couplings $\alpha_4$ and $\alpha_5$ at $\sqrt{s} = 3$ TeV is shown in figure 7.20a. This result shows the sensitivity after the application of preselection and MVA, described in sections 7.5.1 and 7.5.2, purposed to remove the included background channels. These contours give a 68% confidence limit on the measurement of $\alpha_4$ and $\alpha_5$ for CLIC operating at $\sqrt{s} = 3$ TeV of

$$-0.0010 < \alpha_4 < 0.0011, \quad (7.15)$$
$$-0.0007 < \alpha_5 < 0.0007. \quad (7.16)$$

Figure 7.21 shows how the 68% confidence region for the $\sqrt{s} = 3$ TeV analysis varies with the uncertainty in the cross-section for the signal, $e^+e^- \rightarrow \nu\bar{\nu}qqqq$, and dominant background processes, $e^\pm \gamma_{BS} \rightarrow \nu_eqqqq$. These contours were produced using a nuisance parameter as discussed in section 7.7.1. Once again, these systematic uncertainties have a small effect on the reported sensitivity of CLIC to the anomalous gauge couplings because of the shape of the $M_{VV}$ distribution.
The Sensitivity of CLIC to Anomalous Gauge Couplings through Vector Boson Scattering

Figure 7.20: $\chi^2$ sensitivity distributions from a fit to $M_{VV}$ at $\sqrt{s} = 3$ TeV. Results include the effect of backgrounds after the application of a series of preselection cuts and MVA. (a) $\chi^2$ sensitivity contours in $\alpha_4$ and $\alpha_5$ space. (b) $\chi^2$ as a function of $\alpha_4$ assuming $\alpha_5 = 0$. (c) $\chi^2$ as a function of $\alpha_5$ assuming $\alpha_4 = 0$. 
The Sensitivity of CLIC to Anomalous Gauge Couplings through Vector Boson Scattering

Figure 7.21: The 68% confidence region including the effect of uncertainties in the cross-section for (a) the signal process $e^+e^- ightarrow \nu\nu qqqq$ and (b) the dominant background processes $e^\pm \gamma_{BS} \rightarrow \nu e qqqq$. 

\[ \alpha_0.005 - 0 = 0\% \]
\[ \alpha_0.005 - 0 = 1\% \]
\[ \alpha_0.005 - 0 = 2\% \]

\[ \sigma_{e^+e^- \rightarrow \nu\nu qqqq} = 0\% \]
\[ \sigma_{e^+e^- \rightarrow \nu\nu qqqq} = 1\% \]
\[ \sigma_{e^+e^- \rightarrow \nu\nu qqqq} = 2\% \]

\[ \sigma_{e^\pm \gamma_{BS} \rightarrow \nu e qqqq} = 50\% \]
Chapter 8

Summary

“These are the things God has revealed to us by his Spirit.”
— 1 Corinthians 2:10

The work presented in this thesis has contributed to the future linear collider experiments in terms of detector design, event reconstruction and demonstration of physics potential.

A number of capacitively coupled pixel detectors were prototyped and tested, using both lab and test beam measurements, to determine whether they were viable for use in the CLIC vertex detector. The performance of these prototyped devices was extremely good. As an offset between the sensor and readout ASICs could be accidentally introduced to the devices during the manufacturing procedure, a number of devices were examined that contained a known offset. Even devices containing an offset of up to $\frac{1}{4}$ of a pixel were found to have comparable performance to the ideally aligned devices. Although modifications would be required for the final design of the sensor and readout ASICs to fully optimise performance, the technique of capacitive coupling of sensor to readout ASICs was found to be viable for use at the future linear collider experiments.

An optimisation study of the calorimeter design for use at the future linear collider was performed. This study clarified those detector parameters that are crucial for achieving outstanding performance in the particle flow paradigm. Furthermore, this work has made it possible to make informed decisions about the detector design that minimise the cost, while retaining outstanding jet energy resolutions. Reliability in the conclusions drawn
from this study could only be achieved by employing the calibration procedure that was developed for the linear collider simulation.

Development of novel software techniques, which make full use of the segmentation of the linear collider calorimeters, led to a significant improvement in the energy resolution of the linear collider detector. This improvement would be extremely expensive to obtain if it were achieved by modifying the design of the calorimeters, therefore, as well as extending the physics reach of the detector, a significant cost reduction was made.

The final study presented in this thesis determined the sensitivity of the CLIC experiment to the anomalous gauge couplings $\alpha_4$ and $\alpha_5$ through the vector boson scattering process. The signal final state $\nu\nuqqqq$ was selected for this analysis based on the relative sensitivities of final states showing sensitivity to these couplings. Background processes were then selected based on whether they could be confused with the signal. An event selection procedure was applied to separate the signal and backgrounds. The significance obtained from this event selection was 52.7 (90.6) for CLIC running at 1.4 (3) TeV. Finally, a $\chi^2$ fit was applied to the distribution of the invariant mass of the system to determine the sensitivity of the CLIC experiment to the anomalous gauge couplings. The sensitivity manifested itself in the form of event weights for the signal final state. Using this procedure the two $\sigma$ confidence limits on the couplings, assuming the corresponding coupling is zero, were found to be

\[-0.0125 < \alpha_4 < 0.0159, \quad (8.1)\]
\[-0.0090 < \alpha_5 < 0.0114, \quad (8.2)\]

at 1.4 TeV and

\[-0.0015 < \alpha_4 < 0.0016, \quad (8.3)\]
\[-0.0010 < \alpha_5 < 0.0011, \quad (8.4)\]

at 3 TeV. These limits should be compared to the current best limits available, which come from the ATLAS experiment at the LHC [80]. That analysis examined $W^\pm Vjj$ events, where $V$ is either $W^\pm$ or $Z$, using $\sim 20 \text{ fb}^{-1}$ of Run 1 data at $\sqrt{s} = 8$ TeV and found two $\sigma$ confidence limits on the anomalous gauge couplings of

\[-0.024 < \alpha_4 < 0.030, \quad (8.5)\]
\[-0.028 < \alpha_5 < 0.033. \quad (8.6)\]
CLIC can significantly improve on these measurements by a factor of approximately 2 (10) at 1.4 (3) TeV [80]. This is a large gain in sensitivity, which indicates just one aspect of the physics capabilities of the linear collider. Furthermore, it should be emphasised that additional gains in sensitivity can be made for CLIC by examining states sensitive to anomalous gauge couplings other than $\nu\nuqqqq$ e.g. $\nu lqqqq$ and $l lqqqq$. This study adds further weight to the argument for the construction of a linear collider.
Colophon

This thesis was made in \LaTeX\ using the “hepthesis” class [93]. Feynman diagrams were produced using TikZ-Feynman [94].
Bibliography


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