Measuring acoustic complexity in continuously varying signals: how complex is a wolf howl?

AUTHORS

Arik Kershenbaum¹, Éloïse C. Déaux², Bilal Habib³, Brian Mitchell⁴, Vicente Palacios⁵, Holly Root-Gutteridge⁶, Sara Waller⁷

¹Department of Zoology, University of Cambridge, UK
²Department of Biological Sciences, Macquarie University, Sydney, 2109, Australia
³Department of Animal Ecology and Conservation Biology, Wildlife Institute of India, Dehradun, India
⁴The Rubenstein School of Environment and Natural Resources, University of Vermont, USA
⁵Instituto Cavanilles de Biodiversidad y Biología Evolutiva. University of Valencia, Spain
⁶Department of Biology, Syracuse University, USA
⁷Montana State University, Bozeman, MT, USA

CORRESPONDING AUTHOR: Arik Kershenbaum, arik.kershenbaum@gmail.com
Communicative complexity is a key behavioural and ecological indicator in the study of animal cognition. Much attention has been given to measures such as repertoire size and syntactic structure in both bird and mammal vocalisations, as large repertoires and complex call combinations may give an indication of the cognitive abilities both of the sender and receiver. However, many animals communicate using a continuous vocal signal that does not easily lend itself to be described by concepts such as "repertoire". For example, dolphin whistles and wolf howls both have complex patterns of frequency modulation, so that no two howls or whistles are quite the same. Is there a sense in which some of these vocalisations may be more "complex" than others? Can we arrive at a quantitative metric for complexity in a continuously varying signal? Such a metric would allow us to extend familiar analyses of communicative complexity to those species where vocal behaviour is not restricted to sequences of stereotyped syllables. We present four measures of complexity in continuous signals (Wiener Entropy, Autocorrelation, Inflection Point Count, and Parsons Entropy), and examine their relevance using example data from members of the genus *Canis*. We show that each metric can lead to different conclusions regarding which howls could be considered complex or not. Ultimately, complexity is poorly defined and researchers must compare metrics to ensure that they reflect the properties for which the hypothesis is being tested.

KEYWORDS: Autocorrelation, Canids, Communication, Complexity, Entropy
INTRODUCTION

Vocal complexity is considered an important property of animal communication (Freeberg and Krams 2015; Larson 2004; McCowan et al. 2002; Pollard and Blumstein 2012), despite being poorly defined, with little agreement how complexity should be quantified (Edmonds 1999). Despite this, complexity has been used to explain different aspects of animal behaviour. For instance, evidence exists in several bird species that females choose mates at least partially on the basis of the complexity of male song (Darolová et al. 2012; Hiebert et al. 1989), and other males may use complexity cues to make conflict escalation decisions (Leitão et al. 2006). It has been postulated that birdsong complexity acts as an index signal; being positively correlated with nutritional competence and cognitive abilities, and negatively correlated with early life stress (Nowicki et al. 2002). Similarly, recent studies have suggested that a correlation exists between communicative complexity and social complexity, such that species with more complex social systems also have more complex communicative interactions (Freeberg and Krams 2015; Krams et al. 2012; Pollard and Blumstein 2012). This in turn could shed light on possible evolutionary pathways to the development of language as an adaptation of highly complex social groups in early hominins (Seyfarth and Cheney 2014).

Communicative complexity can also have practical implications for the identification of and discrimination between similar sub-populations where complexity varies geographically (Briefer et al. 2010; Kershenbaum et al. 2012). While many species can distinguish between the vocalisations of in-group and out-group individuals, e.g. wolves Canis lupus (Palacios et al. 2015; Zaccaroni et al. 2012), elephants Loxodonta africana (O’Connell-Rodwell et al. 2007), and multiple bird species (Nakagawa et al. 2001; Radford 2005), it is often not clear what vocal cues are being used to make this discrimination, and complexity characteristics may play a role (Briefer et al. 2008). Geographic differences in vocal complexity may be particularly noticeable where ecological conditions lead to differences in food availability, cognitive development, and hence vocal repertoire size (Byers and Kroodsma 2009; Kipper et al. 2006; Pfaff et al. 2007). In parallel, researchers can make use of differences in repertoire size, for example, to distinguish between sub-populations of birds and
mammals (Gwilliam et al. 2008; Pitcher et al. 2012). Clearly, vocal complexity is an important
phenomenon with far ranging implications for the study of animal communication.

Previous studies of vocal complexity have focussed largely on birdsong, because of three essential
properties that make this communication modality particularly tractable: (a) most birdsong can be
divided into discrete syllables or notes (Marler and Slabbekoorn 2004); (b) there exists a simple
metric - repertoire size - for measuring purported complexity (Byers and Kroodsma 2009); and (c) the
well-established role of birdsong in mate choice provides the opportunity for manipulative as well as
correlative experiments to be carried out, quite clearly demonstrating the role of song complexity in
enhancing fitness (Searcy 1992). Even when birdsong is open-ended so that repertoire cannot
adequately be defined, for example in the northern mockingbird *Mimus polyglottos* (Gammon 2014),
the discrete nature of the song syllables means that other measurements of communicative complexity
can be used, most notably Shannon entropy (Briefer et al. 2010; Da Silva et al. 2000; Kershenbaum
2013). Although some animals from other taxa have vocal communication systems that are similarly
discrete, e.g. rock hyrax *Procavia capensis* (Kershenbaum et al. 2012), or are closed-ended, e.g.
several primates (Cäsar et al. 2012; Zuberbühler 2002), outside of passerine birds they are the
exception rather than the rule. Indeed, the vocal communication system of some species consists
entirely of signals whose properties are continuously varying, and with such signals the existing
concepts of complexity (e.g. based on repertoire and entropy) cannot be applied. For example,
considerable empirical evidence supports the existence of semantic information in the whistles of
bottlenose dolphins *Tursiops truncatus*; in particular, the use of signature whistles to signal individual
However, dolphin whistles are relatively long and unbroken tonal signals that are continuously
frequency modulated. Similarly, wolf howls are continuous frequency modulated signals that have
been shown to contain individual identity information (Palacios et al. 2007; Root-Gutteridge et al.
2014; Tooze et al. 1990), but cannot be classified into stereotyped categories. One possible
interpretation of complexity, which may or may not be intuitive to the reader, is that the frequency of
a *simple* signal varies little, or predictably, with time, whereas a *complex* signal varies greatly, and
unpredictably. The reader may examine Figure 1, which shows several examples of wolf howls and
decide which howls are simple and which complex. However, although some howls may appear
intuitively more complex than others, the lack of an objective definition of complexity for such
signals renders the judgement unhelpful. As such, previous studies of information content in
continuously varying animal vocal signals have had to make use of alternative techniques, example
for measuring similarity between pairs of vocalisations, rather than quantifying characteristics of the
vocalisations themselves (Kershenbaum et al. 2013).

Complexity in itself is poorly defined (Edmonds 1999), and as a result any particular use of the term
in examining animal behaviour is liable to be criticised. In particular, information theoretical
definitions of complexity based on concepts such as entropy are often regarded with suspicion by
biologists, because the most complex (highest entropy) signals are in fact random signals - something
that most ethologists would consider to be non-complex (Suzuki et al. 2005). We agree that defining
complexity is difficult, however we hope to mitigate this difficulty somewhat by insisting that
researchers should examine and understand what kind of signal a particular definition of complexity
would deem either complex or non-complex. It would then be possible to determine whether a
particular definition of complexity meets the demands of discriminating between signals in relation to
the study’s hypotheses.

In this paper, we aim to show how concepts of continuous complexity can be measured using different
approaches, and we illustrate how use of a particular complexity metric can lead to different
conclusions from using other metrics. We present four candidate complexity metrics and compare
their performance against each other, identifying which kinds of signals each particular metric would
indicate to be “complex” or “not complex”. We recognise that in drawing up a proposed metric of
complexity for continuous signals, it is inevitable that subjective interpretations of the complexity or
simplicity of a signal must necessarily influence the decisions a researcher makes in designing an
experiment. Rather than attempt to avoid this subjective tendency, we hope to formalise it somewhat,
by presenting a range of complexity metrics, along with illustration of their significance for signals of
different types. That way, researchers can choose a metric that captures the "kind of" complexity for
which they are searching; providing a quantitative measure for an essentially subjective property.
METHODS

We describe below four quantitative metrics that have been previously used for quantifying complexity in continuous signals: Wiener Entropy, Autocorrelation, Inflection Point Count, and Parsons Entropy. For each of these metrics, we define how they are computed and explain in what way they could be considered to be measures of complexity in a continuous signal, giving detailed examples using simulated waveforms as shown in Figure 2. As each of our proposed metrics measures quite different features of acoustic signals, it is constructive to illustrate the behaviour of each metric by showing examples of signals with both high and low metric values, as this provides an indication of which signal features are being emphasised by each metric. We do this using example acoustic signals drawn from an empirical data set consisting of 2,014 coded canid howls from 16 different species and subspecies, as described in a previous work (Kershenbaum et al. 2016). We calculated each complexity metric for each howl and then plot on a time-frequency graph the three howls with the lowest metric values, and the three howls with the highest values.

Wiener Entropy

Wiener Entropy, or spectral flatness, has been proposed as a measure of the complexity of birdsong elements, and has been applied to the analysis of increasing complexity during the process of song learning in juvenile birds (Baker and Logue 2003; Tchernichovski et al. 2000). Wiener Entropy is a measure of the extent to which a signal contains a mixture of frequencies, as opposed to a single frequency or tone. As such, a value of 0 represents a single sine wave, and a value of 1 indicates white noise, in which all frequencies are equally represented. To calculate the Wiener Entropy, we first perform a Fourier transform to calculate the spectral power $P$ present at each of $N$ distinct frequency bins. The formal definition is given as the ratio of the geometric means of the spectral powers to the arithmetic mean:
Wiener Entropy can be applied to the signal spectrum, where $P(f)$ corresponds to the FFT of the input waveform (column C in Figure 2), or to the spectrographic representation of the signal, where $P(f)$ corresponds to $F(t)$ (column B in Figure 2). The former definition also measures the entropic contribution of background noise and discards any information on temporal variation in frequency, therefore, we calculate Wiener Entropy only on the dominant signal frequency, i.e. column B in Figure 2. In addition, we square-root transform this metric for normality.

**Autocorrelation**

Autocorrelation (Figure 2, column D) measures the self-similarity of a signal, and so quantifies the extent to which the signal contains repetitions of the same pattern, or is varied without similarity (Stoica and Moses 2005). The autocorrelation sequence of a signal consisting of $N$ discrete samples, $F(1...N)$ is measured by shifting the signal by time lag $l$ and calculating the correlation between the shifted and the unshifted signals:

$$ac(l) = \sum_{n=0}^{N} F(n) \cdot F(n - l)$$

The Autocorrelation metric is then calculated as the sum of $ac(l)$ for all $l$. As with Weiner entropy, we measure the repetitiveness of the howl frequency modulation, rather than flatness *per se*, by calculating autocorrelation on the dominant signal frequency in the spectrogram. In addition, we log-transform this metric for normality.

**Inflection points**

A number of studies, particularly with cetaceans, have measured vocal complexity by counting the number of inflection points in a vocalisation (Janik et al. 1994; May-Collado and Wartzok 2008). A more complex signal, in this context, is a signal in which the frequency is changing direction (rising/falling) often (e.g. Figure 2, panel 4B). In keeping with these studies, we define the number of
inflection points as the count of changes in gradient direction of the vocalisation. To ensure that this complexity metric takes continuous values, we divide the number of inflection points by the length of the signal, which also standardises the metric, to remove the correlation between vocalisation length and complexity that would otherwise be present. In addition, we square-root transformed this metric for normality. Simplified Matlab code for counting the number of inflection points in a vector $X$ is shown below:

1. \( g = \text{gradient}(X); \)
2. \( s = \text{sign}(g); \)
3. \( d = g(1:end-1) \neq g(2:end); \)
4. \( C = \text{sum}(d); \)

The above algorithm (1) measures the gradient at each point, (2) determines the sign of the gradient (positive, increasing, or negative, decreasing), (3) tests whether the sign of the gradient at this point is different from the sign of the gradient at the next point, which would indicate a change in direction, and (4) counts the number of such changes in direction.

**Parsons Entropy**

The Parsons code is a reduced representation of a varying signal, used primarily for music retrieval systems (Downie 2003; Parsons and Levin 1975). However, a recent study showed that dolphin signature whistles can be represented as Parsons codes, while maintaining much of the individual identity information (Kershenbaum et al. 2013). To convert a continuous signal to a Parsons code, we divide the signal into a fixed number of slices, and record whether the frequency from one slice to the next is rising, falling, or remaining constant. To increase the descriptive power of the Parsons code, we can distinguish between rises and falls of different magnitude, classifying the 10% of the largest magnitude changes as “big rise” or “big drop” (Müllensiefen and Friele 2004; Pauws 2002), and thus dividing changes in pitch into seven categories: big rise, medium rise, small rise, no change, small drop, medium drop, big drop (Figure 3). The implementation of this coding is described more fully in (Kershenbaum et al. 2013). The continuous signal has now been converted into a series of discrete characters from a finite alphabet of seven step categories, and so we can then calculate the simple
Shannon entropy (Cover and Thomas 1991) as is often done with stereotyped signals, where \( P(n) \) is the probability of occurrence of step category \( n \). We refer to this entropy metric that measures the unpredictability of the Parsons code as the “Parsons Entropy”.

\[
PE = - \sum_{n=1}^{7} P(n) \log_7 P(n)
\]

As each of the four metrics Wiener Entropy, Autocorrelation, Inflection Point Count, and Parsons Entropy are all purported to measure the same property - signal complexity - the metrics may potentially measure similar features of the acoustic signals. Therefore, we test directly for correlation between the different metrics by calculating the Pearson's correlation coefficient between each pair of metrics. All analyses were carried out in Matlab R2014b (The Mathworks, Inc).

RESULTS

**Complexity of simulated waveforms**

The values of the four metrics for each of the five waveforms in Figure 2 are shown in Figure 4. All metrics gave the constant frequency (1) the lowest score, representing the simplest waveform. The random waveform (5) received the highest scores from three of the metrics, Wiener Entropy, Inflection Point Count, and Parsons Entropy, indicating (as with traditional entropy measures) that randomness is interpreted as high complexity. Autocorrelation was low for the random waveform.

The oscillating frequency (4) scored highly for complexity with all metrics (especially Autocorrelation and Parsons Entropy), consistent with the intuitive interpretation of this as a complex signal. However, the two-frequency waveform (2) received similar Wiener Entropy and Autocorrelation scores to constantly increasing frequency (3), but notably higher scores than (3) for Inflection Point Count and Parsons Entropy. Indeed, both Inflection Point Count and Parsons Entropy considered the constantly increasing frequency (3) to be approximately as complex as the constant frequency (1).
Complexity of wolf howls

Examples of howls with the lowest, and highest values of each of the complexity metrics are shown in Figure 5. Wiener Entropy and Autocorrelation illustrate the difficulty of traditional metrics as indicators of complexity. It is not clear that the howls with the lowest Wiener Entropy or Autocorrelation are necessarily less complex by subjective interpretation than those with high Wiener Entropy or Autocorrelation. A low Wiener Entropy score is achieved by a signal possessing a single frequency (i.e. approximating a part of a sine wave), whereas high Wiener Entropy score is achieved by flat signals, which are transformed by FFT to a mixture of a large number of frequencies, and hence high entropy. Neither appear to be particularly complex by intuitive definition. Low Autocorrelation scores are achieved by irregular but not repetitive signals, and such irregularity is a promising trait of complexity; however high Autocorrelation scores are achieved by signals with a single frequency, which does not appear to be either complex or simple.

In contrast, the number of inflection points seems an intuitive measure of complexity, as high Inflection Point Count howls are very varied, whereas low Inflection Point Count howls appear simpler. However, some howls with low Inflection Point Count still have considerable variation in frequency. Parsons Entropy also detects subjectively complex howls, and those with low Parsons Entropy appear subjectively simple; specifically, the frequency of these simple howls varies monotonically.

Colinearity among complexity metrics

Weak correlations existed between all metrics (Figure 6), however, a stronger negative correlation was found between Wiener Entropy and Parsons Entropy ($R=-0.38$). Taken with the other differences found between metrics, there does not appear to be grounds for describing any pair of metrics as co-varying, and the metrics appear to be measuring different aspects of complexity.

DISCUSSION
We have illustrated the similarities and differences between four different metrics, each of which could be considered a quantitative measure of complexity in a continuously varying signal. Despite the poorly defined nature of signal complexity, we have provided the reader with both quantitative comparisons, and qualitative illustrations of the result of using each of these metrics in the evaluation of the complexity of simulated signals and natural canid howls.

All metrics distinguished clearly between a constant flat frequency, and a randomly varying signal, with all metrics except Autocorrelation placing the random signal at the most complex end of the quantitative scale. Researchers will need to consider whether or not the characterisation of a random signal as "complex" (a definition taken from the field of entropy and information theory) is consistent with the hypotheses that they are testing. In contrast to Wiener Entropy, Inflection Point Count, and Parsons Entropy, Autocorrelation gave a higher score to a regularly varying signal than to the random one. Parsons Entropy also gave a much higher value for a regularly varying signal than for the flat, rising, and random frequencies, indicating that Parsons Entropy, which measures changes in slope, is a good metric for measuring the extent to which a signal changes with time - either regularly or irregularly. When examining actual howls qualitatively, both Inflection Point Count and Parsons Entropy appeared to distinguish between howls that the authors felt looked "simple" (i.e. varying in frequency in a constant way) and those that looked "complex" (i.e. varying in an inconsistent way with time), although clearly this subjective distinction may not be globally applicable. When examining the example howls for low and high Wiener Entropy and Autocorrelation, there did not appear to be as much of a subjective difference in complexity. However, the essence of these results is to provide the comparison, rather than to impose subjective conclusions, and Figure 5 makes this comparison clearly.

Despite the fact that complexity in vocal signals of any kind remains poorly defined (Kershenbaum 2013), the concept of complexity is still widely used for investigating questions of proximal behaviour (Darolová et al. 2012; Demartsev et al. 2014; Gustison and Bergman 2016), ultimate fitness (Freeberg et al. 2012; Ord et al. 2012; Pollard and Blumstein 2012), and drivers of the evolution of social systems (Bergman and Beehner 2015; Freeberg and Krams 2015; Krams et al. 2012). Even if we
accept the definition of entropy as a proxy for complexity (Doyle et al. 2008), it is not clear how such a metric can be applied to continuously varying signals. We have shown that multiple approaches are possible, each with its benefits and disadvantages. For example, counting the number of inflection points is a useful method for identifying highly frequency modulated signals, but can become overwhelmed in the presence of a highly random signal. Parsons Entropy may suffer less from this constraint, as the signal is divided into discrete segments. Wiener Entropy measures the noisiness of a signal, but can misinterpret a simple upsweep as complex as it contains many frequencies, albeit spread through time. Autocorrelation is a powerful tool for detecting repetition, but returns a low value for asymmetric changes in frequency.

We have no objective gold-standard of complexity to compare to our metrics and to indicate whether a particular metric truly captures the property of complexity or not. Yet we believe that our study has merit precisely because it allows quantification of the subjective measure for which researchers may be searching. If an oscillating signal is the nature of complexity being tested, then Inflection Point Count or Parsons Entropy may be the best metric to use. However, if for a particular hypothesis, randomness is best rejected as not complex, then perhaps Autocorrelation is a better-suited metric. It is vital for researchers to understand the implications of their choice of a particular complexity metric, rather than to make use of a metric whose properties may be unknown, and perhaps surprising. Our approach of defining a quantitative metric also has the advantage of enabling clearer comparisons between different study systems that may use similar but non-quantitative assessments of vocalisation type. Qualitative descriptions of frequency modulations in continuous signals, e.g. "flat", "rising", "step up" (Hallberg 2007; Palacios et al. 2007) are useful, but difficult to compare between studies.

Vocal signal complexity is likely an important property in the communication of many species, including birds (Briefer et al. 2010; Darolová et al. 2012; Freeberg 2008; Kipper et al. 2006; Leitão et al. 2006), amphibians (Larson 2004; Narins and Capranica 1978), terrestrial mammals (Demartsev et al. 2014; Gustison and Bergman 2016; Schlenker et al. in press) and cetaceans (Doyle et al. 2008; Ferrer-i-Cancho and McCowan 2009; Garland et al. 2013; Nash and Bowles 2011), and also human language (Ferrer-i-Cancho 2005; Ferrer-i-Cancho and Solé 2003; Kershenbaum et al. 2014; Montemurro and Zanette 2015). Complexity is a difficult property to measure, particularly when the
signals are continuously varying, rather than a sequence of discrete notes. To investigate questions such as the connection between social complexity and vocal complexity, appropriate measures of complexity must be found (Freeberg and Krams 2015; Kershenbaum 2013). Species with continuously varying vocalisations, such as wolves and dolphins, share many of the properties of particular interest in the investigation of the evolution of communication. Wolves and dolphins in particular are highly social and cooperative, as well as having intelligent problem-solving abilities (wolves: Mech and Boitani 2010; Mech et al. 2015, dolphins: Gazda et al. 2005; King and Janik 2015; Wells 2003), and so it would be unfortunate if a quantitative assessment of their vocal complexity were neglected.

We have shown that multiple metrics do exist for capturing the complexity of these vocalisations, and we have provided quantitative tools to assess the suitability of the different types of metrics. We encourage researchers to make use of such quantitative measures when testing hypotheses in these and similar species, thereby extending the investigation of complex communication in animals to those species not previously amenable to quantitative analysis.

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References


Figure 1. Six examples of wolf howls, with time on the x-axis and frequency on the y-axis. All howls show some pattern of rising and falling frequencies, but the determination of which howl is most complex appears wholly subjective.
Figure 2. Simulated sounds demonstrating the relationship between waveform, spectrogram, spectrum, and autocorrelation. The first column (A) shows five different waveforms: (1) constant frequency sine wave; (2) rapid doubling in frequency; (3) constantly increasing in frequency; (4) oscillating frequency; and (5) random waveform. The second column (B) shows the frequency of the waveform with time. Column (C) shows overall spectra for these waveforms: a single peak where one frequency is present (1), two peaks where two frequencies exist (2), a range of frequencies in (3) and (4), and all frequencies present in the case of white noise (5). Column (D) shows the autocorrelation of the spectrogram.
Figure 3: Example of a Parsons code representation of a vocal signal (blue). The time-course is divided into ten equal sections (red), and the frequency change for each section is recorded only as "big rise", "small rise", "no change", "small drop", or "big drop".
Figure 4. Complexity scores for the four metrics, for each of the five waveforms shown in Figure 2.
The bar charts are normalised for comparison by subtracting the mean and dividing by the standard deviation.
Figure 5. Examples of howls with low metric values (top row), and high metric values (bottom row), for each of the four metric types. Each plot shows those three sample howls with the highest or lowest values for each particular metric (not necessarily the same sample howls for each metric). The y-axis indicates frequency deviation from the median value of each howl, to allow a clear comparison between howls of differing frequencies.
Figure 6. Pairwise correlation between each of the metrics. The red line indicates line of best fit, and $R$ and $p$ values given in the title of each plot are for Pearson correlation.