Exploring the International Linkages of the Euro Area: a Global VAR Analysis

Stephane Dees, Filippo di Mauro,
M. Hashem Pesaran and L. Vanessa Smith

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Stephane Dees
European Central Bank

Filippo di Mauro
European Central Bank

M. Hashem Pesaran
University of Cambridge and USC

L. Vanessa Smith
University of Cambridge

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Abstract

This paper presents a global model linking individual country vector error-correcting models in which the domestic variables are related to the country-specific variables as an approximate solution to a global common factor model. This global VAR is estimated for 26 countries, the euro area being treated as a single economy. This paper proposes two important extensions of previous research (see Pesaran, Schuermann and Weiner, 2004). First, it provides a theoretical framework where the GVAR is derived as an approximation to a global unobserved common factor model. Also using average pair-wise cross-section error correlations, the GVAR approach is shown to be quite effective in dealing with the common factor interdependencies and international comovements of business cycles. Second, in addition to generalised impulse response functions, we propose an identification scheme to derive structural impulse responses. We focus on identification of shocks to the US economy, particularly the monetary policy shocks, and consider the time profiles of their effects on the euro area. To this end we include the US model as the first country model and consider alternative orderings of the US variables. Further to the US monetary policy shock, we also consider oil price, US equity and US real output shocks.

Keywords: *Global VAR (GVAR), Global interdependencies, global macroeconomic modeling, impulse responses.*

JEL Classification: *C32, E17, F47*

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1 Introduction

Several developments over the past decade have drawn considerable attention to international business cycle linkages among major economies and regions. In particular the question of whether, and to what extent, the recent U.S. slowdown has influenced economic activity elsewhere in the world, especially in the euro area, has been controversial.

At the root of such discussions is the observation that the recent experience with business cycle synchronization seems to have been very different from those before. In particular, there have been remarkable differences in economic activity and business cycles across the major economies in the 1990s and several influential papers in the literature have presented evidence for a lower degree of synchronization since the 1990s.

By contrast, other strands in the literature argue that a rapidly rising degree of financial market integration has induced a closer financial and real international interdependence. Kose, Otrok and Whiteman (2003), using a Bayesian latent factor model in output, consumption and investment for 63 countries find evidence of a world business cycle. Monfort, Renne, Rüffer and Vitale (2003) show that G-7 countries share common dynamics in real economic activity, with clearly identifiable common swings across countries. Data also reveal an important effect of oil price developments in increasing business comovements. Finally, strong and increasing unilateral spill-over effects from North-America area to the European area are being found, often interpreted as being caused by the process of globalization.

In order to bridge the gap between the purely statistical analyses and the traditional modelling approaches, the present paper studies the transmission mechanisms of shocks at the world level using a global VAR (GVAR). Such a framework is able to account for various transmission channels, including not only trade relationships but also financial linkages, most notably through interest rates, stock prices and exchange rates, which have proved to be particularly relevant over the recent past.[1]

Building on the work of Pesaran, Schuermann and Weiner (2004), hereafter PSW, this paper presents a global model covering 33 countries grouped into 25 countries and a single euro area economy comprising 8 of the 11 countries that joined euro in 1999. The 26 economies in the present version of the GVAR model are linked through economy-specific vector error-correcting models in which the domestic and foreign variables are simultaneously inter-related, thus providing a general, yet practical, global modelling framework for a quantitative analysis of the relative importance of different shocks and channels of transmission mechanisms for the analysis of the comovements of outputs, inflation, interest rates, exchange rates and equity prices. To deal with the modelling issues that arise from the creation of the euro area (a single exchange rate and short term interest rate post 1999), the GVAR model presented in this paper is estimated with the euro area being treated as a single economy. This turns out to be economet-

[1] See, for example, Anderton et al. (2004) for an overview.

rically justified and allows us to consider the impact of external shocks on the euro area as a whole without the danger of being subject to possible inconsistencies that could arise if the different economies in the euro area were modeled separately. The effects of external shocks on the euro area will be examined based on different simulations using generalized as well as structural impulse response functions.

Compared to the previous version of the GVAR developed by PSW, the current version, in addition to increasing the geographical coverage, also extends the estimation period, and includes long-term as well as short-term interest rates, thus allowing more fully for the possible effects of bond markets on output, inflation and equity prices.

The present paper also provides a theoretical framework where the GVAR is derived as an approximation to a global unobserved common factor model. Also using average pair-wise cross-section error correlations, the GVAR approach is shown to be quite effective in dealing with the common factor interdependencies and international comovements of business cycles. Second, in addition to generalized impulse responses, we show how to use the GVAR model for the purpose of 'structural' identification. We focus on identification of shocks to the U.S. economy, particularly the monetary policy shocks, and consider the time profiles of their effects on the euro area. Further to the U.S. monetary policy shock, we also consider the effects of shocks to oil prices, U.S. equity prices and U.S. real output on the euro area and the rest of the world.

The plan of the paper is as follows: Section 2 presents the GVAR approach to model international linkages and Section 3 gives details on the version of the GVAR used in this paper. Section 4 examines the ability of the model to account for interdependencies and international comovements by computing pair-wise cross section correlations of the endogenous variables and the associated residuals. Section 5 derives generalized impulse response functions for the analysis of country-specific and global shocks. Section 6 considers the problem of structural identification of shocks to the U.S. economy and their consequences for euro area in particular. Section 7 checks the robustness of the GVAR results to the choice of trade weights by estimating a model using time varying weights. Section 8 discusses the issue of structural breaks in the context of the GVAR model. Section 9 offers some concluding remarks. Appendix A provides a summary of data sources used and Appendix B gives generalized impulse response figures. Detailed results not reported in the main text can be found in a Supplement provided by the authors on request.

2 Modelling International Transmissions: A GVAR Approach

One of the most striking features of the business cycles across countries are the patterns of comovement of output, inflation, interest rates and real equity prices. These comovements have become more pronounced over the past two decades due to increased economic and financial integration, with important implications for macroeconomic policy spillovers across countries. The extent of comovement of real GDP across countries has been empirically investigated by a number of authors, both by considering bivariate correlation of real GDP across countries and by decomposing the variations of real GDP into common and country-specific shocks. Multivariate and multicountry analysis have also been undertaken in the context of G-7 economies. For example, Gregory, Head and Raynauld (1997) using Kalman filtering and dynamic factor analysis provide a decomposition of aggregate output, consumption and investment for G-7 countries into factors that are (i) common across all countries, (ii) common to the aggregates within a given country, and (iii) specific to the individual aggregates. Other similar decompositions have also been attempted by Canova and Marrinan (1998), Lumsdaine and Prasad (2003) and Kose *et al.* (2003).[2]

There are clearly many channels through which the international transmissions of business cycles can take place. In particular, they could be due to common observed global shocks (such as changes in oil prices), they could arise as a result of global unobserved factors (such as the diffusion of technological progress or regional political developments), or could be due to specific national or sectoral shocks.

Unobserved factor models with a large number of macroeconomic variables have recently gained popularity with the work of Stock and Watson (2002). A related literature on dynamic factor models has also been developed by Forni and Reichlin (1998) and Forni, Hallin, Lippi, and Reichlin (2000). The factor models, estimated using principal components, are generally used to summarize by a small set of factors the empirical content of a large number of variables. Although unobserved factor models have important applications in forecasting, the identification of factors is often problematic, especially when we wish to give them an economic interpretation.[3] It is also likely that even when all such "common" factors are taken into account, there will be important residual interdependencies due to policy and trade spillover effects that remain to be explained.

Therefore, a fairly detailed global framework would be needed if we are to investigate the relative importance of such diverse sources of comovements in the world economy, and their impacts on the euro area. For this purpose we make use of the global vector autoregressive model (GVAR) recently developed by PSW.

To motivate the GVAR model for the analysis of the international transmission mechanisms and to relate it to the unobserved factor models, suppose there are $N+1$ countries (or regions) in the global economy, indexed by $i = 0, 1, ..., N$, where country 0 serves as the numeraire country (which we take as the U.S., but could be any other country). The aim is to model a number of country-specific macroeconomic variables such as real GDP, inflation, interest rates and

[2] Other related references include Norrbin and Schlagenhauf (1996), Artis, Kontolmis and Osborn (1997), Bergman, Bordo and Jonung (1998), Clark and Shin (2000), and Kose (2002).

[3] For an attempt at structural identification of factor models see Forni, Lippi and Reichlin (2003).

exchange rates collected in the vector $\mathbf{x}_{it}$, over time, $t = 1, 2..., T$, and across the $N+1$ countries. Given the general nature of interdependencies that might exist in the world economy, it is clearly desirable that all the country-specific variables $\mathbf{x}_{it}$, $i = 0, 1, ..., N$, and observed global factors (such as oil prices) are treated endogenously. The following general factor model provides a good starting point and allows us also to relate the GVAR approach to the more familiar factor models used in the literature primarily for the analysis of G-7 economies.

Denote the observed global factors by the $m_d \times 1$ vector $\mathbf{d}_t$, and the unobserved global factors by the $m_f \times 1$ vector $\mathbf{f}_t$, and assume that[4]

$$\mathbf{x}_{it} = \boldsymbol{\delta}_{i0} + \boldsymbol{\delta}_{i1} t + \boldsymbol{\Gamma}_{id}\mathbf{d}_t + \boldsymbol{\Gamma}_{if}\mathbf{f}_t + \boldsymbol{\xi}_{it}, \text{ for } i = 0, 1, ..., N;\ t = 1, 2, ..., T, \quad (1)$$

where $\boldsymbol{\Gamma}_i = (\boldsymbol{\Gamma}_{id}, \boldsymbol{\Gamma}_{if})$ is the $k_i \times m$, matrix of factor loadings, $m = m_d + m_f$, $\boldsymbol{\xi}_{it}$ is a $k_i \times 1$ vector representing the country-specific effects involving lagged values of $\mathbf{x}_{it}$ or country-specific dummy variables capturing major institutional and political upheavals, and $\boldsymbol{\delta}_{i0}$ and $\boldsymbol{\delta}_{i1}$ are the coefficients of the deterministics, here intercepts and linear trends. Other deterministics, such as seasonal dummies, can also be included in the model. The vector of observed global variables could include international variables such as oil or other commodity prices, world expenditure on R&D, or other indicators of global technology such as the number of international patents registered in the U.S..

Unit root and cointegration properties of $\mathbf{x}_{it}$, $i = 0, 1, ..., N$, can be accommodated by allowing the global factors, $\mathbf{h}_t = (\mathbf{d}_t', \mathbf{f}_t')'$, and/or the country-specific factors, $\boldsymbol{\xi}_{it}$, to have unit roots. More specifically, we assume that

$$\Delta\mathbf{h}_t = \Lambda(L)\,\boldsymbol{\eta}_t,\ \boldsymbol{\eta}_t \sim IID(\mathbf{0}, \mathbf{I}_m), \quad (2)$$

$$\Delta\boldsymbol{\xi}_{it} = \boldsymbol{\Psi}_i(L)\,\mathbf{v}_{it}, \quad \mathbf{v}_{it} \sim IID\left(\mathbf{0}, \mathbf{I}_{k_i}\right), \quad (3)$$

where L is the lag operator and

$$\Lambda(L) = \sum_{\ell=0}^{\infty} \underset{m\times m}{\Lambda_\ell} L^\ell, \quad \boldsymbol{\Psi}_i(L) = \sum_{\ell=0}^{\infty} \underset{k_i\times k_i}{\boldsymbol{\Psi}_{i\ell}} L^\ell. \quad (4)$$

The coefficient matrices, Λ_ℓ and $\boldsymbol{\Psi}_{i\ell}$, $i = 0, 1, ..., N$, are absolute summable, so that $Var(\Delta\mathbf{f}_t)$ and $Var(\Delta\boldsymbol{\xi}_{it})$ are bounded and positive definite, and $[\boldsymbol{\Psi}_i(L)]^{-1}$ exists. In particular we require that

$$Var(\Delta\boldsymbol{\xi}_{it}) = \sum_{\ell=0}^{\infty} \boldsymbol{\Psi}_{i\ell}\boldsymbol{\Psi}_{i\ell}' \leq \mathbf{K} < \infty, \quad (5)$$

where $\mathbf{K}$ is a fixed bounded matrix.

First differencing (1) and using (3) we have

$$[\boldsymbol{\Psi}_i(L)]^{-1}(1-L)(\mathbf{x}_{it} - \boldsymbol{\delta}_{i0} - \boldsymbol{\delta}_{i1} t - \boldsymbol{\Gamma}_{id}\mathbf{d}_t - \boldsymbol{\Gamma}_{if}\mathbf{f}_t) = \mathbf{v}_{it}.$$

[4] Dynamic factor models of Forni and Lippi (1997) can also be accommodated by including lagged values of $\mathbf{d}_t$ and $\mathbf{f}_t$ as additional factors via suitable extensions of $\mathbf{d}_t$ and $\mathbf{f}_t$. For example, $\mathbf{f}_t$ in (1) can be replaced by $\mathbf{f}_t^* = (\mathbf{f}_t', \mathbf{f}_{t-1}', ..., \mathbf{f}_{t-p_f}')'$.

Using the approximation

$$(1-L)\left[\boldsymbol{\Psi}_i(L)\right]^{-1} \approx \sum_{\ell=0}^{p_i} \boldsymbol{\Phi}_{i\ell} L^{\ell} = \boldsymbol{\Phi}_i(L, p_i),$$

we obtain the following approximate VAR(p_i) model:

$$\boldsymbol{\Phi}_i(L, p_i)\left(\mathbf{x}_{it} - \boldsymbol{\delta}_{i0} - \boldsymbol{\delta}_{i1} t - \boldsymbol{\Gamma}_{id}\mathbf{d}_t - \boldsymbol{\Gamma}_{if}\mathbf{f}_t\right) \approx \mathbf{v}_{it}. \tag{6}$$

Without the unobserved common factors, $\mathbf{f}_t$, the model for the i^{th} country decouples from the rest of the country models and each country model can be estimated separately using the econometric techniques developed in Harbo *et al.* (1998) and Pesaran, Shin and Smith (2000) with $\mathbf{d}_t$ treated as weakly exogenous. With the unobserved common factors included, the model is quite complex and its econometric analysis using Kalman filtering techniques would be quite involved unless N is very small. When N is relatively large a simple, yet effective, alternative would be to follow Pesaran (2004a) and proxy $\mathbf{f}_t$ in terms of the cross section averages of country-specific variables, $\mathbf{x}_{it}$, and the observed common effects, $\mathbf{d}_t$. To see how this procedure could be justified in the present more complicated context, initially assume $k_i = k$ and use the same set of weights, w_j, $j = 0, 1, ..., N$, to aggregate the country-specific relations defined by (1) to obtain

$$\begin{aligned}
\sum_{j=0}^{N} w_j \mathbf{x}_{jt} &= \sum_{j=0}^{N} w_j \boldsymbol{\delta}_{j0} + \left(\sum_{j=0}^{N} w_j \boldsymbol{\delta}_{j1}\right) t + \left(\sum_{j=0}^{N} w_j \boldsymbol{\Gamma}_{jd}\right) \mathbf{d}_t \\
&\quad + \left(\sum_{j=0}^{N} w_j \boldsymbol{\Gamma}_{jdf}\right) \mathbf{f}_t + \sum_{j=0}^{N} w_j \boldsymbol{\xi}_{jt},
\end{aligned}$$

or

$$\mathbf{x}_t^* = \boldsymbol{\delta}_0^* + \boldsymbol{\delta}_1^* t + \boldsymbol{\Gamma}_d^* \mathbf{d}_t + \boldsymbol{\Gamma}_f^* \mathbf{f}_t + \boldsymbol{\xi}_t^*. \tag{7}$$

Also, note from (3) that

$$\boldsymbol{\xi}_t^* - \boldsymbol{\xi}_{t-1}^* = \sum_{j=0}^{N} w_j \boldsymbol{\Psi}_j(L)\, \mathbf{v}_{jt}. \tag{8}$$

But using Lemma A.1 in Pesaran (2004a), it is easily seen that for each t the left hand side of (8) will converge to zero in quadratic mean as $N \to \infty$, if (5) holds, the country specific shocks, $\mathbf{v}_{jt}$, are independently distributed across j, and if the weights, w_j, satisfy the atomistic conditions

$$\text{(i): } w_j = O\left(\frac{1}{N}\right), \text{ (ii): } \sum_{j=1}^{N} |w_j| < K, \tag{9}$$

where K is a fixed constant. Under these conditions (for each t)

$$\boldsymbol{\xi}_t^* - \boldsymbol{\xi}_{t-1}^* \overset{q.m.}{\to} 0,$$

and hence

$$\boldsymbol{\xi}_t^* \overset{q.m.}{\to} \boldsymbol{\xi}^*,$$

where $\boldsymbol{\xi}^*$ is a time-invariant random variable. Using this result in (7) and assuming that the $k \times m_f$ average factor loading coefficient matrix, $\boldsymbol{\Gamma}_f^*$, has full column rank (with $k \geq m_f$) we obtain

$$\mathbf{f}_t \overset{q.m.}{\to} \left(\boldsymbol{\Gamma}_f^{*\prime}\boldsymbol{\Gamma}_f^*\right)^{-1}\boldsymbol{\Gamma}_f^*\left(\mathbf{x}_t^* - \boldsymbol{\delta}_0^* - \boldsymbol{\delta}_1^* t - \boldsymbol{\Gamma}_d^*\mathbf{d}_t - \boldsymbol{\xi}^*\right),$$

which justifies using the observable vector $\{1, t, \mathbf{d}_t, \mathbf{x}_t^*\}$ as proxies for the unobserved common factors.[5] Substituting this result in (6), for N sufficiently large we have

$$\boldsymbol{\Phi}_i\left(L, p_i\right)\left(\mathbf{x}_{it} - \tilde{\boldsymbol{\delta}}_{i0} - \tilde{\boldsymbol{\delta}}_{i1} t - \tilde{\boldsymbol{\Gamma}}_{id}\mathbf{d}_t - \tilde{\boldsymbol{\Gamma}}_{if}\mathbf{x}_t^*\right) \approx \mathbf{v}_{it}, \tag{10}$$

where $\tilde{\boldsymbol{\delta}}_{i0}, \tilde{\boldsymbol{\delta}}_{i1}$, $\tilde{\boldsymbol{\Gamma}}_{id}$ and $\tilde{\boldsymbol{\Gamma}}_{if}$ are given in terms of $\boldsymbol{\delta}_{i0}, \boldsymbol{\delta}_{i1}, \boldsymbol{\Gamma}_{id}, \boldsymbol{\Gamma}_{if}$, $\boldsymbol{\delta}_0^* + \boldsymbol{\xi}^*$, $\boldsymbol{\delta}_1^*, \boldsymbol{\Gamma}_d^*$, and $\boldsymbol{\Gamma}_f^*$.

In practice, the number of countries, $N+1$, may not be sufficiently large, and the individual countries not equally important in the global economy. The country-specific shocks might also be cross sectionally correlated due to spatial or contagion effects that are not totally eliminated by the common factors, $\mathbf{d}_t$ and $\mathbf{f}_t$. Finally, k_i, the number of country-specific variables, need not be the same across i. For example, some markets may not exist or might not be sufficiently developed in some of the countries. Even if we focus on the same set of variables to model across countries, there will be one less exchange rate than there are countries in the global model. The GVAR framework developed in PSW addresses these considerations by using country-specific weights, w_{ij}, rather the the common weights w_j used above, in construction of cross section averages. Specifically, instead of using the same $\mathbf{x}_t^*$ in all country models PSW use

$$\mathbf{x}_{it}^* = \sum_{j=0}^{N} w_{ij}\mathbf{x}_{jt}, \text{ with } w_{ii} = 0, \tag{11}$$

in the i^{th} country model. The weights, w_{ij}, $j = 0, 1, ..., N$ could be used to capture the importance of country j for country i^{th} economy. Geographical patterns of trade provide an obvious source of information for this purpose and could also be effective in mopping up some of the remaining spatial dependencies. The weights could also be allowed to be time-varying so long as they are pre-determined. This could be particularly important in the case of rapidly expanding emerging economies with their fast changing trade relations with the rest of the world. The use of the country-specific weights also allows a simple solution to the problem of k_i, the number of country-specific variables, being different across i. It would be sufficient to attach zero weights to the missing

[5] In a much simpler context Pesaran (2004a) shows that it would still be valid to use $\{1, t, \mathbf{d}_t, \mathbf{x}_t^*\}$ as a proxy for $\mathbf{f}_t$ even if the rank condition is not satisfied. It seems reasonable to believe that the same would apply here.

variable in country i, with the remaining weights being re-balanced to add up to unity.

With the above considerations in mind, the GVAR counter part of (10) may now be written more generally as the individual country VARX$^*(p_i, q_i)$ models:

$$\mathbf{\Phi}_i (L, p_i) \mathbf{x}_{it} = \mathbf{a}_{i0} + \mathbf{a}_{i1} t + \mathbf{\Upsilon}_i (L, q_i) \mathbf{d}_t + \mathbf{\Lambda}_i (L, q_i) \mathbf{x}^*_{it} + \mathbf{u}_{it}, \tag{12}$$

for $i = 0, 1, ..., N$, where for estimation purposes $\mathbf{\Phi}_i (L, p_i)$, $\mathbf{\Upsilon}_i (L, q_i)$ and $\mathbf{\Lambda}_i (L, q_i)$ can be treated as unrestricted. These country-specific models can now be consistently estimated separately, treating $\mathbf{d}_t$ and $\mathbf{x}^*_{it}$ as weakly exogenous, which is compatible with a certain degree of weak dependence across $\mathbf{u}_{it}$. The weak exogeneity of these variables can then be tested in the context of each of the country-specific models.[6]

Once the individual country models are estimated, all the $k = \sum_{i=0}^{N} k_i$ endogenous variables of the global economy, collected in the $k \times 1$ vector $\mathbf{x}_t = (\mathbf{x}'_{0t}, \mathbf{x}'_{1t}, ..., \mathbf{x}'_{Nt})'$, need to be solved simultaneously. PSW show how this can be done in the case where $p_i = q_i = 1$. In the present more general context we first re-write (12) as[7]

$$\mathbf{A}_i(L, p_i, q_i)\mathbf{z}_{it} = \boldsymbol{\varphi}_{it}, \text{ for } i = 0, 1, 2, ..., N \tag{13}$$

where

$$\begin{aligned} \mathbf{A}_i(L, p_i, q_i) &= [\mathbf{\Phi}_i (L, p_i),\ -\mathbf{\Lambda}_i (L, q_i)],\ \mathbf{z}_{it} = \begin{pmatrix} \mathbf{x}_{it} \\ \mathbf{x}^*_{it} \end{pmatrix}, \\ \boldsymbol{\varphi}_{it} &= \mathbf{a}_{i0} + \mathbf{a}_{i1} t + \mathbf{\Upsilon}_i (L, q_i) \mathbf{d}_t + \mathbf{u}_{it}. \end{aligned}$$

Let $p = \max(p_0, p_1, ..., p_N, q_0, q_1, ..., q_N)$ and construct $\mathbf{A}_i(L, p)$ from $\mathbf{A}_i(L, p_i, q_i)$ by augmenting the $p - p_i$ or $p - q_i$ additional terms in powers of L by zeros. Also note that

$$\mathbf{z}_{it} = \mathbf{W}_i \mathbf{x}_t, \quad i = 0, 1, 2, ..., N, \tag{14}$$

where $\mathbf{W}_i$ is a $(k_i + k^*_i) \times k$ matrix, defined by the country specific weights, w_{ji}.

With the above notations (13) can be written equivalently as

$$\mathbf{A}_i(L, p)\mathbf{W}_i \mathbf{x}_t = \boldsymbol{\varphi}_{it},\ i = 0, 1, ..., N,$$

and then stack to yield the VAR(p) model in $\mathbf{x}_t$:

$$\mathbf{G}(L, p) \mathbf{x}_t = \boldsymbol{\varphi}_t, \tag{15}$$

where

$$\mathbf{G}(L, p) = \begin{pmatrix} \mathbf{A}_0(L, p)\mathbf{W}_0 \\ \mathbf{A}_1(L, p)\mathbf{W}_1 \\ \vdots \\ \mathbf{A}_N(L, p)\mathbf{W}_N \end{pmatrix},\ \boldsymbol{\varphi}_t = \begin{pmatrix} \boldsymbol{\varphi}_{0t} \\ \boldsymbol{\varphi}_{1t} \\ \vdots \\ \boldsymbol{\varphi}_{Nt} \end{pmatrix}. \tag{16}$$

[6] For further details see PSW.

[7] Here we are assuming that $\mathbf{d}_t$ is globally exogenous. But it is easy to adapt the solution approach to allow for the case where $\mathbf{d}_t$ is included in one of the models as endogenous.

The GVAR(p) model, (15), can now be solved recursively, and used for forecasting or generalized impulse response analysis in the usual manner. The issue of structural impulse response analysis poses special problems in the context of the GVAR model and will be dealt with in Section 6.

3 The GVAR Model (1979-2003)

The version of the GVAR model developed in this paper covers 33 countries, where 8 of the 11 countries that originally joined euro on January 1, 1999 are grouped together, and the remaining 25 countries are modeled individually (see Table 1). The present GVAR model, therefore, contains 26 countries/regions. The original PSW model contained 11 countries/regions based on 25 countries. With increased country coverage, the countries in the present GVAR model account for 90% of world output as compared to 80% covered by the 11 countries/regions in PSW.

Table 1: Countries and Regions in the GVAR Model

Unites States	**Euro Area**	**Latin America**
China	Germany	Brazil
Japan	France	Mexico
United Kingdom	Italy	Argentina
	Spain	Chile
Other Developed Economies	Netherlands	Peru
Canada	Belgium	
Australia	Austria	
New Zealand	Finland	
Rest of Asia	**Rest of W.Europe**	**Rest of the World**
Korea	Sweden	India
Indonesia	Switzerland	South Africa
Thailand	Norway	Turkey
Philippines		Saudi Arabia
Malaysia		
Singapore		

The models are estimated over the period 1979(2)-2003(4). This considerably extends the 11 country/region models estimated in PSW over the shorter period 1979(2)-1999(4), most notably including the first years of EMU. The variables included in the current version of the GVAR differ also from those considered by PSW. In order to capture more fully the possible effects of bond markets on output and inflation we now include, wherever possible, both a short rate (ρ_{it}^S), as well as a long rate of interest (ρ_{it}^L). However, given the data limitations and problems associated with compiling comparable money supply measures we have decided against the inclusion of real money balances in the current version. Other variables included are real output (y_{it}), the rate of inflation, ($\pi_{it} = p_{it} - p_{i,t-1}$), the real exchange rate ($e_{it} - p_{it}$), and real equity prices (q_{it}), when available. More specifically

$$\begin{array}{c} y_{it} = \ln\left(GDP_{it}/CPI_{it}\right), \quad p_{it} = \ln(CPI_{it}), \\ q_{it} = \ln(EQ_{it}/CPI_{it}), \;\; e_{it} = \ln(E_{it}), \\ \rho_{it}^{S} = 0.25 * \ln(1 + R_{it}^{S}/100), \;\; \rho_{it}^{L} = 0.25 * \ln(1 + R_{it}^{L}/100), \end{array} \tag{17}$$

where

$$\begin{array}{rcl} GDP_{it} & = & \text{Nominal Gross Domestic Product of country } i \\ & & \text{during period } t, \text{ in domestic currency}, \\ CPI_{it} & = & \text{Consumer Price Index in country } i \text{ at time } t, \\ & & \text{equal to 1.0 in a base year (1995)}, \\ EQ_{it} & = & \text{Nominal Equity Price Index}, \\ E_{it} & = & \text{Exchange rate of country } i \text{ at time } t \text{ in terms of } U.S. \text{ dollars}, \\ R_{it}^{S} & = & \text{Short rate of interest per annum, in per cent (typically a three month rate)} \\ R_{it}^{L} & = & \text{Long rate of interest per annum, in per cent (typically a ten year rate)} \end{array}$$

The country-specific foreign variables, $y_{it}^*, \pi_{it}^*, q_{it}^*, \rho_{it}^{*S}, \rho_{it}^{*L}$, were constructed using trade weights. Baxter and Kouparitsas (2004) in studying the determinants of business cycle comovements conclude that bilateral trade is the most important source of inter country business cycle linkages.[8] Initially, we use fixed trade weights based on the average trade flows computed over the three years 1999-2001. Allowing for time-varying trade weights is straightforward and is considered in Section 7.

The time series data for the euro area was constructed by cross section weighted averages of $y_{it}, \pi_{it}, q_{it}, \rho_{it}^{S}, \rho_{it}^{L}$, over Germany, France, Italy, Spain, Netherlands, Belgium, Austria and Finland, using the average Purchasing Power Parity GDP weights, also computed over the 1999-2001 period.

With the exception of the U.S. model, all models include the country-specific foreign variables, $y_{it}^*, \pi_{it}^*, q_{it}^*, \rho_{it}^{*S}, \rho_{it}^{*L}$ and the log of oil prices (p_t^o), as weakly exogenous. In the case of the U.S. model, oil prices are included as an endogenous variable, with $e_{US,t}^* - p_{US,t}^*$, $y_{US,t}^*$, and $\pi_{US,t}^*$ as weakly exogenous. Given the importance of the U.S. financial variables in the global economy, the U.S.-specific foreign financial variables, $q_{US,t}^*$, $R_{US,t}^{*S}$ and $R_{US,t}^{*L}$, were not included in the U.S. model as they are unlikely to be weakly exogenous with respect to the U.S. domestic financial variables. The U.S.-specific foreign output and inflation variables, $y_{US,t}^*$ and $\pi_{US,t}^*$, were , however, included in the U.S. model (which were not included by PSW) in order to capture the possible second round effects of

[8] Imbs (2004) also provides further evidence on the effect of trade on business cycle synchronization. He concludes that whilst specialization patterns have a sizable effect on business cycles, trade continues to play an important role in this process. He also notes that economic regions with strong financial links are significantly more synchronized. Focusing on global linkages in financial markets, Forbes and Chinn (2004) also show that direct trade appears to be one of the most important determinants of cross-country linkages.

external shocks on the U.S.. Given the importance of the U.S. for the global economy, initially it was thought that the inclusion of $y^*_{US,t}$ and $\pi^*_{US,t}$ as weakly exogenous in the U.S. model might result in the violation of the weak exogeneity assumption. However, as reported below this turns out not to be the case.

In this paper, as the focus is mainly on the impact of external shocks on the euro area economy, we will from now concentrate the presentation of the results to countries/regions with special relevance to the euro area: United States, China, Japan, euro area, United Kingdom and rest of Western Europe. A more detailed set of results are available in a Supplement that can be obtained from the authors on request.

3.1 Trade Weights and Aggregation Weights

The trade shares used to construct the country-specific foreign variables (the "starred" variables) are given in the 26 by 26 trade share matrix provided in the Supplement. Table 2 below presents the trade shares for our eight focus economies (seven countries plus euro area itself composed of eight countries), with a 'Rest' category showing the trade shares with the remaining 10 countries in our sample. First considering the euro area, we can see that the U.S., the U.K. and the rest of Western Europe have a similar share in euro area trade (around 1/5) accounting together for almost two third of total euro area trade. Other important information that emerges from the trade matrix includes the very high share of the euro area in the trade of the U.K. and the rest of Western Europe (more than half of the trade relationships of these countries are with euro area countries). Hence, these countries are key in the transmission of shocks to the euro area via third market, or through second-round effects.

Table 2: Trade Weights Based on Direction of Trade Statistics

Country/						Rest of W.Europe			Rest*
Region	U.S.	E.A.	China	Japan	U.K.	Sweden	Switz.	Norway	
U.S.	0.000	0.155	0.073	0.124	0.052	0.008	0.012	0.004	0.571
E.A.	0.227	0.000	0.056	0.072	0.238	0.057	0.090	0.028	0.232
China	0.236	0.164	0.000	0.248	0.029	0.010	0.007	0.003	0.304
Japan	0.319	0.132	0.128	0.000	0.032	0.007	0.009	0.003	0.371
U.K.	0.180	0.537	0.020	0.042	0.000	0.027	0.028	0.023	0.143
Sweden	0.104	0.517	0.025	0.035	0.115	0.000	0.017	0.099	0.088
Switz.	0.113	0.670	0.015	0.039	0.066	0.015	0.000	0.004	0.079
Norway	0.090	0.449	0.020	0.030	0.181	0.132	0.008	0.000	0.089

Note: Trade weights are computed as shares of exports and imports displayed in rows by region such that a row, but not a column, sums to one.

*"Rest" gathers the remaining countries. The complete trade matrix used in the GVAR model is given in a Supplement that can be obtained from the authors on request. Source: Direction of Trade Statistics, 1999-2001, IMF.

Although we estimate models at a country level (the euro area being considered here as a single economy), we also wish to derive regional responses to shocks. Hence, for the rest of Western Europe (and also for rest of Asia, Latin America, Other Developed Countries and rest of the world), we will aggregate impulse response functions by using weights based on the PPP valuation of country GDPs, which are thought to be more reliable than weights based on U.S. dollar GDPs.

3.2 Unit Root Tests

Although the GVAR methodology can be applied to stationary and/or integrated variables, here we follow PSW and assume that the variables included in the country-specific models are integrated of order one (or $I(1)$). This allows us to distinguish between short run and long run relations and interpret the long run relations as cointegrating. Therefore, we begin by examining the integration properties of the individual series under consideration. In view of the widely accepted poor power performance of traditional Dickey-Fuller (DF) tests, we report unit root t-statistics based on weighted symmetric estimation of ADF type regressions introduced by Park and Fuller (1995). These tests, henceforth WS, exploit the time reversibility of stationary autoregressive processes in order to increase their power performance. Leybourne *et al.* (2004) and Pantula *et al.* (1995) provide evidence of superior performance of the WS test statistic compared to the standard ADF test or the GLS-ADF test proposed by Elliot *et al.* (1996). The lag length employed in the WS unit root tests is selected by the Akaike Information Criterion (AIC) based on standard ADF regressions. Table 3 presents WS statistics for the level, first difference and the second differences of all the country-specific domestic variables in the GVAR model, namely the domestic variables plus the oil prices, whilst Table 4 summarizes the test results for the country-specific foreign variables.[9]

Real output, interest rates (short and long), exchange rates and real equity prices (domestic and foreign) are $I(1)$ across the focus countries, with two notable exceptions. First, real output in the U.K. appears borderline $I(0)/I(1)$ according to the WS statistics, although ADF tests indicate that U.K. real output is $I(1)$. Second, e^* in the U.S. model is an $I(2)$ variable. As in PSW, we deal with this problem by including the real exchange rate $(e - p)$ instead of the nominal exchange rate variable, e, in the different country-specific models. Unit root tests applied to $(e-p)$ and $(e^* - p^*)$ indicate that these variables are $I(1)$ in all cases. Finally, consumer price indices turn out to be $I(2)$, so that inflation (Δp and Δp^*) appears to be $I(1)$ across all countries. The test results also generally support the unit root hypothesis in the case of the variables for the remaining countries except for $(e-p)$ and (e^*-p^*) for Canada and (e^*-p^*) for Mexico. See Supplement available by the authors on request.

[9]Details of the computation of the WS statistics can be obtained from the authors on request.

Table 3: Unit Root Test Statistics for Domestic Variables

Domestic Variables	U.S.	E.A.	China	Japan	U.K.	Sweden	Switz.	Norway
	Country/Region[a]							
y	-2.76	-2.44	-3.75	-1.35	-3.64	-2.83	-2.36	-2.60
Δy	-6.93	-4.60	-3.34	-3.46	-3.21	-14.51	-6.77	-5.43
$\Delta^2 y$	-6.99	-8.12	-9.90	-14.30	-11.98	-8.90	-7.32	-8.02
p	-0.11	-2.01	-3.61	-0.60	-0.39	-0.77	-1.48	-1.51
Δp	-0.07	0.28	-3.16	-0.54	-0.61	-1.18	-1.81	-1.38
$\Delta^2 p$	-13.91	-11.15	-4.90	-13.45	-6.44	-12.20	-12.77	-12.98
q	-2.07	-3.05	-	-1.46	-1.47	-2.50	-1.23	-3.06
Δq	-7.52	-4.26	-	-6.79	-8.46	-7.14	-9.39	-5.53
$\Delta^2 q$	-8.68	-12.43	-	-6.83	-7.46	-11.15	-7.96	-7.10
e	-	-2.45	-0.88	-2.43	-2.40	-2.80	-2.73	-2.10
Δe	-	-7.17	-9.08	-4.25	-7.96	-3.87	-7.92	-7.60
$\Delta^2 e$	-	-8.90	-8.46	-9.09	-8.38	-7.01	-7.70	-7.26
ρ^S	-1.17	-1.26	-1.20	-1.61	-1.76	-1.95	-1.77	-1.77
$\Delta\rho^S$	-3.63	-5.51	-7.68	-5.65	-10.95	-10.16	-5.14	-10.73
$\Delta^2\rho^S$	-10.40	-9.34	-7.17	-7.79	-7.60	-8.40	-7.71	-7.64
ρ^L	-3.84	-2.71	-	-1.95	-3.59	-3.18	-2.41	-0.98
$\Delta\rho^L$	-7.75	-4.58	-	-8.87	-7.74	-6.34	-6.22	-6.73
$\Delta^2\rho^L$	-7.05	-7.59	-	-9.88	-8.11	-7.07	-7.51	-7.28
p^0	-2.86	-	-	-	-	-	-	-
Δp^0	-5.61	-	-	-	-	-	-	-
$\Delta^2 p^0$	-8.00	-	-	-	-	-	-	-
$e-p$	-	-2.09	-2.22	-2.09	-2.61	-2.58	-2.18	-2.20
$\Delta(e-p)$	-	-7.48	-4.02	-8.11	-7.90	-3.89	-7.95	-7.70
$\Delta^2(e-p)$	-	-8.96	-13.00	-9.60	-8.40	-12.03	-12.47	-7.11

Note: The WS statistics are based on univariate AR(p) specifications in the level of the variables with p≤ 5, and the statistics for the level, first differences and second differences of the variables are all computed based on the same sample period, namely, 1980Q2-2003Q4. The WS statistics for all level variables are based on regressions including a linear trend, except for the interest rate variables. The 95% critical value of the WS test for a regression with a linear trend is -3.24, and for a regression with an intercept only is -2.55.

[a] The unit root test statistics for all the countries are given in a Supplement that can be obtained from the authors on request.

Table 4: Unit Root Test Statistics for Foreign Variables

Foreign	Country/Region[a]							
Variables	U.S.	E.A.	China	Japan	U.K.	Sweden	Switz.	Norway
y^*	-3.84	-2.72	-1.68	-2.45	-2.50	-2.63	-2.45	-2.96
Δy^*	-5.20	-4.75	-5.83	-5.66	-4.86	-4.88	-4.65	-5.43
$\Delta^2 y^*$	-5.84	-6.72	-13.23	-6.60	-6.51	-6.32	-6.11	-6.75
p^*	-1.02	-0.26	-0.60	-0.79	-0.77	-0.92	-0.81	-1.11
Δp^*	-1.25	-1.68	-0.53	-0.94	-0.22	-0.05	-0.53	-0.45
$\Delta^2 p^*$	-11.75	-5.26	-11.54	-4.35	-13.60	-5.90	-13.00	-5.56
q^*	-	-2.32	-2.12	-2.38	-2.94	-2.80	-2.86	-2.71
Δq^*	-	-6.98	-6.55	-7.28	-4.47	-4.53	-4.39	-7.18
$\Delta^2 q^*$	-	-11.18	-10.92	-11.00	-12.11	-12.31	-12.25	-11.94
e^*	1.18	-0.87	-2.16	-1.26	-2.50	-1.86	-2.50	-2.53
Δe^*	-2.38	-6.86	-7.29	-7.20	-7.21	-7.33	-7.21	-7.21
$\Delta^2 e^*$	-10.71	-7.16	-9.34	-7.77	-9.07	-9.09	-9.07	-8.87
ρ^{*S}	-	-1.38	-1.26	-1.01	-0.77	-0.93	-0.98	-0.98
$\Delta \rho^{*S}$	-	-9.63	-4.65	-5.34	-7.46	-8.10	-7.91	-9.81
$\Delta^2 \rho^{*S}$	-	-9.60	-8.82	-8.62	-8.81	-9.12	-9.22	-8.45
ρ^{*L}	-	-3.88	-2.79	-3.17	-2.24	-2.26	-2.30	-2.46
$\Delta \rho^{*L}$	-	-5.50	-4.93	-5.56	-5.05	-5.06	-4.92	-5.25
$\Delta^2 \rho^{*L}$	-	-7.83	-7.33	-7.57	-7.13	-7.46	-7.09	-7.58
p^0	-	-2.86	-2.86	-2.86	-2.86	-2.86	-2.86	-2.86
Δp^0	-	-5.61	-5.61	-5.61	-5.61	-5.61	-5.61	-5.61
$\Delta^2 p^0$	-	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00	-8.00
$e^* - p^*$	-2.40	-1.99	-1.61	-1.93	-1.95	-2.05	-2.03	-2.02
$\Delta(e^* - p^*)$	-8.26	-7.34	-7.55	-6.93	-7.47	-7.55	-7.49	-7.43
$\Delta^2(e^* - p^*)$	-10.90	-7.46	-9.47	-9.89	-9.08	-9.05	-9.07	-8.83

Note: The WS statistics are based on univariate AR(p) specifications in the level of the variables with p≤ 5, and the statistics for the level, first differences and second differences of the variables are all computed based on the same sample period, namely, 1980Q2-2003Q4. The WS statistics for all level variables are based on regressions including a linear trend, except for the interest rate variables. The 95% critical value of the WS test for a regression with a linear trend is -3.24, and for a regression with an intercept only is -2.55.

[a] The unit root test statistics for all the countries are given in a Supplement that can be obtained from the authors on request.

3.3 Specification and Estimation of the Country-Specific Models

Based on the unit root test results and the available variables we specify different country-specific models as follows. First, for the euro area, Japan, the UK, and countries belonging to the rest of Western Europe, we include real output (y), inflation rate (Δp), short-term interest rate (ρ^S), long-term interest rate (ρ^L), real equity prices (q) and real exchange rate ($e - p$) as endogenous variables and foreign real output (y^*), foreign inflation (Δp^*), foreign real equity prices (q^*), foreign interest rates (short - ρ^{*S} - and long - ρ^{*L} -) and oil prices (p^o) as weakly exogenous variables. In the case of China, owing to data constraints, real equity prices and long-term interest rates are excluded from the set of endogenous variables. The U.S. model contains y, Δp, ρ^S, ρ^L, q and oil prices (p^o), as the endogenous variables. The U.S. dollar exchange rate is determined outside the U.S. model. As in PSW the only exchange rate included in the U.S. model is the foreign real exchange rate variable, ($e^*_{US} - p^*_{US}$) which is treated as weakly exogenous. The inclusion of oil prices in the U.S. model as endogenous, allows the evolution of the global macroeconomic variables to influence oil prices, a feature which was absent from the PSW version which treated oil prices as weakly exogenous in all country-specific models. Furthermore, unlike the PSW version, the present specification includes U.S.-specific foreign real output (y^*_{US}) and foreign inflation (Δp^*_{US}) as weakly exogenous variables. This allows for the U.S. model to be more fully integrated in the world economy and hence to take a more satisfactory account of second round effects in the global economic system as a whole. It is, of course, important that the weak exogeneity of these variables in the U.S. model are tested, and this is done below.

Once the variables to be included in the different country models are specified, the corresponding cointegrating VAR models are estimated and the rank of their cointegrating space determined. Initially we select the order of the individual country VARX*(p_i, q_i) models. It should be noted that p_i, the lag order of the domestic variables, and q_i the lag order of the foreign ('star') variables in VARX* models need not be the same. In the empirical analysis that follows we entertain the case where the lag order of the domestic variables, p_i, is selected according to the Akaike information criterion. Due to data limitations, the lag order of the foreign variables, q_i, is set equal to one in all countries with the exception of the U.S., the euro area and the U.K.. For the same reason, we do not allow $p_{\max i}$ or $q_{\max i}$ to be greater than two. We then proceed with the cointegration analysis, where the country specific models are estimated subject to reduced rank restrictions. To this end, the error-correction forms of the individual country equations given by (12) are derived.

The rank of the cointegrating space for each country/region was computed using Johansen's trace and maximal eigenvalue statistics as set out in Pesaran, Shin and Smith (2000) for models with weakly exogenous $I(1)$ regressors, in the case where unrestricted constants and restricted trend coefficients are included in the individual country error correction models.

Table 5 presents the cointegration rank statistics for the euro area, Japan, the U.K. and the rest of Western Europe. Tables 6 and 7 present these statistics for China and the U.S., respectively. The order of the VARX* models as well as the number of cointegration relationships are presented in Table 8. Among the countries of interest, the VARX* models have an order of 2 for domestic variables (except for Switzerland and Japan whose lag order is 1) and 1 for foreign variables. For the U.S. and the euro area, the main countries of focus, as well as the main trading partner of the euro area, the U.K., we decided to allow for richer dynamics in the associated VARX* models by setting $q_i = 2$. This decision was corroborated by the residual serial correlation test results shown in Table 9.

As regards the number of cointegrating relationships, we find 4 for Japan, 3 for U.K.[10], Sweden and Switzerland, 2 for the euro area, Norway and the U.S. and 1 for China. The cointegration results are based on the trace statistic (at the 95% critical value level), which is known to yield better small sample power results compared to the maximal eigenvalue statistic.

Table 5: Cointegration Rank Statistics for the euro area, Japan, the UK and the rest of Europe

				Country/Region[a]				Critical Values	
H_0	H_1	E.A.	Japan	U.K.	Sweden	Switz.	Norway	95%	90%
Maximum Eigenvalue Statistics									
$r = 0$	$r = 1$	77.82	199.44	97.68	78.23	97.35	110.91	63.52	60.13
$r \leq 1$	$r = 2$	55.97	83.88	69.90	68.97	76.39	65.11	57.13	53.86
$r \leq 2$	$r = 3$	38.37	43.91	49.51	39.06	56.46	46.16	50.64	47.46
$r \leq 3$	$r = 4$	31.74	39.11	35.33	33.87	33.85	28.87	43.94	40.89
$r \leq 4$	$r = 5$	24.06	33.66	26.02	27.92	20.19	20.45	36.84	33.91
$r \leq 5$	$r = 6$	15.80	19.46	14.43	18.92	16.77	12.44	28.81	25.98
Trace Statistics									
$r = 0$	$r > 1$	243.76	419.45	292.87	266.98	301.01	283.95	197.7	190.81
$r < 1$	$r \geq 2$	165.94	220.01	195.19	188.75	203.66	173.04	156.44	150.23
$r \leq 2$	$r \geq 3$	109.97	136.13	125.29	119.78	127.27	107.92	119.03	113.57
$r \leq 3$	$r \geq 4$	71.60	92.23	75.79	80.72	70.81	61.76	85.44	80.74
$r \leq 4$	$r \geq 5$	39.85	53.12	40.45	46.85	36.96	32.89	55.5	51.66
$r \leq 5$	$r \geq 6$	15.80	19.46	14.43	18.92	16.77	12.44	28.81	25.98

[a] Test results for the remaining countries are provided in a Supplement that can be obtained from the authors on request.

[10] In a similar modelling approach, Garratt, Lee, Pesaran, and Shin (2003) find 5 cointegration relationships for the U.K. model. However, this different outcome may be due to the fact that they use a much larger dataset. We also allowed for 5 cointegration relationships for the U.K. model. The results were very similar.

Table 6: Cointegration Rank Statistics for China

H_0	H_1	China	Critical Values 95%	90%
Maximum Eigenvalue Statistics				
$r = 0$	$r = 1$	104.47	50.64	47.46
$r \leq 1$	$r = 2$	29.29	43.94	40.89
$r \leq 2$	$r = 3$	23.38	36.84	33.91
$r \leq 3$	$r = 4$	13.56	28.81	25.98
Trace Statistics				
$r = 0$	$r > 1$	170.70	119.03	113.57
$r < 1$	$r \geq 2$	66.24	85.44	80.74
$r \leq 2$	$r \geq 3$	36.94	55.5	51.66
$r \leq 3$	$r \geq 4$	13.56	28.81	25.98

Table 7: Cointegration Rank Statistics for the US

H_0	H_1	U.S.	Critical Values 95%	90%
Maximum Eigenvalue Statistics				
$r = 0$	$r = 1$	92.49	54.24	51.08
$r \leq 1$	$r = 2$	56.87	47.99	44.96
$r \leq 2$	$r = 3$	32.77	41.66	38.76
$r \leq 3$	$r = 4$	20.37	35.19	32.43
$r \leq 4$	$r = 5$	16.18	28.43	25.83
$r \leq 5$	$r = 6$	6.76	20.98	18.56
Trace Statistics				
$r = 0$	$r > 1$	225.45	158.01	151.94
$r < 1$	$r \geq 2$	132.96	122.96	117.56
$r \leq 2$	$r \geq 3$	76.08	91.81	87.09
$r \leq 3$	$r \geq 4$	43.31	64.54	60.53
$r \leq 4$	$r \geq 5$	22.94	41.03	37.76
$r \leq 5$	$r \geq 6$	6.76	20.98	18.56

Table 8: VARX* Order and Number of Cointegration Relationships in the Country-Specific Models

	VARX*(p_i, q_i)		# Cointegrating
Country[a]	p_i	q_i	Relationships
United States	2	2	2
Euro Area	2	2	2
China	2	1	1
Japan	1	1	4
United Kingdom	2	2	3
Sweden	2	1	3
Switzerland	1	1	3
Norway	2	1	2

[a] Test results for the remaining countries are provided in a Supplement that can be obtained from the authors on request..

Table 9: F Statistics for Tests of Residual Serial Correlation for Country-Specific VARX* Models

	VARX*(p_i, q_i)			Domestic Variables						
Countries	p_i	q_i		y	Δp	q	$e-p$	ρ^S	ρ^L	p^o
U.S.	2	1	F (4,73)	0.88	2.15	1.51	-	2.70*	0.64	2.98*
	2	2	F (4,70)	0.53	1.84	0.99	-	3.08*	1.83	2.66*
E.A.	2	1	F (4,67)	0.76	6.45*	1.68	1.41	1.48	1.40	-
	2	2	F (4,61)	0.75	6.16*	0.82	0.99	1.58	1.14	-
China	2	1	F(4,71)	1.52	3.83*	-	1.33	4.81*	-	-
	2	2	F (4,65)	1.57	2.71*	-	1.29	3.65*	-	-
Japan	1	1	F (4,73)	2.74*	1.77	3.74*	1.79	3.83*	0.51	-
	1	2	F (4,67)	1.85	1.68	3.18*	2.00	3.69*	1.25	-
U.K.	2	1	F (4,67)	1.04	4.57*	0.77	1.48	0.19	1.13	-
	2	2	F (4,61)	1.09	2.98*	1.82	1.53	1.01	1.41	-
Sweden	2	1	F (4,67)	1.44	0.31	3.16*	0.31	0.86	2.02	-
	2	2	F (4,61)	1.81	0.06	0.49	0.84	1.24	1.35	-
Switz.	1	1	F (4,73)	0.56	3.39*	6.59*	0.87	1.74	6.28*	-
	1	2	F (4,67)	1.29	3.32*	4.09*	0.63	1.81	7.53*	-
Norway	2	1	F (4,67)	3.25*	3.51*	1.40	0.91	1.60	2.12	-
	2	2	F (4,61)	3.26*	3.47*	0.78	1.38	3.72*	1.85	-

Note: * denotes statistical significance at the 5% level or less.

3.4 Testing Weak Exogeneity

The final step in our estimation procedure concerns the test of the weakly exogeneity of the country-specific foreign variables (the "starred" variables) and the oil prices. Weak exogeneity is tested along the lines described in Johansen (1992) and Harbo et al (1998). This involves a test of the joint significance of the estimated error correction terms in auxiliary equations for the country-specific foreign variables, $\mathbf{x}_{it}^*$. In particular, for each l^{th} element of $\mathbf{x}_{it}^*$ the following regression is carried out

$$\Delta x_{it,l}^* = \mu_{il} + \sum_{j=1}^{r_i} \gamma_{ij,l} ECM_{i,t-1}^j + \sum_{k=1}^{s_i} \varphi_{ik,l} \Delta \mathbf{x}_{i,t-k} + \sum_{m=1}^{n_i} \vartheta_{im,l} \Delta \widetilde{\mathbf{x}}_{i,t-m}^* + \epsilon_{it,l}$$

where $ECM_{i,t-1}^j$, $j = 1, 2, ..., r_i$ are the estimated error correction terms corresponding to the r_i cointegrating relations found for the i^{th} country model and $\Delta\widetilde{\mathbf{x}}_{i,t-m}^* = (\Delta x_{i,t-m}'^*, \Delta(e_{i,t-m}^* - p_{i,t-m}^*))'$. Note that in the case of the U.S. the term $\Delta(e_{i,t-k}^* - p_{i,t-k}^*)$ is implicitly included in $\Delta x_{i,t-k}'^*$. The test for weak exogeneity is an F test of the joint hypothesis that $\gamma_{ij,l} = 0$, $j = 1, 2, ..., r_i$ in the above regression. The lag orders s_i and n_i, need not be the same as the orders p_i and q_i of the underlying country-specific VARX* models. We carried out two sets of experiments, one set using the lag orders of the underlying VARX* models given in 8, and in another set of experiments we set $s_i = p_i$ and $n_i = 2$ for all countries. In both cases the exogeneity hypothesis could not be rejected for most of the variables being considered. Under the former specification of the lag orders 9 out of 153 cases were found to be significant at the 5% level, whilst under the latter only 3 out of 153 exogeneity tests turned out to be statistically significant. The test results for this case are summarized in Table 10.

For the set of focus countries, as can be seen from this table, the weak exogeneity assumptions are not rejected with the exception of foreign output in Sweden and foreign inflation in Norway, which indicates rejection at the 5% significance level. This does not seem to us to be too serious a violation and could have arisen due to insufficient dynamics.[11] We would have been much more concerned if the weak exogeneity assumptions were rejected in the case of the U.S. or the euro area models, for example. But as can be seen from Table 10, the weak exogeneity of foreign variables and oil prices are not rejected in the euro area model. Aggregation of the euro area countries in a single model could have violated the weak exogeneity assumptions that underlie GVAR modelling. However, the tests suggest that the foreign euro area-specific variables can be considered as weakly exogenous. The same applies to the foreign variables $(y_{US}^*, \Delta p^*, e_{US}^* - p_{US}^*)$ included in the U.S. model. As expected foreign real equity prices and foreign interest rates (both short and long term) cannot be considered as weakly exogenous and have thus not been included in the U.S. model.

[11] Indeed, once n_i is set equal to 3 for these countries the test results are no longer statistically significant.

Table 10: F Statistics for Testing the Weak Exogeneity of the Country-specific Foreign Variables and Oil Prices

		Foreign Variables						
Country[a]		y^*	Δp^*	q^*	ρ^{*S}	ρ^{*L}	p^o	$e^* - p^*$
United States	F(2 , 75)	0.16	1.22	-	-	-	-	1.90
Euro Area	F(2 , 67)	0.09	0.00	2.36	0.22	2.07	2.31	-
China	F(1 , 72)	0.22	1.76	3.96	0.10	1.19	0.33	-
Japan	F(4 , 71)	0.60	1.02	0.58	0.63	0.62	2.30	-
United Kingdom	F(3 , 66)	1.88	0.24	0.39	1.11	0.83	1.04	-
Sweden	F(3 , 66)	2.47*	1.49	0.09	0.40	0.31	0.49	-
Switzerland	F(3 , 72)	0.31	0.44	0.34	0.85	0.35	0.37	-
Norway	F(2 , 67)	0.80	2.95*	0.23	0.51	0.54	1.25	-

[a]Test results for the remaining countries are provided in a Supplement that can be obtained from the authors on request.

3.5 Contemporaneous Effects of Foreign Variables on Their Domestic Counterparts

Table 11 presents the contemporaneous effects of foreign variables on their domestic counterparts. These values can be interpreted as impact elasticities between domestic and foreign variables. Most of these elasticities are significant and have a positive sign, as expected.

These elasticities are very informative as regards the international linkages between the domestic and foreign variables. Focusing on the euro area, we can see that a 1% change in foreign real output in a given quarter leads to an increase of 0.5% in euro area real output within the same quarter. Similar foreign output elasticities are obtained across the different regions, though the effect is slightly weaker for the U.S.. The relatively large and statistically significant elasticity estimate obtained in the case of the euro area largely reflects the high degree of trade openness of the euro area economy.

We can also observe a high elasticity between long-term interest rates, ρ^L and ρ^{*L}, implying relatively strong comovements between euro area and foreign bond markets. More importantly, the contemporaneous elasticity of real equity prices is significant and slightly above one. Hence, the euro area stock markets would seem to overreact to foreign stock price changes, although the extent of over-reaction is not very large and is statistically significant only marginally. Similar results are also obtained for Sweden and Norway. Contemporaneous financial linkages are likely to be very strong amongst the European economies through the equity and the bond market channels.

In contrast, we find rather low elasticities for inflation. For the euro area the foreign inflation elasticity is 0.12 and is not statistically significant, suggesting that in the short run the euro area prices are not much affected by changes in foreign prices. The same is also true for the U.S., and to lesser extent, for the

U.K. inflation rates. For the remaining focus countries foreign inflation effects are much larger and are statistically significant.

Another interesting feature of the results are the very weak linkages that seem to exist across short-term interest rates (Sweden being an exception) and the high, significant relationships across long-term rates. This clearly shows a much stronger relation between bond markets than between monetary policy reactions.

Table 11: Contemporaneous Effects of Foreign Variables on their Domestic Counterparts

	Domestic Variables				
Country[a]	y	Δp	q	ρ^S	ρ^L
United States	0.34*	0.05	-	-	-
	(0.10)	(0.06)			
Euro Area	0.50*	0.12	1.16*	0.09*	0.62*
	(0.10)	(0.08)	(0.08)	(0.02)	(0.08)
China	-0.01	1.21	-	0.12	-
	(0.14)	(0.66)		(0.07)	
Japan	0.48*	0.50*	0.60*	-0.02	0.48*
	(0.16)	(0.09)	(0.12)	(0.05)	(0.11)
United Kingdom	0.44*	0.47	0.87*	0.24	0.74*
	(0.14)	(0.25)	(0.07)	(0.16)	(0.14)
Sweden	1.18*	1.21*	1.17*	1.20*	0.95*
	(0.33)	(0.21)	(0.11)	(0.28)	(0.13)
Switzerland	0.47*	0.51*	0.70*	0.14*	0.41*
	(0.12)	(0.14)	(0.13)	(0.06)	(0.07)
Norway	0.79	1.07*	1.02*	0.15	0.56*
	(0.42)	(0.16)	(0.12)	(0.11)	(0.14)

Note: * denotes statistical significance at the 5% level or less. Standard errors are in parentheses.

[a] Test results for the remaining countries are provided in a Supplement that can be obtained from the authors on request.

4 Pair-wise Cross Section Correlations: Variables and Residuals

One of the key assumptions of the GVAR modelling approach is that the "idiosyncratic" shocks of the individual country models should be cross sectionally "weakly correlated", so that $Cov(\mathbf{x}_{it}^{*}, u_{it}) \rightarrow 0$, as $N \rightarrow \infty$ and as a result the weak exogeneity of the foreign variables is ensured. Direct tests of weak exogeneity assumptions discussed above indirectly support the view that the idiosyncratic shocks could only be weakly correlated. In this section we provide direct evidence on the extent to which this is likely to be true. The basic idea is similar to the cross section dependence test proposed in Pesaran (2004b). By conditioning the country-specific models on weakly exogenous foreign variables, viewed as proxies for the "common" global factors, it is reasonable to expect that the degree of correlation of the remaining shocks across countries/regions will be modest. These residual interdependencies, as mentioned in the introduction, could reflect policy and trade spillover effects.

As a simple diagnostic of the extent to which the country specific foreign variables have been effective in reducing the cross-section correlation of the variables in the GVAR model, we have computed average pair-wise cross-section correlations for the levels and first differences of the endogenous variables of the model, as well as those of the associated residuals over the estimation period, 1979-2003. We also computed average pair-wise cross section correlations of the residuals obtained after reestimating all of the indivudual country specific models over the same period excluding the foreign star variables, including oil as endogenous in all the countrty models.[12]

Table 12: Average Pair-wise Cross-Section Correlations of Real Output and Associated Model's Residuals

Country	Levels	1st Difference	VAR Residuals	VARX* Residuals
United States	0.96	0.15	0.04	-0.04
Euro Area	0.96	0.14	0.11	-0.01
China	0.96	0.02	-0.01	-0.01
Japan	0.92	0.03	-0.03	-0.07
United Kingdom	0.95	0.08	0.07	0.01
Sweden	0.96	0.07	0.07	0.01
Switzerland	0.93	0.13	0.08	0.01
Norway	0.96	0.08	0.05	0.01

Note: VAR residuals are based on cointegrating VAR models with domestic variables only and oil prices. VARX* residuals refer to the country models with country specific foreign variables.

The results are summarized in Tables 12 to 17, for real outputs, inflation, real equity prices, real exchange rate, short-term interest rate and the long-term interest rates, respectively.

Table 13: Average Pair-wise Cross-Section Correlations of Inflation and Associated Model's Residuals

[12] For each country model we used the same VAR order as that specified in Table (8) and selected the number of cointegrating relationships based on Johansen's trace statistic applied to the individual VAR models (excluding the star variables).

Country	Levels	1st Difference	VAR Residuals	VARX* Residuals
United States	0.41	0.12	0.15	0.02
Euro Area	0.40	0.11	0.13	0.00
China	0.00	-0.02	0.05	-0.05
Japan	0.31	0.01	0.06	0.00
United Kingdom	0.37	0.06	0.11	0.00
Sweden	0.37	0.06	0.11	0.00
Switzerland	0.31	0.07	0.10	0.05
Norway	0.31	0.07	0.11	0.03

Note: See the note to Table 12.

Table 14: Average Pair-wise Cross-Section Correlations of Real Equity Prices and Associated Model's Residuals

Country	Levels	1st Difference	VAR Residuals	VARX* Residuals
United States	0.59	0.39	0.34	-0.02
Euro Area	0.58	0.42	0.39	-0.08
Japan	0.37	0.31	0.21	-0.09
United Kingdom	0.61	0.40	0.38	-0.03
Sweden	0.57	0.38	0.36	-0.01
Switzerland	0.54	0.26	0.19	-0.05
Norway	0.61	0.36	0.33	0.01

Note: See the note to Table 12.

Table 15: Average Pair-wise Cross-Section Correlations of Real Exchange Rates and Associated Model's Residuals

Country	Levels	1st Difference	VAR Residuals	VARX* Residuals
Euro Area	0.62	0.31	0.26	0.28
China	-0.22	0.06	0.06	0.03
Japan	0.59	0.22	0.19	0.17
United Kingdom	0.62	0.28	0.22	0.19
Sweden	0.59	0.28	0.21	0.19
Switzerland	0.63	0.27	0.25	0.28
Norway	0.62	0.31	0.27	0.28

Note: See the note to Table 12.

Table 16: Average Pair-wise Cross-Section Correlations of Short-Term Interest Rates and Associated Model's Residuals

Country	Levels	1st Difference	VAR Residuals	VARX* Residuals
United States	0.38	0.10	0.05	0.00
Euro Area	0.49	0.16	0.08	0.03
China	0.32	0.03	0.00	-0.04
Japan	0.47	0.06	0.03	-0.01
United Kingdom	0.51	0.12	0.09	0.01
Sweden	0.46	0.04	0.04	-0.01
Switzerland	0.33	0.09	0.08	0.00
Norway	0.40	0.03	0.02	-0.01

Note: See the note to Table 12.

Table 17: Average Pair-wise Cross-Section Correlations of Long-Term Interest Rates and Associated Model's Residuals

Country	Levels	1st Difference	VAR Residuals	VARX* Residuals
United States	0.75	0.40	0.31	-0.03
Euro Area	0.78	0.45	0.34	-0.06
Japan	0.76	0.28	0.26	-0.05
United Kingdom	0.78	0.39	0.29	-0.01
Sweden	0.80	0.37	0.28	0.07
Switzerland	0.59	0.37	0.31	0.02
Norway	0.72	0.28	0.19	0.02

Note: See the note to Table 12.

The average cross section correlations are generally high for the level of the endogenous variables and fall as first differences of these variables are considered. The results vary widely across variables and less so across countries, with inflation and real exchange rate for China being the exceptions. Output levels, sharing common trends, show the highest degree of cross section correlations of around 92%-96%. This is followed by long-term interest rates (59%-80%), real equity prices (37%-61%), and short-term interest rates (32%-51%). The effect of first differencing on cross section correlations differ widely over variables as well as countries, and is most pronounced in the case of the output series. Average cross section correlations of output changes, Δy_{it}, range between 2% for China to 15% for the U.S., as compared to cross section correlations of output levels of 96% for both of these economies. Similar outcomes are also observed in the case of inflation and short-term interest rates. By comparison, first differencing of equity prices and long-term interest rates has only limited effect on cross section correlations. For example, the average cross section correlations of equity prices fall from 37%-61% to 26%-42% as one moves from levels of equity prices to their first differences. Overall, there is significant evidence of cross country correlations for the variables in the GVAR model, although the extent of this correlation depends on the variable, whether it is transformed to stationarity by first differencing, and the country.

Turning to the cross section correlation of the residuals from the VARX* models (including domestic and foreign star variables), it is quite striking that except for real exchange rates these correlations are very small and do not depend on the choice of the variable or country. This is particularly apparent in the case of the equity and bond markets where the cross section correlation of the residuals ranges between -8% and +1%, as compared to the values in the range 37% and 61% (or 26% and 42%) if cross section correlations of the levels (or first differences) are considered. The model has clearly been successful in capturing the common effects driving bond and equity markets. The real exchange rate variable presents an important exception which requires further consideration.

With regard to the cross section correlations of the residuals from the individual country models that include only the domestic variables, their value appears to lie between that of the first-differenced variables and the residuals from the VARX* models. Exceptions are noted in the case of inflation, where the correlations of the residuals from the individual country models excluding the star variables are slightly higher than those based on the first-differenced variables, and for the real exchange rates where the correlations of the residuals from the VARX* models and VAR models (excluding the star variables), are virtually identical.

Overall, the cross section correlation results show the importance of country-specific variables in dealing with often significant dependencies that exist across macroeconomic variables. Although, these results do not constitute a formal statistical test of the importance of the foreign variables in the GVAR model, they do provide an important indication of their usefulness in modelling global interdependencies. The results also show that once country-specific models are formulated conditional on foreign variables, there remains only a modest degree of correlations across the shocks from different regions.

5 Generalized Impulse Response Functions

To study the dynamic properties of the global model and to assess the time profile of the effects of shocks to foreign variables on the euro area economy, we investigate the implications of four different external shocks:

- A one standard error positive shock to oil prices
- A one standard error negative shock to U.S. real equity prices
- A one standard error negative shock to U.S. real GDP
- A one standard error positive shock to U.S. interest rates

We also discuss how global shocks can be defined whitin the GVAR framework, and briefly discuss the main effects of global shocks to real equity prices and real output on euro area.

In this section we make use of the Generalized Impulse Response Function (GIRF), proposed in Koop, Pesaran and Potter (1996) for non-linear models and discussed in further details in Pesaran and Shin (1998) for vector error correcting models. The GIRF is an alternative to the Orthogonalized Impulse Responses (OIR) of Sims (1980). The OIR approach requires the impulse responses to be computed with respect to a set of orthogonalized shocks, whilst the GIR approach considers shocks to individual errors and integrates out the effects of the other shocks using the observed distribution of all the shocks without any orthogonalization. Unlike the OIR, the GIRF is invariant to the ordering of the variables and the countries in the GVAR model, which is clearly an important consideration. Even if a suitable ordering of the variables in a given country model can be arrived at from economic theory or general *a priori* reasoning, it is not clear how to order countries in the application of the OIR to the GVAR model.

In the absence of strong a priori beliefs on ordering of the variables and/or countries in the GVAR model, the GIRFs provide useful information with respect to changes in oil prices, equity prices and even interest rates. Although, the approach is silent as to the reasons behind the changes, the GIRFs can be quite informative about the dynamics of the transmission of shocks from the rest of the world to the euro area.

In the discussion of the results, we focus only on the first two years following the shock. This seems a reasonable time horizon over which the model presents credible results. We have, however, included figures (see Appendix B) of the GIRFs over a longer period. The figures display results over 40 quarters and are intended as visual aids for the analysis of model's convergence properties. The figures clearly show that the model is stable (the endogenous variables return gradually to their long run solution in a reasonable amount of time). This is also supported by the eigenvalues of the GVAR model, which are 268 in total given that the maximum lag order of the individual country/region models is 2. The model contains 134 endogenous variables and the rank of the global cointegrating matrix is at most 63. Hence, the global system should have at least 71 eigenvalues (i.e. $134 - 63$), that fall on the unit circle. The GVAR satisfies these properties and indeed has 71 eigenvalues equal to unity, with the remaining 197 eigenvalues having moduli all less than unity.[13]

5.1 Shock to Oil Prices

Table 22 presents the GIRFs of a positive one standard error shock to oil prices on the regions of interest over the first two years. A one standard error positive shock results in a 13-14% increase per quarter in the price of oil.

On impact the oil price shock has diverse effects on real output across countries. In the U.S., the U.K., the euro area and the rest of Western Europe, the increase in oil prices has a negative impact on real output. This negative effect persists in the U.S., the euro area and the U.K. after 2 years. The U.S. seems to be the most affected economy. This result is consistent with other studies showing the higher dependence of the U.S. economy to oil than the other industrialized economies. The effect in the U.K. is dampened by the fact that the economy is also an oil producer.

The impact is instead positive and significant for Japan and China. For the former, this is a result entirely at odds with general belief based on standard indicators of oil dependency of the Japanese economy. The result for China could be due to her sustained (so far) high levels of output growth over the past decade that has coincided with episodes of high and low oil prices. The high output elasticity of energy demand in China could also be behind the seemingly perverse results which we have obtained. Further empirical investigation is clearly needed.[14]

Regarding inflationary impacts, the oil price shock is less ambiguous. All countries, except China, exhibit an increase in inflation by more than 0.1 percent. Again, the U.S. response is the largest, which is consistent with what we observe on the real side, and in line with a rise in short-term interest rates, triggered in turn by increased inflationary pressures.

As regards financial variables, the increase in oil prices coincides with downward movements in equity prices and increases in long-term interest rates. The

[13] Of these 197 eigenvalues, 128 (64 pairs) are complex, implying the cyclical properties of the impulse response functions. The eigenvalues with the largest complex part are $.042906 \pm 0.734367i$, $.026508 \pm 0.718040i$ and $.124609 \pm 0.638926i$,where $i = \sqrt{-1}$. After the unit roots, the three largest eigenvalues (in moduli) are .897016, .895260 and .884433, implying a reasonable rate of convergence of the model after a shock to its long-run equilibrium. Given the unit eigenvalues of the system, some shocks will have permanent effects on the levels of the endogenous variables.

[14] In light of the distinctive behaviour of the Chinese economy, as a robustness check, we re-estimated the GVAR model excluding China. The results for the euro area and for the other countries of interest remain very similar, confirming the main conclusions of the paper. The only difference that may be considered as significant concerns the behaviour of the exchange rate in Japan, which seems to be quite plausible considering the relatively large share of China in the Japanese trade.

increase in long-term interest rates shows that the bond markets tend to react more to inflation expectations rather than to the growth prospects. Bond and equity market reactions are consistent with each other and are common to all regions.

Interestingly, the real exchange rate reaction is mixed across countries/regions. The yen depreciates rather substantially, as compared to the other currencies. This result may explain the differences already observed regarding the effect of the oil price shock on real output; the depreciation of the yen implying positive effects on competitiveness and hence on exports. This positive effect could then more than compensate the negative impact of oil price increases on economic activity.

5.2 Shock to U.S. Equity Prices

The point estimates of the GIRFs for a one standard error negative shock to U.S. equity prices over a two-year horizon are given in Table 23. This shock is equivalent to a fall of around 6-7% in U.S. real equity prices per quarter. In the U.S., the equity price shock is accompanied by a decline in real GDP by 0.1% on impact, by 0.4% on average over the first year and by 0.6% on average over the second year.

The transmission of the shocks to the other equity markets is rather quick and significant. On impact, equity prices fall by 2.1% in Japan, 5.2% in the euro area, 4.6% in the U.K. and 4.5% in the rest of Western Europe. Over time, the decrease in equity prices converges to the U.S. responses and is even stronger in the case of the euro area and the rest of Western Europe. This shows that markets have tended to overreact to shocks, equity prices in the European markets overshooting the U.S. responses, partly reflecting the higher volatility of the European equity markets as compared to the volatility of the S&P 500 used as the market index for the U.S..

Like in the U.S., real output in euro area is negatively affected by the adverse equity shock, although to a lesser extent. Inflation tends to decrease although the magnitude of the reaction remains very limited. Short-term and long-term interest rates also marginally decrease. The impact on the former is stronger in the U.S. and the U.K. than in the euro area, which may be related to the different reaction functions of monetary authorities to asset price movements in these economies.

Finally, real exchange rates appreciate in the case of the euro area, China and the rest of Western Europe, while they depreciate in the U.K. and Japan. Like in the previous simulations, the Japanese yen tends to depreciate in response to an adverse equity price shock.

5.3 Shock to U.S. Real GDP

Table 24 reports the GIRFs for a one standard error negative shock on U.S. GDP, equivalent to a 0.5-0.6% decrease in U.S. real output. In the simulation, the real output shock is associated with a slight decrease in U.S. inflation and a decrease in real equity prices (by slightly more than 1%) and short-term interest rates (by around 60 basis points). Although this shock cannot be interpreted as an orthogonalized shock, as we have not imposed any structure on the GVAR, we can however interpret it, given the signs of the different responses, as a U.S. demand shock. This simulation also provides insights into the transmission of real shocks across countries.

The transmission of the shock appears to be relatively slow. On impact, real GDP in the euro area, the U.K. and the rest of Western Europe is reduced by about 1/10 the shock on the U.S.. Over time, the shock propagation increases. In the first year, on average, real GDP decreases by around 0.1% in the euro area, the rest of Western Europe and the U.K., and by around 0.05% in Japan. China is the only region benefiting from the negative U.S. shock. In the second year, the response of the shock in the euro area, in the rest of Western Europe and in Japan is around one third of that in the U.S. (around one tenth for the U.K.). Given the weight of the U.S. in the euro area trade (around 20%) and the openness ratio of the euro area, these results clearly indicate that other channels play an important role in the transmission of shocks.

Among the other channels, the financial linkages appear to be of importance. Interestingly, the equity markets in the different regions react strongly to the change in U.S. real GDP growth. Similarly to U.S. equity prices, real equity prices in the other markets decrease by around 1% on average in the first two years. This strong reaction is likely to explain the relatively pronounced response of real output. The U.K. equity market appears instead to be less affected.

Concerning the real exchange rates, all currencies tend to appreciate vis-a-vis the U.S. dollar, the Japanese yen reacting in the same way as the other currencies. The reaction of interest rates is also very similar across countries and regions. Short-term and long-term interest rates decrease by around 20 basis points. It is worth pointing out that the reactions of exchange rates remain very small considering the volatility of exchange rate markets to news related to U.S. GDP. Their responses are also much smaller than those of equity markets. As the Chinese real exchange rate reacts relatively less to the shock, the Chinese economy tends to gain market shares with respect to the countries and regions outside the U.S.. This may partly explain the positive response of China's real GDP.

5.4 Shock to U.S. Short-Term Interest Rate

Table 25 presents results over the two first years of a positive one standard error shock on U.S. short-term interest rates. Although it is difficult to interpret this shock as a monetary policy shock, this simulation shows usual changes in the other variables while interest rates increase. In the U.S., the one standard error positive shock is equivalent to a 0.2% increase in short-term interest rates (i.e. around 80 basis points), measured at quarterly rates. This increase is associated with an increase in real output of 0.2% on impact. Real GDP then decreases gradually and remains at around 0.2% below baseline levels after 8 quarters. Higher inflation also accompanies the interest rate increase, though the magnitude remains limited. Real equity prices fall by 0.5% on average in the first year and by 0.7% in the second year.

This association of changes in U.S. variables has some effects on the other countries and regions. Interest rates tend to rise in the rest of the world although their increase is modest compared to the U.S. interest rate change. Hence, while monetary policy changes in the U.S. and in the rest of the world tend to move in the same direction, the extent of the comovements appears to be limited.

Regarding real output, the U.S. interest rate shock has a negative effect on the other countries, the U.K. being the only exception. Following a U.S. interest rate increase, the real exchange rates depreciate except for China that experience an appreciation in its real exchange rate relative to U.S. dollar. Inflation does not react significantly to the shock. Equity markets follow the U.S. responses, some countries overreacting to the U.S. equity market changes.

5.5 Global Shocks

So far we have considered the effects of variable/country specific shocks, with paricular emphasis on the shocks originating from the U.S. viewed possibly as global shocks, considering the dominent role of the U.S. in the world economy. Whilst such a strategy might be appropriate in the case of shocks to the U.S. equity market, it might be less defensible for other types of shocks. Therefore, it might be desirable to consider the effects of "global" shocks which might not necessary originate from a particular country, but could be common to the world economy as a whole. Examples of such shocks include major developments in technology or global shocks to commodity or equity markets. Apart from explicitly including global effects, such as oil prices, in the GVAR model, it is also possible to consider the effects of global shocks defined as a weighted average of varaible-specific shocks across all the countries in the model. To see how this can be done consider the GVAR model (15), and abstracting from determinstic terms and higher order lags write it as

$$\mathbf{G}\mathbf{x}_t = \mathbf{H}\mathbf{x}_{t-1} + ... + \mathbf{u}_t, \ \mathbf{u}_t \sim IID(\mathbf{0}, \mathbf{\Sigma}_u) \tag{18}$$

with a total of $k = \sum_{i=0}^{N} k_i$ domestic variables for the $N+1$ countries.

A global shock at time t to a specific variable, can now be defined as a shock to say the ℓ^{th} variable in all $N+1$ countries simultaneously aggregated to a single shock using a set of weights reflecting the relative importance of the individual countries in the world economy. For example, using PPP GDP weights a global shock to the ℓ^{th} variable can be defined as

$$u_{\ell t}^{g} = \mathbf{a}_{\ell}' \mathbf{u}_t,$$

where a_ℓ is a $(k \times 1)$ selection vector, $a_\ell = (a_{0\ell}', a_{1\ell}', ..., a_{N\ell}')'$ and $a_{i\ell}$ is the $k_i \times 1$ vector with zero elements except for its element that corresponds to the ℓ^{th} variable which is set equal to w_i, the weight of the i^{th} country in the world economy. By construction $\sum_{i=0}^{N} w_i = 1$.

The generalized impulse response function in the case of a one standard error global shock is given by

$$\psi(h, \mathbf{x} : u_\ell^g) = E(\mathbf{x}_{t+h}|\Omega_{t-1}, u_{\ell t}^g = \sqrt{\mathbf{a}_\ell' \mathbf{\Sigma}_u \mathbf{a}_\ell}) - E(\mathbf{x}_{t+h}|\Omega_{t-1}),$$

and in the case of the above GVAR model is easily seen to be

$$\psi(0, \mathbf{x} : u_\ell^g) = \frac{\mathbf{G}^{-1}\mathbf{\Sigma}_u \mathbf{a}_\ell}{\sqrt{\mathbf{a}_\ell' \mathbf{\Sigma}_u \mathbf{a}_\ell}}. \tag{19}$$

The effect of a one standard error global shock, on expected values of x at time $t+h$, for $h = 1, 2, ...$ can then be obtained recursively by using 19 and solving forward in the light of the difference equation 18.

We examine the time profiles of the effects of two type of global shocks: a negative global shock to real equity prices and a negative global shock to real output. Generalized impulse response functions of the impacts of such shocks on the main variables are provided in a Supplement available from the authors on request. In the case of the global equity shock, the results are very similar to those of a shock to the U.S. equity prices discussed above. This result confirms the predominant role of the U.S. stock market in the equity price developments across countries.

In the case of the global output shock, beyond the fact that the U.S. is relatively less affected (since the shock hits all countries at the same time), the results are broadly similar when compared with those of the shock to U.S. real output. The main difference concerns the impacts of the global output shock on real exchange rates, which tend to depreciate vis-a-vis the U.S. dollar, while they appreciate in most cases when the shock originates in the U.S..

6 Structural Impulse Response Analysis Using the GVAR Model

Structural identification of all the 134 different shocks (the total number of endogenous variables) in the GVAR model will be a formidable undertaking, and might not be necessary since in practice monetary policy, demand and supply shocks are likely to be highly correlated across countries. In what follows we focus on identification of shocks to the U.S. economy, particularly the monetary policy shocks, and consider the time profiles of their effects on the euro area. To this end we include the U.S. model as the first country model and following Sims (1980), consider alternative orderings of the variables within the U.S. model. The outcome of this exercise will be invariant to the ordering of the rest of the variables in the GVAR model, so long as the contemporaneous correlations of these shocks are left unrestricted (both in relation to themselves and with respect to the U.S. shocks). Ordering of the rest of the variables in the GVAR model will be important for the analysis of the U.S. monetary policy shocks, only if short-run over-identifying restrictions are imposed on the parameters of the models.

In the light of the arguments advanced in Sims and Zha (1998), one possible identification scheme for the U.S. pursued below, is to adopt the ordering of the variables in the U.S. model as follows:

$\mathbf{x}_{0t}$ = (oil, short-term interest rate, long-term interest rate, equity prices, inflation and output). It is also assumed that variance matrix of the structural errors ($\boldsymbol{\varepsilon}_{0t}$) associated to these variables are orthogonal.[15]

Consider the $VARX^*(1)$ model for the U.S. denoted by the country index $i = 0$,

$$\mathbf{x}_{0t} = \mathbf{\Phi}_0\mathbf{x}_{0t-1} + \mathbf{\Psi}_{01}\mathbf{x}^*_{0t} + \mathbf{\Psi}_{02}\mathbf{x}^*_{0,t-1} + \mathbf{u}_{0t}. \tag{20}$$

Premultiply (20) by $\mathbf{P}_0$,

$$\mathbf{P}_0 x_{0t} = \mathbf{P}_0\mathbf{\Phi}_0\mathbf{x}_{0,t-1} + \mathbf{P}_0\mathbf{\Psi}_{01}\mathbf{x}^*_{0t} + \mathbf{P}_0\mathbf{\Psi}_{02}\mathbf{x}^*_{0,t-1} + \mathbf{P}_0\mathbf{u}_{0t}$$

where

$$\boldsymbol{\varepsilon}_{0t} = \mathbf{P}_0\mathbf{u}_{0t}$$

are the structural shocks. The identification conditions a là Sims(1980) are given by

$$\begin{aligned} Cov(\boldsymbol{\varepsilon}_{0t}) &: \text{ diagonal} \\ \mathbf{P}_0 &: \text{ lower triangular} \end{aligned}$$

$$\begin{aligned} Cov(\mathbf{u}_{0t}) &= \mathbf{\Sigma}_{u0} = \mathbf{Q}_0'\mathbf{Q}_0 \\ Cov(\boldsymbol{\varepsilon}_{0t}) &= \mathbf{\Sigma}_{\varepsilon 0} = \mathbf{P}_0\mathbf{\Sigma}_{u0}\mathbf{P}_0'. \end{aligned}$$

where $\mathbf{Q}_0$ is the upper Cholesky factor of $\mathbf{\Sigma}_{u0}$. Hence

$$\mathbf{P}_0\mathbf{\Sigma}_{u0}\mathbf{P}_0' = \mathbf{P}_0\mathbf{Q}_0'\mathbf{Q}_0\mathbf{P}_0' = \mathbf{\Sigma}_{\varepsilon 0},$$

and

$$\mathbf{P}_0\mathbf{Q}_0' = \mathbf{\Sigma}_{\varepsilon 0}^{1/2}, \text{ a diagonal matrix.} \tag{21}$$

Consider now the GVAR model (18) and premultiply it by

$$\mathbf{P}_G^0 = \begin{pmatrix} \mathbf{P}_0 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \end{pmatrix} \tag{22}$$

[15] An alternative approach that could be explored is that of Christiano, Eichenbaum and Evans (1999). We could also consider non-recursive identification schemes. The mathematical treatment will be the same. Only the form of P_0 and the variance matrix of ε_{0t} will be different.

to obtain

$$\mathbf{P}_G^0\mathbf{G}x_t = \mathbf{P}_G^0\mathbf{H}\mathbf{x}_{t-1} + ... + \boldsymbol{\varepsilon}_t,$$

where

$$\boldsymbol{\varepsilon}_t = \begin{pmatrix} \boldsymbol{\varepsilon}_{0t} \\ \mathbf{u}_{1t} \\ \vdots \\ \mathbf{u}_{Nt} \end{pmatrix},$$

$$\boldsymbol{\Sigma}_\varepsilon = Cov\left(\boldsymbol{\varepsilon}_t\right) = \begin{pmatrix} V(\boldsymbol{\varepsilon}_{0t}) & Cov(\boldsymbol{\varepsilon}_{0t}, \mathbf{u}_{1t}) & \cdots & Cov(\boldsymbol{\varepsilon}_{0t}, \mathbf{u}_{Nt}) \\ Cov(\mathbf{u}_{1t}, \boldsymbol{\varepsilon}_{0t}) & V(\mathbf{u}_{1t}) & \cdots & Cov(\mathbf{u}_{1t}, \mathbf{u}_{Nt}) \\ \vdots & \vdots & & \vdots \\ Cov(\mathbf{u}_{Nt}, \boldsymbol{\varepsilon}_{0t}) & Cov(\mathbf{u}_{Nt}, \mathbf{u}_{1t}) & \cdots & V(\mathbf{u}_{Nt}) \end{pmatrix} \tag{23}$$

with

$$\begin{aligned} V(\boldsymbol{\varepsilon}_{0t}) &= \boldsymbol{\Sigma}_{\varepsilon,00} = \mathbf{P}_0\widehat{\Sigma}_{u,00}\mathbf{P}_0', \\ Cov(\boldsymbol{\varepsilon}_{0t}, \mathbf{u}_{jt}) &= Cov(\mathbf{P}_0\mathbf{u}_{0t}, \mathbf{u}_{jt}) = \mathbf{P}_0\boldsymbol{\Sigma}_{u,0j}. \end{aligned}$$

Generalized impulse responses with respect to the structural shocks are now defined as

$$\psi(h, \mathbf{x} : \boldsymbol{\varepsilon}) = E(\mathbf{x}_{t+h}|\ \Omega_{t-1}, \mathbf{e}_i'\boldsymbol{\varepsilon}_t = \sqrt{\mathbf{e}_i'\boldsymbol{\Sigma}_\varepsilon\mathbf{e}_i}) - E(\mathbf{x}_{t+h}|\Omega_{t-1}).$$

But, the contemporaneous effects are

$$\mathbf{P}_G^0\mathbf{G}\ E(\mathbf{x}_t|\Omega_{t-1}, \mathbf{e}_i'\boldsymbol{\varepsilon}_t = \sqrt{\mathbf{e}_i'\boldsymbol{\Sigma}_\varepsilon\mathbf{e}_i}) = \mathbf{P}_G^0\mathbf{H}\mathbf{x}_{t-1} + \frac{\boldsymbol{\Sigma}_\varepsilon\mathbf{e}_i}{\sqrt{\mathbf{e}_i'\boldsymbol{\Sigma}_\varepsilon\mathbf{e}_i}}$$

where $\mathbf{e}_i$ is a selection vector applied to all the elements of $\mathbf{x}_t$.

Thus, the contemporaneous effects are given by

$$\psi(0, \mathbf{x} : \boldsymbol{\varepsilon}_0) = \frac{(\mathbf{P}_G^0\mathbf{G})^{-1}\boldsymbol{\Sigma}_\varepsilon\mathbf{e}_i}{\sqrt{\mathbf{e}_i'\boldsymbol{\Sigma}_\varepsilon\mathbf{e}_i}} = \frac{\mathbf{G}^{-1}\left(\mathbf{P}_G^0\right)^{-1}\boldsymbol{\Sigma}_\varepsilon\mathbf{e}_i}{\sqrt{\mathbf{e}_i'\boldsymbol{\Sigma}_\varepsilon\mathbf{e}_i}}.$$

The impulse responses for other horizons can be derived using the same recursive relations used for the computation of the generalized impulse responses.

Under the orthogonalization scheme, $\boldsymbol{\Sigma}_\varepsilon$, defined by (23), is specified as

$$V(\boldsymbol{\varepsilon}_{0t}) = \mathbf{I}_{k_o},$$

while the following options can be entertained

$$Cov(\boldsymbol{\varepsilon}_{0t}, \mathbf{u}_{jt}) = 0, \tag{24}$$

or

$$Cov(\boldsymbol{\varepsilon}_{0t}, \mathbf{u}_{jt}) = \mathbf{P}_0 \boldsymbol{\Sigma}_{u,0j}, \tag{25}$$

for $j = 1, 2, ..., N$. Under this specification, using (21) we have

$$\mathbf{P}_0 = (\mathbf{Q}_0')^{-1},$$

and hence

$$\left(\mathbf{P}_G^0\right)^{-1} = \begin{pmatrix} \mathbf{Q}_0' & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \end{pmatrix}.$$

The covariance specification, (24), imposes over-identifying restrictions, and should be used with care. By contrast (25) does not impose any further restrictions, and ensures that the impulse responses of structural shocks to the U.S. economy will be invariant to any re-ordering of the variables in the rest of the GVAR model. Also the structural impulse responses of the shocks to the oil prices (the first variable in the $VARX^*$ model of the U.S.) will be the same as the corresponding generalized impulse responses.[16] The same, however, will not be true of the other structural impulse responses. Note also that these results will not hold under the restricted (over-identified) covariance specification (24).

6.1 Comparisons of Structural and Generalized Impulse Responses

We shall continue to consider structural shocks corresponding to the four shock scenarios discussed in Section 5, and focus only on their effects on the euro area variables. We will start first with the exact identification scheme, and then consider the sensitivity of the results to alternative orderings of the variables in the U.S. model.

6.1.1 Shock to Oil Prices

As shown in Pesaran and Shin (1998), a shock to the oil price (the first variable in the U.S. model) will yield the same IRFs in the structural exactly identified case as in the case of the GIRFs. The IRFs will, however, differ in the overidentified case. Figure 1 shows the impacts of a positive one s.d. shock to oil prices on U.S. and euro area variables, comparing the exactly-identified and the over-identified cases. For output in the U.S., the response is very similar in these two cases, while for the euro area, the over-identified response is slightly less negative in the short-run. For inflation, equity prices and interest rates, the IRFs are broadly equivalent irrespective of the identification method. Only exchange rate reactions seem to differ: in the exactly identified case, the real appreciation of the euro is less pronounced compared with the overidentified one in the short term; the euro even depreciates after one year.

[16] See, Pesaran and Shin (1998).

6.1.2 Shock to U.S. Equity Prices

As shown in Figure 2, identification of the equity price shock does not change significantly the shape of the impulse response functions. The only significant difference concerns oil prices which decrease twice as much as in the non-identified case. This difference partly explains the larger response of inflation in the case of structural shocks. Indeed, for the U.S., while in the GIRF case inflation increases in the short term before decreasing after 3 quarters, the fall in the inflation rate is faster and larger.

Comparing exactly identified and overidentified impulse responses (Figure 3) does not yield significant differences. The impact on interest rates tends to be slightly larger in the overidentified case, both in the U.S. and in the euro area. The impact on output and on equity prices is also more pronounced in the overidentified case for the euro area. This difference may also explain the reduction in the real appreciation of the euro when going from the exactly identified case to the overidentified one.

6.1.3 Shock to U.S. Real GDP

While under the two previous shock scenarios the impulse responses did not change much, there are noticable differences between the GIRF and the identified case. As shown in Figure 4, the U.S. real GDP shock is smaller in the exactly identified case compared with the non-structural one : U.S. real GDP decreases by 0.4 percent and converges progressively towards a 0.25% difference with respect to the baseline, while the reduction in the case of the non-structural shock amounts to 0.4% in the long run). In the euro area, the impact on real GDP is also smaller. However, the multiplier between the U.S. and the euro area response on GDP is broadly similar (around 0.4). The inflation response tends to be large in the short run, and larger compared with the non-identified shock. Another interesting result concerns the interest rates impacts. Compared with the non-structural impulse responses, the structural one features a rather modest decrease in interest rates, both in the U.S. (five times less) and in the euro area (twice less). Unlike the non-structural impulse responses, the exactly identified shocks imply a real appreciation of the euro in the short run. The equity prices tend also to increase in the euro area in the short term, before decreasing in the long run.

The comparison between the structural impulse responses (between exactly identified and overidentified) is shown in Figure 5. Although the shock on U.S. GDP is broadly equivalent in both cases, the overidentified shocks tend to trigger larger responses on euro area GDP, U.S. interest rates and oil prices. On the contrary, the responses are less pronounced for equity prices. Like in the non-identified case, the long run response of euro area inflation is negative, while it is positive in the exactly identified case.

6.1.4 U.S. Monetary Policy Shocks and Sensitivity of the Impulse Responses to Alternative Orderings

The identification of the shock to U.S. short-term interest rates allows us to treat it as a U.S. monetary policy shock. This shock leads to similar results when comparing non-structural and structural (exactly identified) responses (see Figure 6). U.S. real output increases in the short-run before decreasing permanently by around 0.2%. The main differences concern equity prices, where the responses are smaller in the structural case and oil prices, where the structural responses feature a small decrease in oil prices, while the non-structural responses feature an increase in oil prices.

The overidentified shock triggers rather different responses (see Figure 7). While for the U.S., the results are similar, their impact on the euro area are significantly different. First, real GDP and inflation increase, while they both decreased in the exactly identified case. The interest rate reaction is also greater in the overidentified case. On the other hand, both exchange rate and oil prices are barely affected by the overidentified shock. These results should be viewed with caution since the over-identifying restrictions might not be supported by the available evidence.

We performed sensitivity analysis of the U.S. monetary policy shocks by experimenting with different orderings (see Figure 8). We define ordering A, the one examined in the preceding analysis, as $\mathbf{x}_{0t}^{A} =$ (oil,short-term interest rate, long-term interest rate, equity prices, inflation and output). Two alternative orderings are investigated, namely ordering B defined as $\mathbf{x}_{0t}^{B} =$ (oil,equity, short-term interest rate, long-term interest rate, output and inflation) and ordering C defined as $\mathbf{x}_{0t}^{C} =$ (oil, equity, long-term interest rate, short-term interest rate, output and inflation).

While orderings A and B yield similar impulse responses, ordering C implies some noticeable differences. First, while the interest rate shock is slightly smaller than the ones implied by the other orderings, the impact on real GDP is greater. On the contrary, the financial variables are less affected (the equity prices barely change and the long-term interest rate increases only marginally). Oil prices, however, exhibit a sharper decrease compared to the other two orderings.

The short-term positive response, which is present irrespective of the ordering, may appear difficult to justify in the context of a monetary policy shock. However, Christiano, Eichenbaum and Evans (1999) show that such a response can be expected when output comes after the monetary policy variables in the ordering of variables, which is actually the case in orderings A, B and C. Moreover, the short-term positive effects on inflation we find is consistent with the finding that took the name in the literature of price puzzle (Sims, 1992; Eichenbaum, 1992). However, this puzzle is present despite the inclusion of the oil price in the model, contradicting Sims and Zha (1998), who show that the price puzzle often disappears when introducing a commodity price. It is clearly worth considering other identification schemes developed in the VAR literature in order to evaluate the extent to which the price puzzle is a "reality" or simply a reflection of the particular identification strategy adopted.

7 Robustness of the GVAR Results to Time-Varying Weights

The preceding analysis was carried out using fixed trade weights on the grounds that changes in trade weights tend to be rather gradual and secular changes in trade weights are often counter acted by the comovements of the macroeconomic variables so that the foreign-specific variables computed using fixed and variable trade weights are often very close. To check the robustness of our results to the choice of trade weights we also estimated the GVAR model using rolling three-year moving averages of the annual trade weights.[17] But before presenting some of these results it would be instructive first to provide some evidence on the relationship of the two measures, $\mathbf{x}_{it}^{*}$ (based on fixed weights) and $\mathbf{x}_{it}^{**}$ (based on the time-varying weights). Since both measures are likely to be I(1), in Tables 26, 27 and 28 we summarize the correlation coefficients of the levels as well as their first differences. In terms of the levels the two measures are very high, in many cases close to unity. In terms of their first difference, the correlations are not as high, particularly in the case of nominal magnitudes such as inflation and interest rates. Given these results, it seems unlikely that the main conclusions of the paper would be much affected by choice of the trade weights.

Table 18: Correlation Coefficients of Country Specific Foreign Output and Inflation using Fixed and Time-Varying Trade Weights

	Output		Inflation	
Country	Levels	1st Difference	Levels	1st Difference
United States	0.999	0.931	0.885	0.677
Euro area	0.998	0.908	0.865	0.501
China	0.999	0.910	0.426	0.026
Japan	0.999	0.880	0.768	0.420
United Kingdom	1.000	0.984	0.948	0.729
Sweden	1.000	0.983	0.889	0.597
Switzerland	1.000	0.991	0.901	0.592
Norway	1.000	0.984	0.912	0.646

To check this conjecture we re-estimated the GVAR model, allowing for p_i in the individual country VARX* models to be unrestricted and q_i to be the same as in the fixed weights case, and obtained very similar number of cointegrating relations.[18] The differences between the two sets of results were Japan (3 cointegrating relations as compared to 4 previously), United Kingdom (2 instead of 3), and Sweden (2 instead of 3). See Table 29. We obtained the same number of cointegrating relations for the remaining countries.

[17] The process of computing time-varying trade weights was initialized by using the same set of weights for the first three years of our sample period.

[18] We also considered the case where p_i is unrestricted and $q_i = 1$ for all countries. The results were very similar to those presented here.

Table 19: Correlation Coefficients of Country Specific Foreign Real Equity Prices and Real Exchange Rates using Fixed and Time-Varying Trade Weights

	Real Equity Prices		Real Exchange Rates	
Country	Levels	1st Difference	Levels	1st Difference
United States	-	-	0.955	0.816
Euro area	0.814	0.996	0.814	0.859
China	0.868	0.980	0.868	0.669
Japan	0.852	0.985	0.852	0.629
United Kingdom	0.981	0.999	0.981	0.955
Sweden	0.975	0.999	0.975	0.956
Switzerland	0.994	0.999	0.994	0.971
Norway	0.868	0.996	0.868	0.893

Table 20: Correlation Coefficients of Country Specific Foreign Short and Long Term Interest Rates using Fixed and Time-Varying Trade Weights

	Short-Term Interest Rates		Long-Term Interest Rates	
Country	Levels	1st Difference	Levels	1st Difference
United States	-	-	-	-
Euro area	0.987	0.993	0.999	0.998
China	0.962	0.931	0.993	0.942
Japan	0.993	0.989	0.998	0.994
United Kingdom	0.999	0.989	0.999	0.998
Sweden	0.991	0.979	1.000	0.999
Switzerland	0.995	0.981	1.000	0.999
Norway	0.997	0.979	0.999	0.992

Turning to the impact effects of the foreign variables, the estimates (together with their standard errors) are summarized in Table 30, and are comparable to the corresponding estimates based on fixed trade weights given in Table 11. The two estimates are generally close and yield similar qualitative conclusions. This is particularly so in the case of real equity prices and long-term interest rates. The results for output are also very close with the exception of the estimates obtained for Norway. Not surprisingly, the results have been affected most in the case of China, where none of the estimates based on the time-varying weights are now statistically significant, as compared to the two statistically significant estimates obtained when using the fixed weights.

Similar conclusions are also reached if one considers average pair-wise cross-section correlations of the residuals or the impulse responses under the two weighting schemes. Estimates of the average cross-section correlations of the residuals for the two sets of weights are summarized in Tables 31 to 36.

We also compared a variety of impulse responses based on fixed and time-varying trade weights, and found that in general, the results are very close. Even when they differ quantitatively, the main conclusions reached using the

Table 21: VARX* Order and Number of Cointegration Relationships in the Country-Specific Models using Fixed and Time-Varying Trade Weights

	Fixed Weights			Time-Varying Weights		
	VARX*(p_i, q_i)		#Cointegrating	VARX*(p_i, q_i)		#Cointegrating
Country	p_i	q_i	Relationships	p_i	q_i	Relationships
U.S.	2	2	2	2	2	2
E.A.	2	2	2	2	2	2
China	2	1	1	2	1	1
Japan	1	1	4	1	1	3
U.K.	2	2	3	2	2	2
Sweden	2	1	3	2	1	2
Switz.	1	1	3	1	1	3
Norway	2	1	2	2	1	2

fixed weights tend to hold qualitatively.

Overall, the main results of the paper seem to be reasonably robust to the choice of the trade weights. In view of these results, it might be a good idea to combine the simplicity of the fixed weights with the up-to-date nature of the time varying weights by selecting three sets of weights to be used at the start, in the middle, and towards the end of the sample. Trade weights that vary continuously could mask the underling movements of the macroeconomic variables that go into the construction of the foreign variables.

8 GVAR and Structural Breaks

One of the fundamental problems facing econometric modelling is the possibility of structural breaks. The problem is likely to be particularly acute in the case of emerging economies that are subject to significant political and social change. The GVAR model is clearly not immune to this problem. Unfortunately, despite the great deal of recent research in this area, there is little known about how best to model breaks. Even if in-sample breaks are identified using Bayesian or classical procedures, there are insurmountable difficulties in allowing for the possibility of future breaks in forecasting and policy analysis. See, for example, Stock and Watson (1996), Clements and Hendry (1998, 1999) and Pesaran, Pettenuzzo and Timermann (2004). It is, therefore, perhaps not surprising that so far we have been silent on this important and troubling issue. It is an area that deserves special attention, which goes well beyond the scope of the present paper. Longer time series might also be needed for this purpose.

However, the fact that country-specific models within the GVAR framework are specified conditional on foreign variables should help in alleviating the structural problem somewhat. For example, suppose that univariate equity return equations are subject to breaks roughly around the same time in different economies. This could arise due to a stock market crash in the U.S. with strong spill over effects to the rest of the world. However, since equity return equations in the country-specific models are specified conditional on the U.S. equity returns, they need not be subject to similar breaks, and in this example the structural break problem could be confined to the U.S. model. This phenomenon is related to the concept of "co-breaking" introduced in macroeconometric modelling by Hendry (1996), and examined further by Hendry and Mizon (1998). The structure of the GVAR can readily accommodate co-breaking and suggests that the VARX* models that underlie the GVAR might be more robust to the possibility of structural breaks as compared to country VAR models. The analysis of structural breaks can then focus on the U.S. or other economic regions from which the break(s) might have originated.

9 Concluding Remarks

This paper updates and extends the GVAR model of Pesaran, Schuermann and Weiner (2004) in a number of directions, provides an unobserved common factor interpretation of the country-specific foreign variables included in the GVAR, and shows how the model can be used for structural impulse response analysis.

Compared to the original version of the GVAR, the current version extends the geographical coverage from 11 country/regions to 26 countries with the euro area being treated as a single economy, updates the estimation period to the end of 2003 (from end of 1999 previously), includes the long-term interest rate as an endogenous variable in country-specific models, and includes oil prices as an endogenous variable in the U.S. model rather than treat it as a global exogenous variable. Also, the U.S. model now allows for feedback effects from changes in output and inflation outside the U.S. variables.

The current version, therefore, captures more fully the interactions in the world economy and includes new channels of transmissions via bond markets, the feedback effects on oil prices from the global economy, and the changes in output and inflation from the rest of the world to the U.S. economy.

Although, the new GVAR model can be used for many different purposes, in this paper we have focussed on the short term and long term implications of external shocks for the euro area economy. We provide impact effects of external changes in interest rates (short and long rates), inflation, output, real equity prices, real exchange rates and oil prices on the euro area and present the time profiles of these shocks using both generalized and structural impulse response functions.

The key to the GVAR modelling is the systematic inclusion of the country-specific foreign variables in the individual country models in order to deal with the common factor dependencies that exist in the world economy. The average pair-wise cross-section correlations computed for the endogenous variables, their first differences, and the residuals from the GVAR model show that very little cross section correlations remain once the effects of foreign variables have been taken into account. This is in line with the results of the tests of weak exogeneity of the foreign variables also reported in the paper.

In addition to generalized impulse response functions, we also consider structural identification of shocks in the global economy, and emphasize that unlike the GIRFs, the results of structural impulse responses in general depend on the order in which different countries are included in the GVAR model. It is partly for this reason that in our structural impulse response analyses we focus on identification of shocks to the U.S. economy, which we order as the first economy in the GVAR model. In particular, we consider the short term and long term effects of a U.S. monetary policy shock on the euro area.

From a policy analysis perspective, a number of interesting results emerge:

The simulations clearly show that financial shocks (equity and bond prices) tend to be transmitted much faster than shocks to real output and/or inflation, and often get amplified as they travel from the U.S. to the euro area and the rest of the world. Equity and bond markets seem to be far more synchronous as compared to the foreign exchange markets. The cross country dependence of exchange rates is around 25% for all countries except China and Japan for which it is lower, and hardly changes as a result of the GVAR modelling that allows for the effect of country specific foreign variables. Further research is clearly needed for a better understanding of the factors that lie behind exchange rate interdependencies. The instability of the cross market correlations over time could also be an important factor in such an explanation.

Transmission of real shocks is rather slow, normally taking 2-3 years, or in some cases even more, before their full impacts are felt. The long run impacts of the real shocks are, however, larger than what might be expected from a simple trade perspective. Regarding output and inflation shocks, the trade linkages appear to work first, leading the shocks to be transmitted in a gradual manner with their effects being spread over 2–3 years. The effects of output shocks across countries is less synchronous than inflation shocks, which is still less synchronous than the effects of shocks to financial variables.

Comparing the effects of the shocks on the euro area economy and the rest of Western Europe, the results show striking similarities. The same applies to the U.K. although to a lesser extent. One noticeable difference concerns the exchange rate response. The U.K. real exchange rate tends to deviate less from the U.S. dollar than the euro.

The model also highlights the importance of second, and even third round effects of the shocks (particularly the financial ones). A shock in the U.S. can be amplified because the U.S. will also be affected over time through the return impacts of output and inflation shocks in the rest of the world. The euro area in turn reacts to the U.S. shocks directly as well as indirectly through the impact of the U.S. shocks on euro area trade partners, and so on. The transmission of shocks does not take place only through trade, but also as importantly through the impact of real shocks on financial variables with subsequent spillover effects on real variables. The GVAR presents a complicated, yet simple to follow, spatio-temporal structure for the analysis of the world economy. To be sure it can be modified and extended further. But it is hoped that the present version makes a further step towards the development of a transparent and coherent framework for the analysis of global interdependencies.

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Table 22: Generalised Impulse Responses of a Negative One Standard Error Shock to Oil Prices

	Quarters									Average	Average
	0	1	2	3	4	5	6	7	8	1st Year	2nd Year
Oil Prices	12.30	13.77	13.69	13.61	13.61	13.62	13.69	13.75	13.75	13.34	13.67
On real output (%)											
U.S.	-0.02	-0.13	-0.19	-0.21	-0.24	-0.26	-0.28	-0.29	-0.31	-0.14	-0.27
Euro Area	-0.03	-0.07	-0.04	-0.04	-0.05	-0.04	-0.04	-0.04	-0.05	-0.05	-0.04
China	0.03	0.05	0.05	0.06	0.06	0.06	0.06	0.05	0.05	0.05	0.06
Japan	0.09	0.14	0.19	0.21	0.22	0.23	0.24	0.25	0.25	0.16	0.24
U.K.	-0.07	-0.05	-0.05	-0.07	-0.08	-0.09	-0.09	-0.11	-0.12	-0.06	-0.09
Rest W.Europe	-0.04	0.02	-0.01	0.01	0.00	0.02	0.01	0.01	0.01	-0.01	0.01
On inflation (%)											
U.S.	0.21	0.16	0.11	0.11	0.11	0.11	0.10	0.11	0.10	0.15	0.11
Euro area	0.11	0.05	0.03	0.04	0.06	0.05	0.05	0.05	0.05	0.05	0.05
China	0.12	-0.07	-0.28	-0.22	-0.10	-0.11	-0.18	-0.18	-0.14	-0.12	-0.14
Japan	0.01	0.12	0.08	0.12	0.12	0.13	0.12	0.13	0.13	0.08	0.13
U.K.	0.18	-0.10	0.06	-0.08	0.04	0.02	0.02	0.02	0.03	0.02	0.02
Rest W.Europe	0.15	0.07	0.11	0.09	0.11	0.09	0.09	0.10	0.09	0.11	0.10
On real equity prices (%)											
U.S.	-1.09	-1.89	-2.17	-2.26	-2.35	-2.37	-2.40	-2.44	-2.46	-1.85	-2.39
Euro Area	-1.36	-3.54	-4.29	-4.65	-4.70	-4.91	-4.99	-5.06	-5.08	-3.46	-4.91
Japan	0.77	-0.46	-1.07	-1.40	-1.45	-1.66	-1.78	-1.86	-1.89	-0.54	-1.69
U.K.	0.09	-0.91	-1.29	-1.45	-1.43	-1.57	-1.59	-1.63	-1.63	-0.89	-1.56
Rest W.Europe	-1.09	-2.91	-3.62	-3.88	-3.93	-4.05	-4.08	-4.14	-4.16	-2.88	-4.05
On short-term interest rate (%)											
U.S.	0.03	0.03	0.03	0.04	0.03	0.03	0.03	0.03	0.03	0.03	0.03
Euro Area	0.01	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.05	0.03	0.04
China	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Japan	0.01	0.04	0.06	0.07	0.08	0.09	0.10	0.10	0.10	0.05	0.09
U.K.	-0.03	0.01	-0.04	-0.03	-0.03	-0.02	-0.02	-0.02	-0.02	-0.02	-0.03
Rest W.Europe	0.00	0.04	0.03	0.03	0.02	0.03	0.03	0.03	0.03	0.03	0.03
On real exchange rate (%)											
Euro Area	0.27	-0.23	0.01	0.16	0.30	0.32	0.36	0.38	0.41	0.05	0.34
China	-0.42	-0.57	-0.48	-0.43	-0.53	-0.60	-0.58	-0.57	-0.61	-0.48	-0.57
Japan	1.28	2.14	2.96	3.51	3.98	4.25	4.48	4.63	4.74	2.47	4.33
U.K.	-0.14	-0.29	-0.67	-0.04	0.58	1.00	1.23	1.39	1.48	-0.28	1.05
Rest W.Europe	0.13	-0.31	-0.52	-0.62	-0.59	-0.59	-0.59	-0.58	-0.57	-0.33	-0.59
On long-term interest rate (%)											
U.S.	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.04	0.04	0.03	0.04
Euro Area	0.03	0.04	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.04	0.06
Japan	0.04	0.04	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.05	0.06
U.K.	0.02	0.02	0.01	0.01	0.02	0.02	0.03	0.03	0.03	0.01	0.02
Rest W.Europe	0.03	0.04	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.04	0.05

Table 23: Generalised Impulse Responses of a Negative One Standard Error Shock to US Equity Prices

	Quarters									Average	Average
	0	1	2	3	4	5	6	7	8	1st Year	2nd Year
Oil Prices	2.06	-0.72	-2.62	-3.08	-3.51	-3.92	-4.20	-4.39	-4.62	-1.09	-4.01
	On real output (%)										
U.S.	-0.11	-0.39	-0.54	-0.60	-0.62	-0.64	-0.63	-0.62	-0.60	-0.41	-0.63
Euro Area	0.03	-0.07	-0.14	-0.23	-0.32	-0.39	-0.44	-0.48	-0.52	-0.10	-0.41
China	0.03	0.05	0.06	0.09	0.12	0.15	0.18	0.22	0.26	0.06	0.17
Japan	0.09	0.05	0.02	-0.06	-0.14	-0.23	-0.31	-0.40	-0.48	0.03	-0.27
U.K.	-0.04	-0.06	-0.16	-0.24	-0.30	-0.34	-0.36	-0.37	-0.37	-0.12	-0.34
Rest W.Europe	-0.14	-0.27	-0.43	-0.54	-0.60	-0.66	-0.70	-0.73	-0.74	-0.34	-0.67
	On inflation (%)										
U.S.	0.05	-0.01	-0.09	-0.09	-0.09	-0.09	-0.10	-0.09	-0.09	-0.04	-0.09
Euro Area	-0.02	-0.03	-0.05	-0.03	-0.02	-0.04	-0.05	-0.05	-0.06	-0.03	-0.04
China	0.24	-0.03	-0.27	-0.09	0.08	0.02	-0.07	-0.03	0.03	-0.04	0.00
Japan	-0.12	0.01	-0.12	-0.06	-0.10	-0.09	-0.12	-0.11	-0.12	-0.07	-0.11
U.K.	-0.02	-0.14	-0.03	-0.21	-0.09	-0.12	-0.14	-0.15	-0.14	-0.10	-0.12
Rest W.Europe	-0.03	0.04	-0.02	-0.05	-0.03	-0.07	-0.09	-0.09	-0.10	-0.01	-0.07
	On real equity prices (%)										
U.S.	-5.71	-7.26	-7.40	-7.24	-7.05	-6.82	-6.59	-6.41	-6.24	-6.90	-6.72
Euro Area	-5.20	-7.46	-8.66	-9.35	-9.76	-10.16	-10.29	-10.28	-10.17	-7.67	-10.12
Japan	-2.14	-4.35	-5.25	-5.49	-5.47	-5.62	-5.64	-5.59	-5.51	-4.31	-5.58
U.K.	-4.60	-5.76	-6.35	-6.33	-6.27	-6.28	-6.12	-5.93	-5.70	-5.76	-6.15
Rest W.Europe	-4.47	-7.39	-8.86	-9.62	-10.08	-10.40	-10.49	-10.49	-10.39	-7.59	-10.37
	On short-term interest rate (%)										
U.S.	0.00	-0.06	-0.10	-0.12	-0.14	-0.16	-0.17	-0.18	-0.19	-0.07	-0.16
Euro Area	-0.01	-0.01	-0.02	-0.04	-0.06	-0.08	-0.10	-0.12	-0.13	-0.02	-0.09
China	0.00	0.00	0.00	0.00	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00
Japan	0.00	0.01	0.01	-0.01	-0.02	-0.03	-0.04	-0.05	-0.06	0.00	-0.03
U.K.	-0.03	-0.02	-0.06	-0.08	-0.10	-0.11	-0.13	-0.14	-0.15	-0.05	-0.12
Rest W.Europe	0.03	0.04	0.03	0.02	0.00	-0.01	-0.03	-0.04	-0.06	0.03	-0.02
	On real exchange rate (%)										
Euro Area	-1.14	-1.69	-1.75	-1.71	-1.62	-1.58	-1.54	-1.52	-1.49	-1.58	-1.56
China	0.07	0.14	0.19	0.09	-0.16	-0.31	-0.36	-0.43	-0.55	0.12	-0.32
Japan	0.03	0.24	0.54	0.61	0.67	0.56	0.53	0.49	0.45	0.35	0.56
U.K.	-0.62	-0.57	-0.53	0.07	0.31	0.30	0.25	0.24	0.23	-0.41	0.28
Rest W.Europe	-1.45	-2.04	-2.28	-2.33	-2.28	-2.25	-2.20	-2.12	-2.03	-2.03	-2.21
	On long-term interest rate (%)										
U.S.	0.01	-0.02	-0.04	-0.05	-0.07	-0.08	-0.09	-0.10	-0.11	-0.02	-0.09
Euro Area	0.00	-0.01	-0.04	-0.05	-0.06	-0.08	-0.09	-0.10	-0.11	-0.02	-0.08
Japan	0.01	-0.01	-0.02	-0.02	-0.03	-0.04	-0.04	-0.05	-0.05	-0.01	-0.04
U.K.	0.02	0.01	-0.01	-0.02	-0.03	-0.04	-0.05	-0.06	-0.07	0.00	-0.04
Rest W.Europe	0.00	-0.01	-0.02	-0.03	-0.05	-0.06	-0.07	-0.08	-0.08	-0.01	-0.06

Table 24: Generalised Impulse Responses of a Negative One Standard Error Shock to US Real GDP

	Quarters									Average	Average
	0	1	2	3	4	5	6	7	8	1st Year	2nd Year
Oil Prices	0.79	-1.16	-1.74	-1.87	-1.93	-2.03	-2.23	-2.35	-2.41	-1.00	-2.14
					On real output (%)						
U.S.	-0.51	-0.56	-0.60	-0.57	-0.56	-0.54	-0.51	-0.49	-0.47	-0.56	-0.53
Euro Area	-0.04	-0.08	-0.10	-0.13	-0.15	-0.16	-0.18	-0.19	-0.19	-0.09	-0.17
China	0.04	0.08	0.11	0.14	0.17	0.20	0.23	0.26	0.28	0.09	0.21
Japan	0.03	0.00	-0.06	-0.10	-0.14	-0.19	-0.22	-0.25	-0.28	-0.03	-0.20
U.K.	-0.07	-0.05	-0.08	-0.08	-0.07	-0.05	-0.04	-0.02	-0.01	-0.07	-0.05
Rest W.Europe	-0.16	-0.09	-0.15	-0.13	-0.14	-0.14	-0.14	-0.14	-0.14	-0.13	-0.14
					On inflation (%)						
U.S.	0.00	-0.03	-0.04	-0.03	-0.03	-0.03	-0.02	-0.02	-0.02	-0.02	-0.03
Euro area	0.04	-0.03	-0.01	0.00	-0.01	-0.01	-0.01	-0.01	-0.02	0.00	-0.01
China	-0.01	0.01	0.04	0.07	0.05	0.04	0.05	0.06	0.05	0.03	0.05
Japan	0.00	-0.03	-0.04	-0.05	-0.06	-0.06	-0.06	-0.07	-0.07	-0.03	-0.06
U.K.	0.02	-0.03	-0.06	-0.02	-0.04	-0.03	-0.02	-0.02	-0.03	-0.02	-0.03
Rest W.Europe	0.05	-0.04	-0.03	-0.02	-0.04	-0.02	-0.03	-0.04	-0.04	-0.01	-0.03
					On real equity prices (%)						
U.S.	-1.08	-1.34	-1.15	-1.12	-1.05	-0.93	-0.81	-0.71	-0.63	-1.17	-0.88
Euro Area	-0.62	-0.96	-0.78	-0.88	-1.08	-1.05	-1.00	-0.95	-0.87	-0.81	-1.02
Japan	-0.31	-0.71	-0.71	-0.66	-0.69	-0.60	-0.53	-0.48	-0.44	-0.60	-0.58
U.K.	0.21	0.08	0.32	0.35	0.32	0.45	0.53	0.61	0.69	0.24	0.48
Rest W.Europe	-1.07	-1.37	-1.18	-1.21	-1.29	-1.26	-1.19	-1.11	-1.03	-1.21	-1.21
					On short-term interest rate (%)						
U.S.	-0.08	-0.13	-0.14	-0.15	-0.16	-0.16	-0.17	-0.17	-0.17	-0.12	-0.16
Euro Area	-0.01	-0.02	-0.04	-0.05	-0.06	-0.07	-0.07	-0.08	-0.09	-0.03	-0.07
China	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
Japan	0.02	0.01	-0.01	-0.02	-0.03	-0.04	-0.05	-0.05	-0.05	0.00	-0.04
U.K.	0.02	-0.01	-0.03	-0.05	-0.05	-0.06	-0.06	-0.07	-0.07	-0.02	-0.06
Rest W.Europe	-0.02	-0.01	-0.04	-0.05	-0.06	-0.07	-0.08	-0.08	-0.09	-0.03	-0.07
					On real exchange rate (%)						
Euro Area	-0.11	-0.22	-0.05	-0.08	-0.12	-0.12	-0.12	-0.13	-0.17	-0.12	-0.12
China	-0.24	-0.34	-0.39	-0.49	-0.54	-0.54	-0.57	-0.60	-0.62	-0.36	-0.56
Japan	-0.43	-0.72	-1.08	-1.38	-1.67	-1.86	-2.01	-2.14	-2.23	-0.90	-1.92
U.K.	-0.10	-0.50	-0.63	-0.60	-0.67	-0.75	-0.83	-0.91	-0.97	-0.45	-0.79
Rest W.Europe	-0.34	-0.37	-0.24	-0.11	-0.06	-0.01	0.04	0.07	0.09	-0.26	0.01
					On long-term interest rate (%)						
U.S.	-0.05	-0.06	-0.07	-0.08	-0.09	-0.09	-0.10	-0.10	-0.11	-0.06	-0.10
Euro Area	-0.01	-0.03	-0.04	-0.05	-0.06	-0.07	-0.07	-0.08	-0.09	-0.03	-0.07
Japan	-0.01	-0.02	-0.03	-0.03	-0.04	-0.04	-0.04	-0.05	-0.05	-0.02	-0.04
U.K.	-0.02	-0.04	-0.05	-0.06	-0.07	-0.07	-0.08	-0.08	-0.09	-0.04	-0.07
Rest W.Europe	-0.01	-0.03	-0.04	-0.05	-0.05	-0.06	-0.07	-0.07	-0.07	-0.03	-0.06

Table 25: Generalised Impulse Responses of a Positive One Standard Error Shock to US Interest rates

	Quarters									average	average
	0	1	2	3	4	5	6	7	8	1st year	2nd year
Oil prices	1.63	2.08	1.90	1.82	1.66	1.53	1.57	1.65	1.57	1.85	1.60
					On real GDP (%)						
US	0.19	0.28	0.13	0.04	-0.02	-0.08	-0.13	-0.16	-0.19	0.16	-0.10
Euro area	0.00	-0.04	-0.08	-0.12	-0.15	-0.17	-0.18	-0.19	-0.20	-0.06	-0.17
China	-0.04	-0.08	-0.11	-0.14	-0.16	-0.18	-0.19	-0.20	-0.21	-0.09	-0.18
Japan	0.09	0.10	0.11	0.06	0.01	-0.04	-0.08	-0.13	-0.18	0.09	-0.06
UK	0.07	0.07	0.08	0.07	0.08	0.09	0.13	0.17	0.22	0.07	0.12
Rest W.Europe	0.03	-0.05	-0.12	-0.18	-0.19	-0.20	-0.22	-0.22	-0.22	-0.08	-0.21
					On inflation (%)						
US	0.06	0.14	0.05	0.04	0.06	0.03	0.02	0.02	0.02	0.07	0.03
Euro area	-0.01	0.00	0.00	-0.01	0.00	-0.02	-0.02	-0.02	-0.02	0.00	-0.02
China	0.36	0.12	-0.28	-0.17	0.07	0.05	-0.08	-0.07	0.01	0.01	-0.01
Japan	-0.07	0.08	-0.05	0.03	0.01	0.03	0.00	0.02	0.01	0.00	0.02
UK	0.07	-0.10	0.01	-0.12	-0.07	-0.08	-0.12	-0.13	-0.12	-0.04	-0.10
Rest W.Europe	0.00	0.00	0.02	-0.01	0.01	-0.02	-0.03	-0.02	-0.02	0.00	-0.02
					On real equity prices (%)						
US	0.02	-0.78	-0.74	-0.61	-0.64	-0.66	-0.64	-0.68	-0.70	-0.53	-0.66
Euro area	-0.03	-1.22	-1.52	-1.64	-1.50	-1.51	-1.48	-1.42	-1.26	-1.10	-1.48
Japan	-0.22	-0.93	-1.39	-1.57	-1.57	-1.82	-2.00	-2.08	-2.07	-1.02	-1.87
UK	-0.22	-0.97	-0.87	-0.63	-0.27	-0.07	0.21	0.48	0.79	-0.67	0.09
Rest W.Europe	0.05	-1.17	-1.66	-1.85	-1.83	-1.82	-1.74	-1.69	-1.55	-1.16	-1.77
					On interest rate (%)						
US	0.19	0.20	0.18	0.19	0.19	0.18	0.18	0.18	0.18	0.19	0.18
Euro area	0.02	0.03	0.03	0.02	0.02	0.02	0.01	0.01	0.01	0.03	0.02
China	0.02	0.03	0.04	0.04	0.03	0.03	0.04	0.04	0.04	0.03	0.03
Japan	-0.02	-0.01	0.00	0.01	0.02	0.02	0.02	0.02	0.02	-0.01	0.02
UK	-0.03	0.01	0.00	0.00	-0.01	-0.02	-0.03	-0.04	-0.04	0.00	-0.03
Rest W.Europe	-0.01	0.02	0.03	0.04	0.03	0.03	0.02	0.02	0.01	0.02	0.02
					On real exchange rate (%)						
Euro area	0.66	0.45	0.39	0.40	0.53	0.58	0.59	0.59	0.65	0.47	0.57
China	-0.75	-0.98	-0.88	-0.86	-1.06	-1.24	-1.28	-1.28	-1.35	-0.87	-1.21
Japan	0.75	1.03	1.99	2.74	3.32	3.59	3.87	4.05	4.21	1.63	3.71
UK	0.07	0.50	0.78	1.35	1.82	2.12	2.45	2.83	3.19	0.68	2.30
Rest W.Europe	0.33	-0.06	-0.25	-0.30	-0.20	-0.14	-0.13	-0.11	-0.05	-0.07	-0.14
					On long-term interest rate (%)						
US	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.05	0.06	0.04	0.05
Euro area	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Japan	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.02
UK	0.01	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.02	0.01
Rest W.Europe	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.02

Table 26: Correlation Coefficients of Country Specific Foreign Output and Inflation using Fixed and Time-Varying Trade Weights

	Output		Inflation	
Country	Levels	1st Difference	Levels	1st Difference
United States	0.999	0.931	0.885	0.677
Euro area	0.998	0.908	0.865	0.501
China	0.999	0.910	0.426	0.026
Japan	0.999	0.880	0.768	0.420
United Kingdom	1.000	0.984	0.948	0.729
Sweden	1.000	0.983	0.889	0.597
Switzerland	1.000	0.991	0.901	0.592
Norway	1.000	0.984	0.912	0.646

Table 27: Correlation Coefficients of Country Specific Foreign Real Equity Prices and Real Exchange Rates using Fixed and Time-Varying Trade Weights

	Real Equity Prices		Real Exchange Rates	
Country	Levels	1st Difference	Levels	1st Difference
United States	-	-	0.955	0.816
Euro area	0.814	0.996	0.814	0.859
China	0.868	0.980	0.868	0.669
Japan	0.852	0.985	0.852	0.629
United Kingdom	0.981	0.999	0.981	0.955
Sweden	0.975	0.999	0.975	0.956
Switzerland	0.994	0.999	0.994	0.971
Norway	0.868	0.996	0.868	0.893

Table 28: Correlation Coefficients of Country Specific Foreign Short and Long Term Interest Rates using Fixed and Time-Varying Trade Weights

	Short-Term Interest Rates		Long-Term Interest Rates	
Country	Levels	1st Difference	Levels	1st Difference
United States	-	-	-	-
Euro area	0.987	0.993	0.999	0.998
China	0.962	0.931	0.993	0.942
Japan	0.993	0.989	0.998	0.994
United Kingdom	0.999	0.989	0.999	0.998
Sweden	0.991	0.979	1.000	0.999
Switzerland	0.995	0.981	1.000	0.999
Norway	0.997	0.979	0.999	0.992

Table 29: VARX* Order and Number of Cointegration Relationships in the Country-Specific Models using Fixed and Time-Varying Trade Weights

	Fixed Weights			Time-Varying Weights		
	VARX*(p_i, q_i)		#Cointegrating	VARX*(p_i, q_i)		#Cointegrating
Country	p_i	q_i	Relationships	p_i	q_i	Relationships
U.S.	2	2	2	2	2	2
E.A.	2	2	2	2	2	2
China	2	1	1	2	1	1
Japan	1	1	4	1	1	3
U.K.	2	2	3	2	2	2
Sweden	2	1	3	2	1	2
Switz.	1	1	3	1	1	3
Norway	2	1	2	2	1	2

Table 30: Contemporaneous Effects of Foreign Variables on their Domestic Counterparts using Time-Varying Trade Weights

	Domestic Variables				
Country	y	Δp	q	ρ^S	ρ^L
United States	0.45*	0.03	-	-	-
	(0.11)	(0.03)			
Euro Area	0.51*	-0.02	1.11*	0.10*	0.70*
	(0.10)	(0.03)	(0.08)	(0.02)	(0.08)
China	-0.14	-0.09	-	0.02	-
	(0.15)	(0.08)		(0.04)	
Japan	0.29*	0.03	0.59*	-0.04	0.44*
	(0.14)	(0.05)	(0.12)	(0.04)	(0.10)
United Kingdom	0.53*	0.25	0.87*	0.12	0.76*
	(0.12)	(0.17)	(0.07)	(0.15)	(0.15)
Sweden	0.92*	0.31*	1.14*	1.18*	0.88*
	(0.37)	(0.10)	(0.11)	(0.34)	(0.14)
Switzerland	0.44*	0.34*	0.69*	0.22*	0.43*
	(0.12)	(0.06)	(0.12)	(0.07)	(0.08)
Norway	0.38	0.39*	1.03*	0.14	0.54*
	(0.37)	(0.10)	(0.12)	(0.12)	(0.14)

Note: * denotes statistical significance at the 5% level or less. Standard errors are in parentheses.

Table 31: Average Pair-Wise Cross-Section Correlations of the Residuals for Output using Fixed and Time-Varying Trade Weights

Country	Fixed Weights Residuals	Time-Varying Weights Residuals
United States	-0.04	-0.04
Euro Area	-0.01	-0.02
China	-0.01	0.01
Japan	-0.07	-0.09
United Kingdom	0.01	0.00
Sweden	0.01	0.01
Switzerland	0.01	0.00
Norway	0.01	0.02

Table 32: Average Pair-Wise Cross-Section Correlations of the Residuals for Inflation using Fixed and Time-Varying Trade Weights

Country	Fixed Weights Residuals	Time-Varying Weights Residuals
United States	0.02	0.03
Euro Area	0.00	0.06
China	-0.05	0.00
Japan	0.00	0.03
United Kingdom	0.00	0.03
Sweden	0.00	0.05
Switzerland	0.05	0.03
Norway	0.03	0.05

Table 33: Average Pair-Wise Cross-Section Correlations of the Residuals for Real Equity Prices using Fixed and Time-Varying Trade Weights

Country	Fixed Weights Residuals	Time-Varying Weights Residuals
United States	-0.02	-0.01
Euro Area	-0.08	-0.07
Japan	-0.09	-0.09
United Kingdom	-0.03	-0.03
Sweden	-0.01	-0.02
Switzerland	-0.05	-0.03
Norway	0.01	0.00

Table 34: Average Pair-Wise Cross-Section Correlations of the Residuals for Real Exchange Rates using Fixed and Time-Varying Trade Weights

Country	Fixed Weights Residuals	Time-Varying Weights Residuals
Euro Area	0.28	0.27
China	0.03	0.03
Japan	0.17	0.20
United Kingdom	0.19	0.20
Sweden	0.19	0.19
Switzerland	0.28	0.27
Norway	0.28	0.27

Table 35: Average Pair-Wise Cross-Section Correlations of the Residuals for the Short-Term Interest Rates using Fixed and Time-Varying Trade Weights

Country	Fixed Weights Residuals	Time-Varying Weights Residuals
United States	0.00	0.01
Euro Area	0.03	0.02
China	-0.04	-0.03
Japan	-0.01	-0.02
United Kingdom	0.01	0.03
Sweden	-0.01	-0.02
Switzerland	0.00	0.01
Norway	-0.01	-0.01

Table 36: Average Pair-Wise Cross-Section Correlations of the Residuals for the Long-Term Interest Rates using Fixed and Time-Varying Trade Weights

Country	Fixed Weights Residuals	Time-Varying Weights Residuals
United States	-0.03	-0.02
Euro Area	-0.06	-0.08
Japan	-0.05	-0.04
United Kingdom	-0.01	-0.02
Sweden	0.07	0.06
Switzerland	0.02	0.02
Norway	0.02	0.01

Figure 1: Structural Impulse Responses of a Positive Unit (+1σ) Shock to Oil Prices

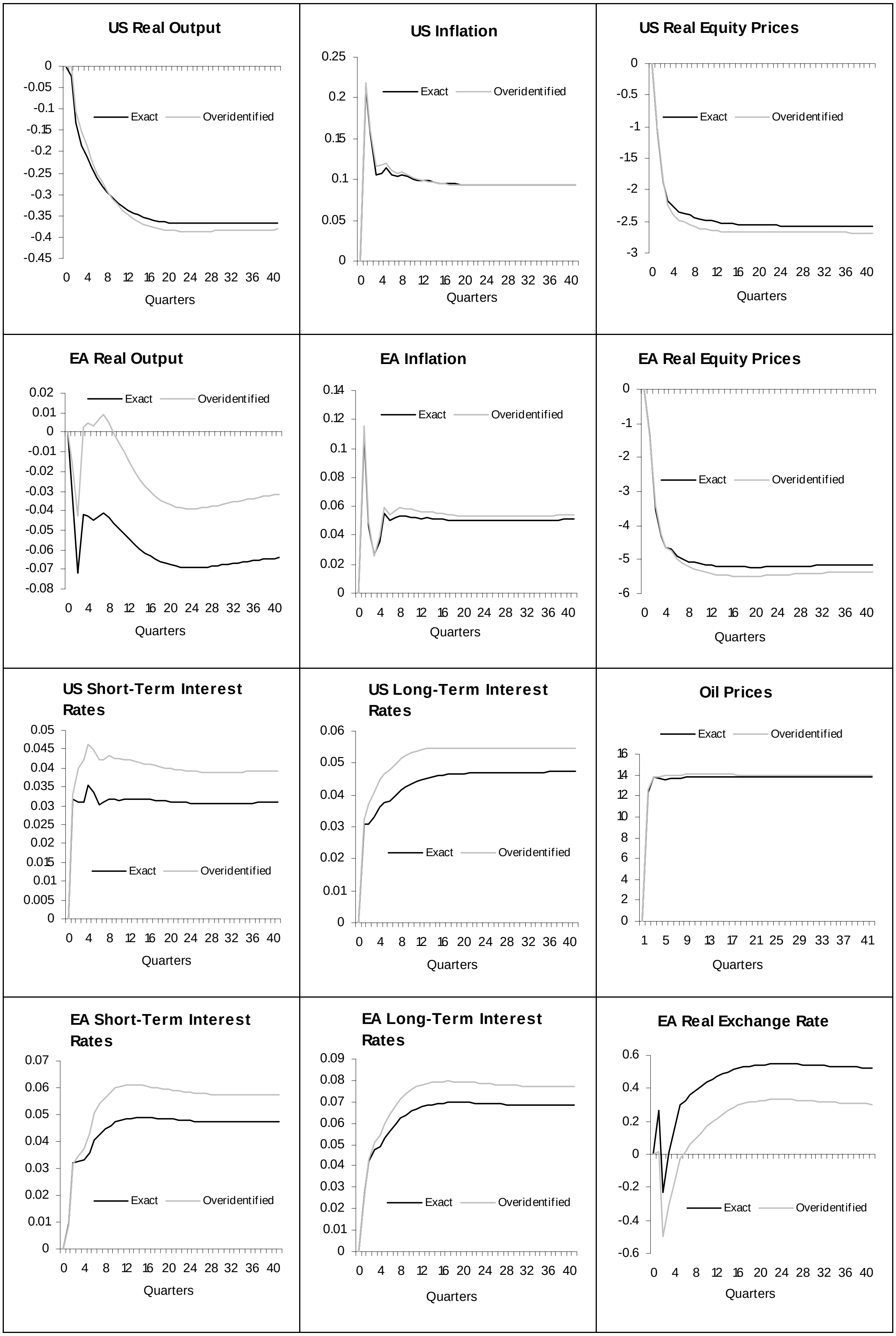


Figure 2: Structural and Non-Structural Impulse Responses of a Negative Unit (-1σ) Shock to US Real Equity Prices

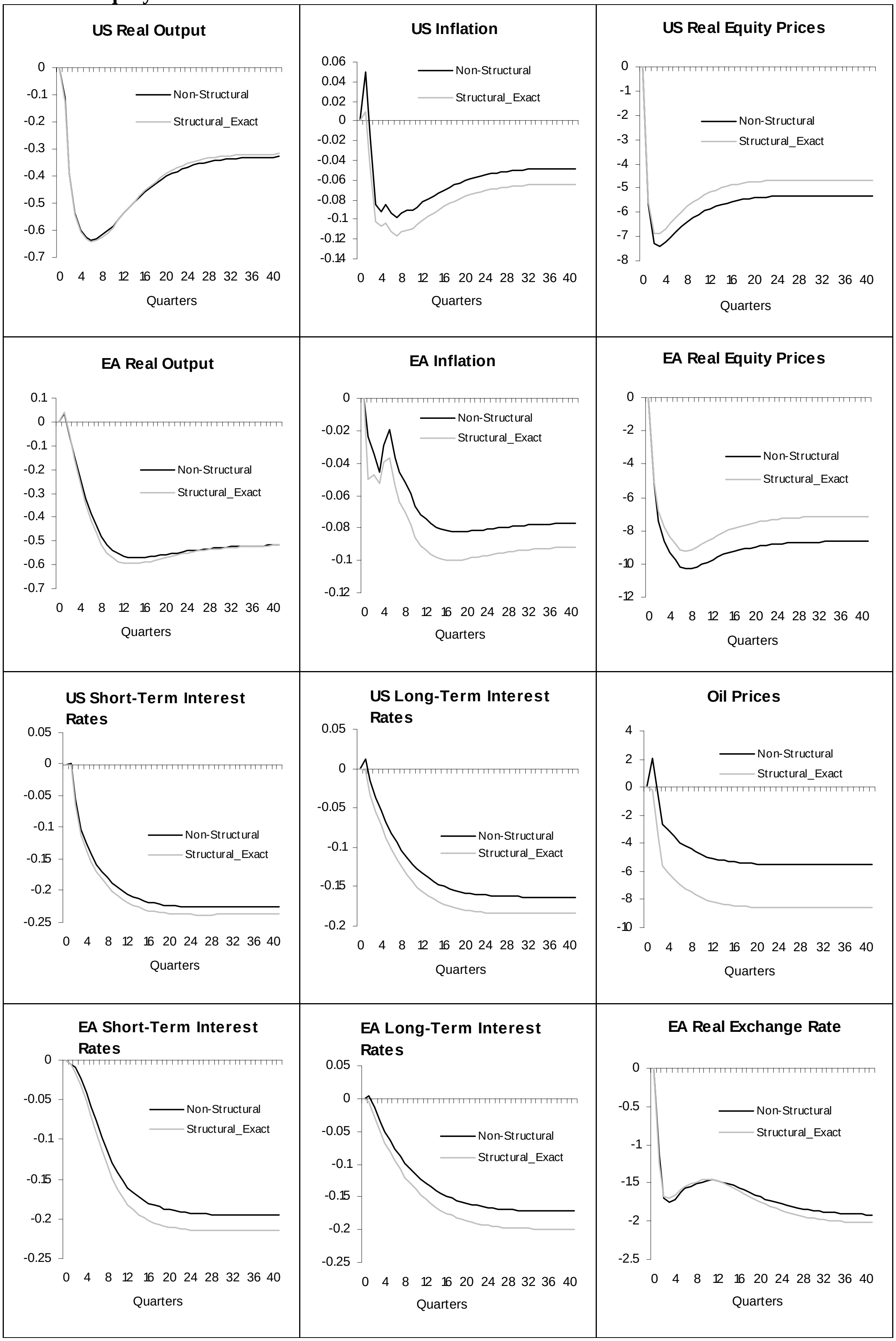


Figure 3: Structural Impulse Responses of a Negative Unit (-1σ) Shock to US Real Equity Prices

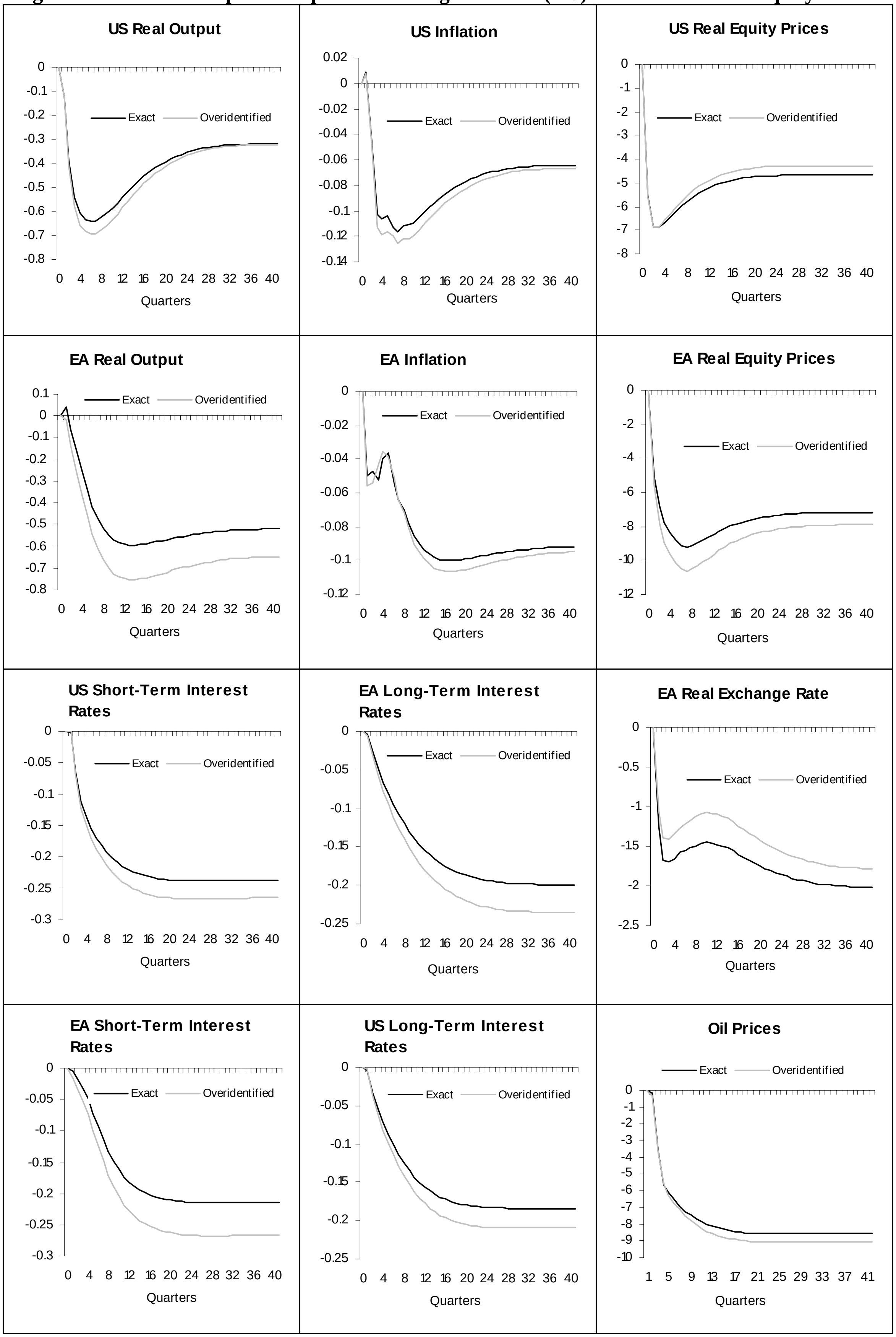


Figure 4: Structural and Non-Structural Impulse Responses of a Negative Unit (-1σ) Shock to US Real Output

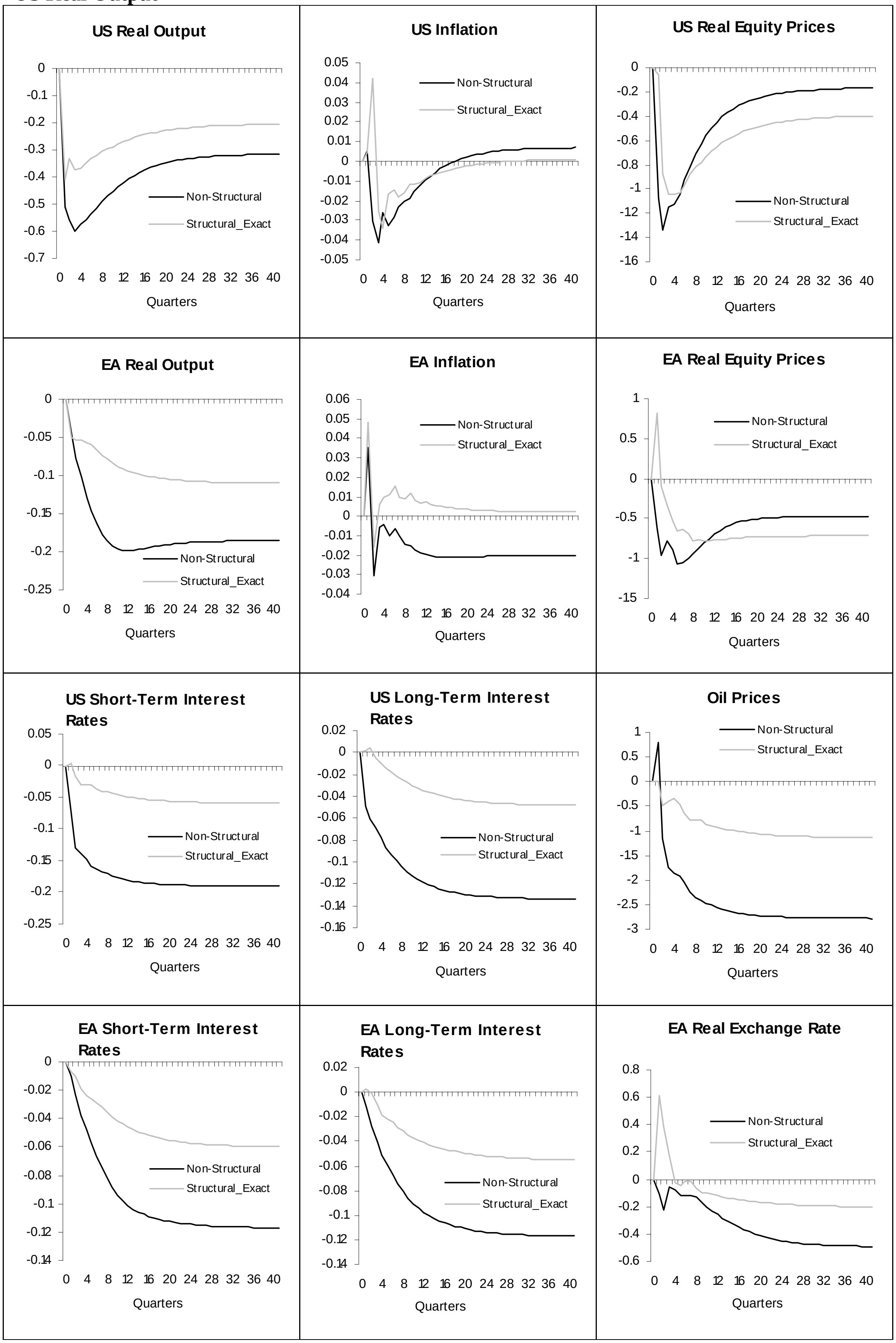


Figure 5: Structural Impulse Responses of a Negative Unit (-1σ) Shock to US Real Output

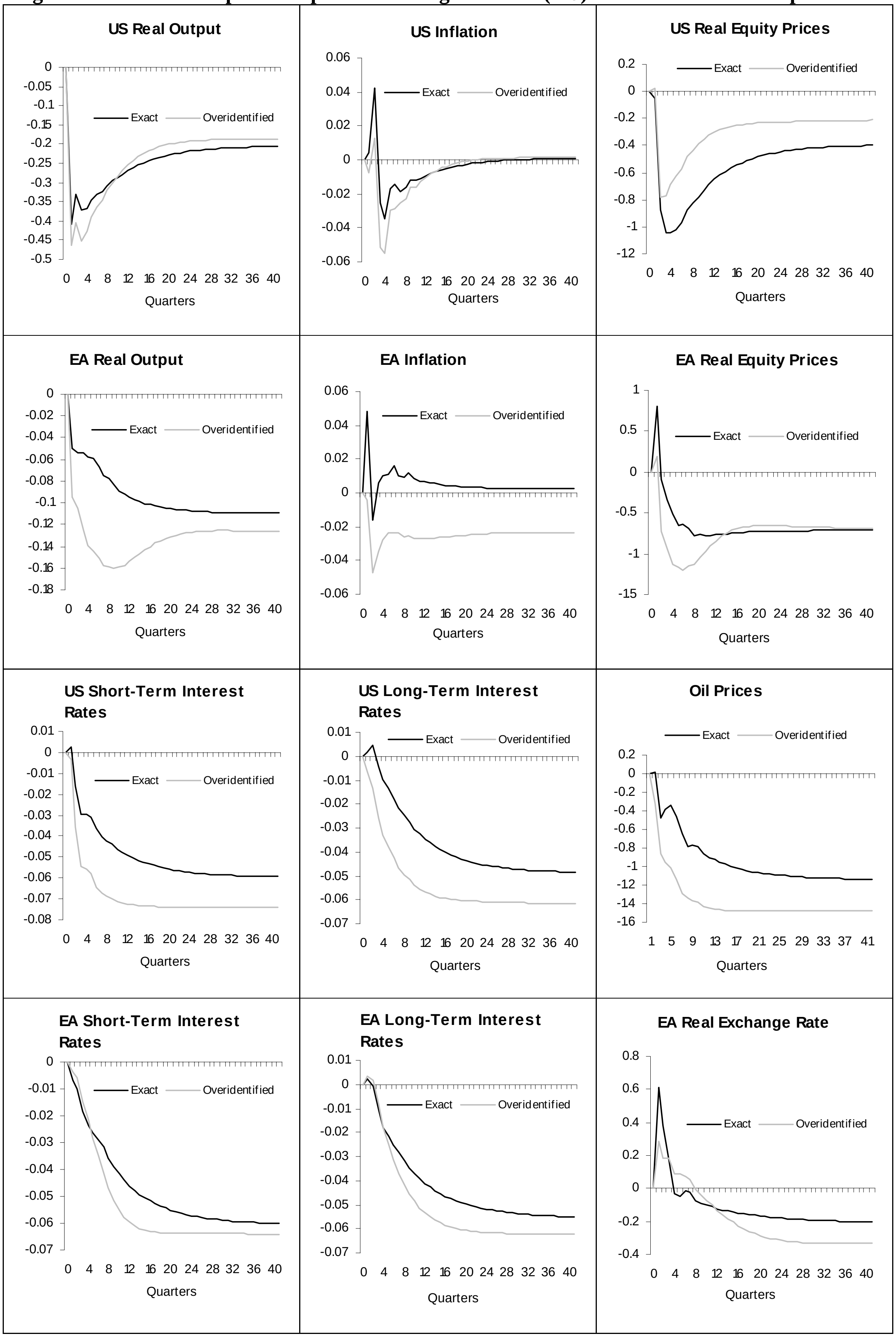


Figure 6: Structural and Non-Structural Impulse Responses of a Positive Unit (+1σ) Shock to US Short-Term Interest Rates

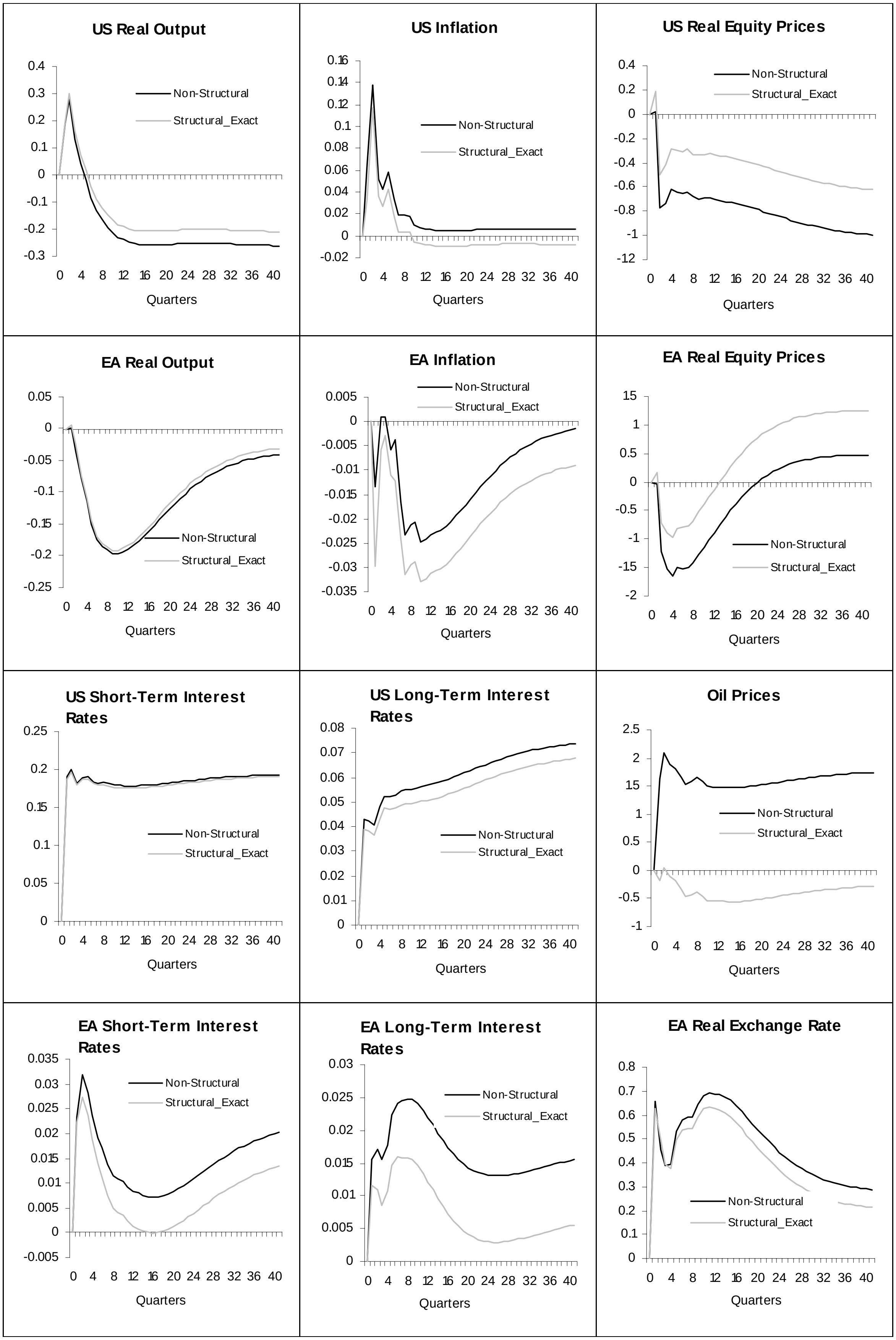


Figure 7: Structural Impulse Responses of a Positive Unit (+1σ) Shock to US Short-Term Interest Rates

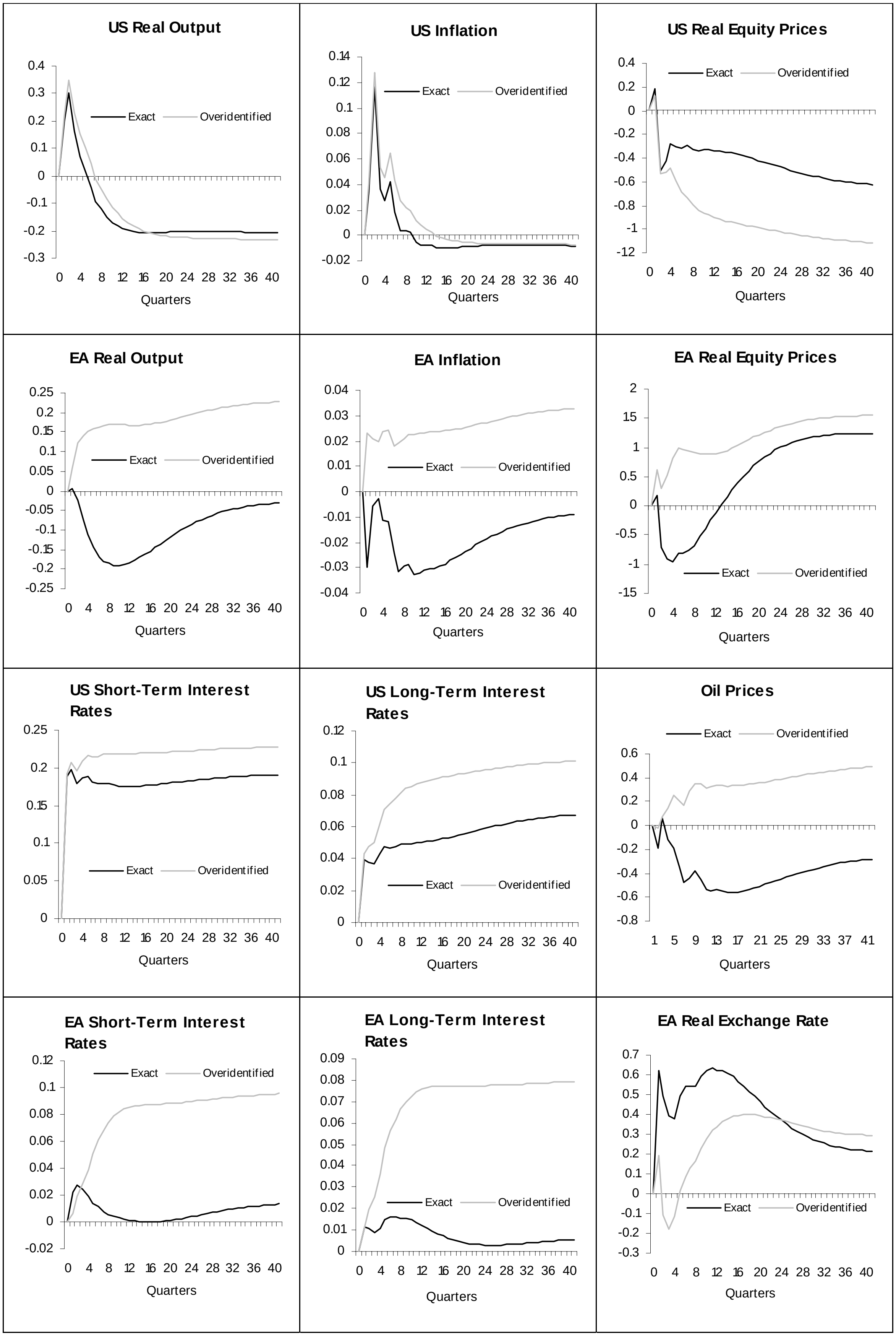
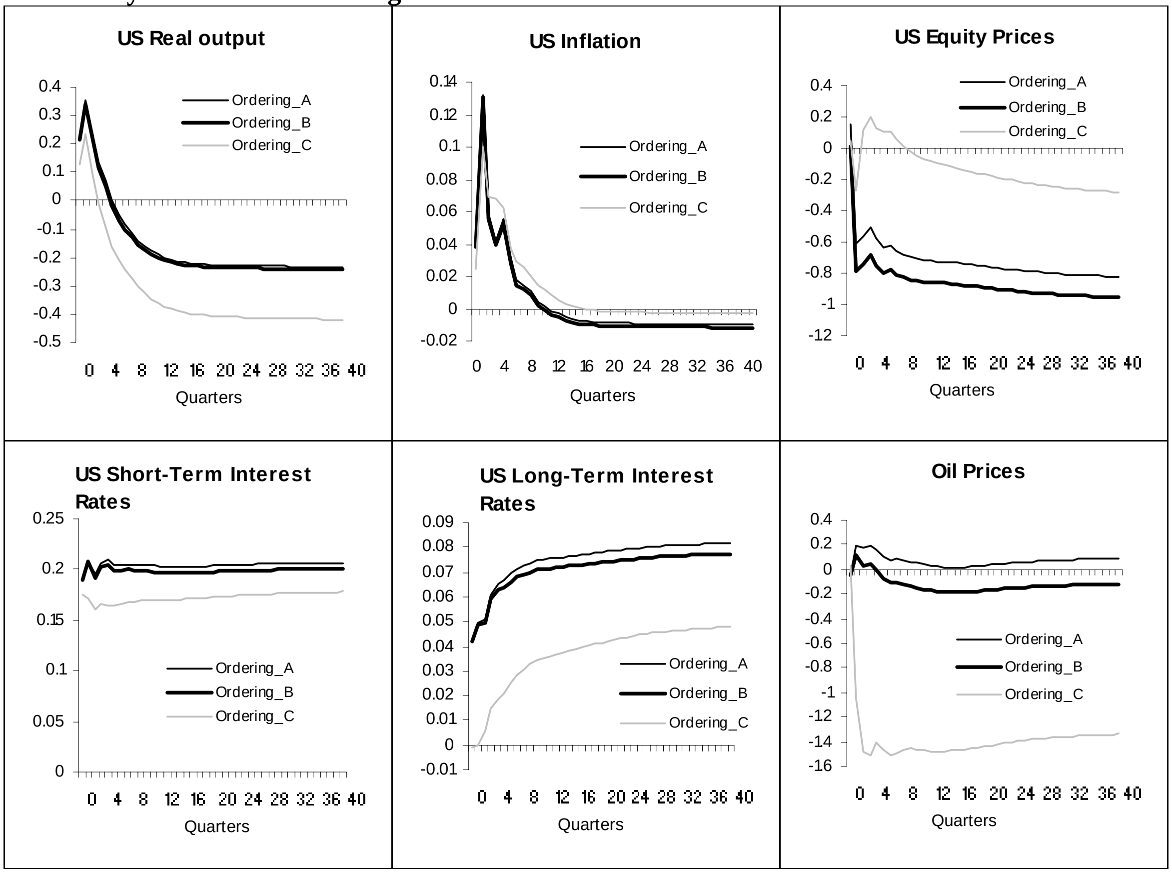


Figure 8: Structural Impulse Responses of a Positive Unit (+1σ) Shock to US Monetary Policy – Sensitivity to Different Orderings



APPENDIX A:

Data Sources and the Interpolation Procedure

The variables used in this paper are Y: Real GDP; CPI: Consumer price index; EQ: Equity price index; E: Exchange rate; R^S: Short-term interest rate; R^L: Long-term interest rate; and P^o: Oil price index.

A.1. Real GDP

The source of all 33 countries is the IMF's International Financial Statistics (IFS) GDP series in 1995 constant prices, except Australia (2001/02), Norway (2001), Singapore (2000), United Kingdom (2000) and United States (2000). France, Germany, Italy, Japan, South Africa, Spain, Netherlands, Switzerland, Australia, New Zealand, Mexico, United Kingdom and United States are all from series br, amd the remaining countries are from series bp. Where recent data were not available, the IFS series were completed with growth rates derived from series provided by Global Insight.

Where quarterly data were not available (i.e. for Argentina, Belgium, Brazil, Chile, China India, indonesia, Malaysia, Mexico, Philipines, Saudi Arabia, Thailand and Turkey), quarterly series were interpolated linearly from the annual series (see A.7). Interpolated series were used only during the periods 1981-1992 for Argentina, 1979 for Belgium, 1979-1989 for Brazil, 1979 for Chile, 1979-1996 for India, 1979-1982 for Indonesia, 1979-1987 for Malaysia, 1979 for Mexico, 1979-1980 for the Philipines, 1979-1992 for Thailand and 1979-1986 for Turkey. Quarterly output series were avialble for the subsequent periods.

The data for Singapore are from Datastream.

For the period before the German unification, in 1990Q4, West German growth rates were used.

The data for Argentina, Austria, Belgium, Brazil, Chile, Finland, India, Indonesia, Korea, Malaysia, Norway, Peru, Philippines, Sweden, Thailand and Turkey were seasonally adjusted. Seasonal adjustment was performed with E-views, using the U.S. Census Bureau's X12 program (for further details, see U.S. Census Bureau, 2000).

A.2. Consumer Price Indices

The data source for all countries was the IFS Consumer Price Index series 64 zf, except China (64 xzf).

A.3. Equity Price Indices

The data source was the IFS series 62 zf (Industrial share prices) for 25 countries (Australia, Austria, Brazil, Canada, Chile, Finland, France, Germany, Italy, Japan, Korea, Mexico, Netherlands, New Zealand, Norway, Peru, Philippines, Saudi Arabia, South Africa, Spain, Sweden, Switzerland, Thailand, United Kingdom, United States). For Norway, Sweden, France, United Kingdom, the IFS data were completed with OECD Main Economic Edicator database (MEI) the IFS data for Austria were completed with Datastream series.

The data source for Belgium, Brazil and Switzerland was Datastream.

The data source for Malaysia, Turkey and China was Bloomberg.

A.4. Exchange Rates

IFS series rf was used for all countries.

A.5. Short-Term Interest Rates

The data source was the IFS series 60 b (Money market - interbank - rate) for 16 countries. For the 8 Euro Area countries (Austria, Belgium, France, Finland, Germany, Italy, Netherlands, and Spain) the ST interest rate was constructed as follows: for 1979Q1-1998Q4, the short-term country-specific inter-bank rate from IFS was used. From 99Q1-01Q4, the overnight EONIA rate was used as the common short-term interest rate for all 8 countries. IFS deposit rate series 60l were used for Argentina, Chile, Saudi Arabia and Turkey. The IFS Treasury Bill rate series 60c were used for Mexico and Philippines. For Sweden, some missing values at the end of the sample were replaced by the series 60 a (the two series are similar over the past). For China, New Zealand and Peru, IFS discount rate 60 were used. For India, Global Insight data were used to complete IFS series.

A.6. Long-Term Interest Rates

A long-term government bond rate was available from the IFS (series 61 zf) for 23 countries. Data from OECD were used to complete gaps in the IFS series for Austria and Sweden. Long-term interest rate series were not available for Argentina, Brazil, Chile, China, India, Indonesia, Peru, Saudi Arabia and Turkey.

A.7. Description of the Interpolation Procedure

Let X_t, $t = 0, 1, 2, ..., T$, be the annual observations compiled as averages of m time-disaggregated observations, x_{it}, $i = 1, 2, ..., m$, $t = 1, 2, ..., T$, such that

$$X_t = \sum_{i=1}^{m} x_{it} \tag{A.1}$$

The objective is to estimate a relatively smooth set of obsevations, x_{it}, $i = 1, 2, ..., m$ that satisfy the above constraint. We confine ourselves to pure interpolation methods (namely without using any related economic time series) and assume that the underlying disaggregated observations are generated by the following time-varying first-order autoregressive process:

$$\begin{aligned} x_{t1} &= \rho_t x_{t-1,m} + \mu_t \\ x_{t2} &= \rho_t x_{t1,m} + \mu_t \\ &\vdots \\ x_{tm} &= \rho_t x_{t,m-1} + \mu_t. \end{aligned}$$

Solving for $x_{t+1,i}$ recursively forward we have

$$x_{t+1,i} = \rho_{t+1}^{i} x_{tm} + \mu_{t+1} \frac{(1 - \rho_{t+1}^{i})}{(1 - \rho_{t+1})}, \text{ for } i = 1, 2, ..., m. \tag{A.2}$$

Substituting these in the constraint (A.1) we find

$$X_t = \rho_{t+1}\frac{(1-\rho_{t+1}^m)}{(1-\rho_{t+1})}x_{tm} + \frac{m\mu_{t+1}}{1-\rho_{t+1}} - \rho_{t+1}\frac{(1-\rho_{t+1}^m)}{(1-\rho_{t+1})^2}\mu_{t+1}.$$

It is easily verified that the interpolations, $x_{t+1,i}$, do in fact exactly add up to the annual data, X_{t+1}.

The uniformly distributed interpolated series, $x_{t+1,i} = X_{t+1}/m$, for $i = 1, 2, ..., m$, correspond to the case where $\rho_{t+1} = 0$. We adopt the geometrically (exponentially) interpolated series which is obtained by setting $\mu_{t+1} = 0$, while other intermediate cases can also be entertained, but in the case of our applications they tend to generate very similar outcomes.

For the exponential interpolation, ρ_{t+1} is computed as the solution to

$$X_{t+1} = \rho_{t+1}\frac{(1-\rho_{t+1}^m)}{(1-\rho_{t+1})}x_{tm}, \tag{A.3}$$

where x_{tm} is the observation at the end of the previous year. This formulation is suitable when interpolating the level of the variables (indices) rather than the growth rates and is applicable to I(1) variables.

To solve for ρ_{t+1}, let $\lambda_{t+1,m} = X_{t+1}/x_{tm}$, and write (A.3) in the expanded form

$$\rho_{t+1}^m + \rho_{t+1}^{m-1} + ... + \rho_{t+1} = \lambda_{t+1,m}, \text{ for } t = 0, 1, ..., \tag{A.4}$$

with

$$\lambda_{1,m} = X_1/x_{0m} = m(X_1/X_0). \tag{A.5}$$

It follows that

$$x_{t+1,i} = x_{tm}\rho_{t+1}^i, \ t = 0, 1, ...; i = 1, 2, ...m. \tag{A.6}$$

To proceed it is required to solve the m^{th} order polynomial equation given by (A.4). For the purpose of our empirical application we are interested in interpolating quarterly observations from annual series, which implies solving the quartic equation (for $m = 4$)

$$\rho_{t+1}^4 + \rho_{t+1}^3 + ... + \rho_{t+1} - \lambda_{t+1,4} = 0. \tag{A.7}$$

To solve the quartic equation of the general form

$$A_4z^4 + A_3z^3 + A_2z^2 + A_1z + A_0 = 0$$

or

$$z^4 + a_3z^3 + a_2z^2 + a_1z + a_0 = 0 \tag{A.8}$$

with $a_i = A_i/A_4$, $i = 0, 1, 2, 3$, we substitute $z = x - a_3/4$ in (A.8) which yields

$$x^4 + px^2 + qx + r = 0, \tag{A.9}$$

where

$$
\begin{aligned}
p &= a_2 - \frac{3}{8}a_3^2,\ q = a_1 - \frac{1}{2}a_2 a_3 + \frac{1}{8}a_3^3 \\
r &= a_0 - \frac{1}{4}a_1 a_3 + \frac{1}{16}a_2 a_3^2 - \frac{3}{256}a_3^4.
\end{aligned}
$$

In order to solve equation (A.9) it needs to be made factorable, which leads to the solution of the following cubic equation

$$u^3 + b_2 u^2 + b_1 u + b_0 = 0, \tag{A.10}$$

where

$$b_2 = -p,\ b_1 = -4r,\ b_0 = 4pr - q^2.$$

The cubic equation (A.10) has only one real root if the disriminant D is greater than zero, where D is defined by

$$D = Q^3 + R^2$$

and

$$Q = \frac{3b_1 - b_2^2}{9},\ R = \frac{9b_1 b_2 - 27b_0 - 2b_2^3}{54}.$$

In this case, $D > 0$, the unique real root is given by

$$u_1 = (R + \sqrt{D})^{1/3} - \frac{Q}{(R + \sqrt{D})^{1/3}} - \frac{1}{3}.$$

Then, by using the above solution to the cubic polynomial, u_1, the following quadratic equations arise

$$
\begin{aligned}
x^2 + \sqrt{u_1 - p}x + \frac{1}{2}u_1 - \frac{q}{2\sqrt{u_1 - p}} &= 0 && \text{(A.11a)} \\
x^2 - \sqrt{u_1 - p}x + \frac{1}{2}u_1 + \frac{q}{2\sqrt{u_1 - p}} &= 0, && \text{(A.11b)}
\end{aligned}
$$

If x_r is a real solution of the pair of quadratics (A.11) then $x_r - a_3/4$ is a real solution to the quartic equation (A.8). Thus a real solution to (A.7) is given by

$$\rho_{t+1} = x_{r,t+1} - 1/4$$

However, multiple real solutions can arise from the solution of the quartic equation defined by (A.8).

Consider two real solutions of (A.8), a and b. Let $\{y_{i1}^a, y_{i2}^a, y_{i3}^a, y_{i4}^a, ...\}$ and $\{y_{i1}^b, y_{i2}^b, y_{i3}^b, y_{i4}^b, ...\}$ be the levels of the interpolated series based on the choice of the roots a and b, respectively. In this case, we define

$$
\begin{aligned}
\Delta_a &= \frac{\left|\ln(y_{i1}^a / y_{i1-1,4}^a)\right| + \left|\ln(y_{i21}^a / y_{i1}^a)\right| + \left|\ln(y_{i41}^a / y_{i3}^a)\right| + ...}{4} \\
\Delta_b &= \frac{\left|\ln(y_{i1}^b / y_{i1-1,4}^b)\right| + \left|\ln(y_{i21}^b / y_{i1}^b)\right| + \left|\ln(y_{i41}^b / y_{i3}^b)\right| + ...}{4}
\end{aligned}
$$

and choose a if $\Delta_a < \Delta_b$, b otherwise.

Appendix B: Additional Impulse Response Functions

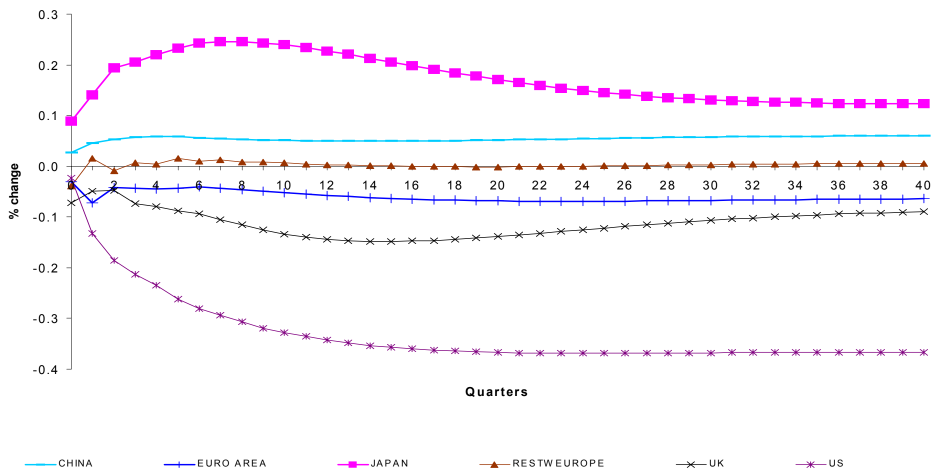


Figure B1: Generalized Impulse Responses of a Positive (+1 s.e.) Shock to Oil Prices on Real Output Across Regions

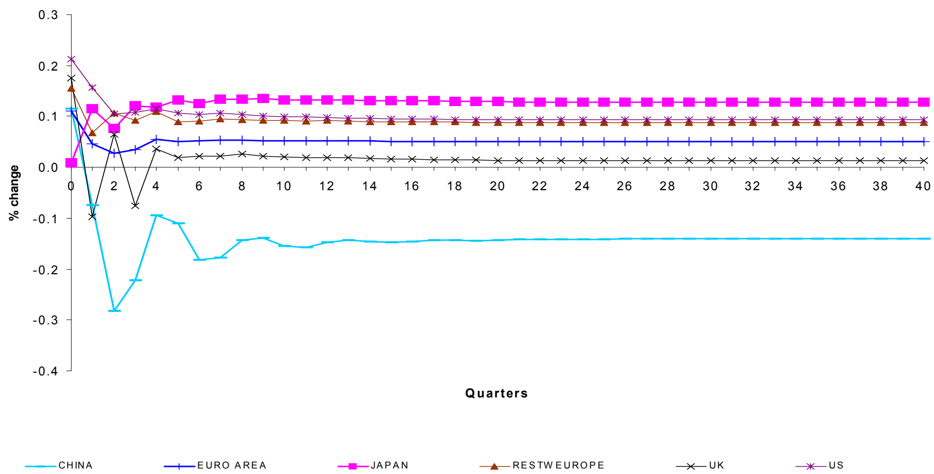


Figure B2: Generalized Impulse Responses of a Positive (+1 s.e.) Shock to Oil Prices on Inflation Across Regions

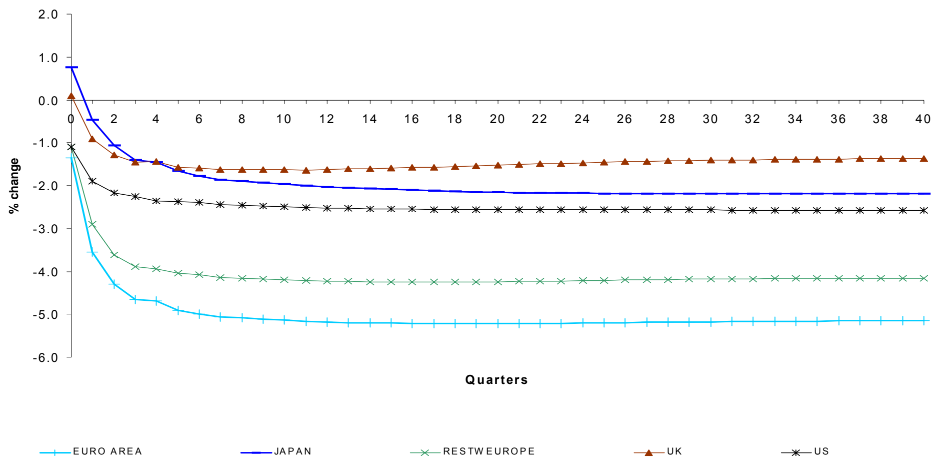


Figure B3: Generalized Impulse Responses of a Positive (+1 s.e.) Shock to Oil Prices on Real Equity Prices Across Regions

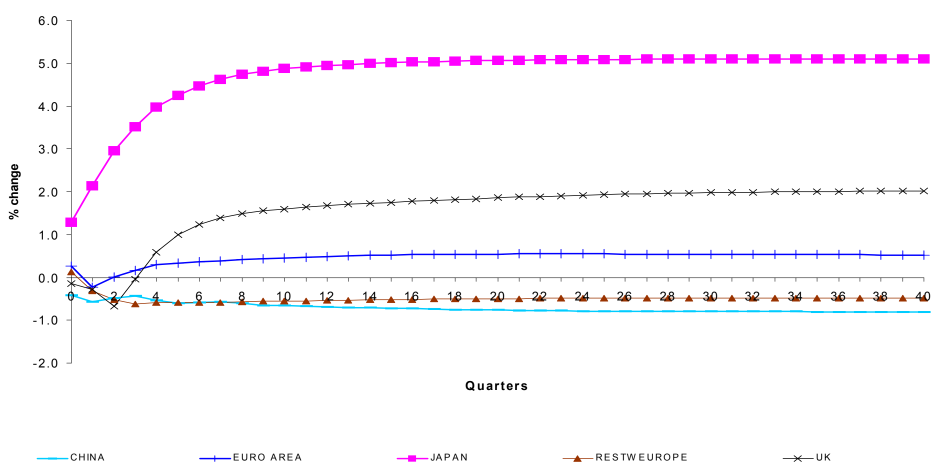


Figure B4: Generalized Impulse Responses of a Positive (+1 s.e.) Shock to Oil Prices on Real Exchange Rates Across Regions

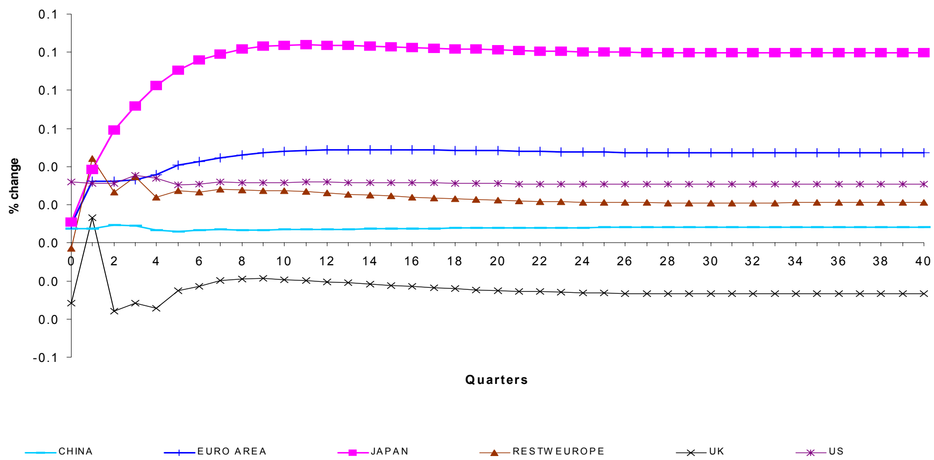


Figure B5: Generalized Impulse Responses of a Positive (+1 s.e.) Shock to Oil Prices on Short-Term Interest Rates Across Regions

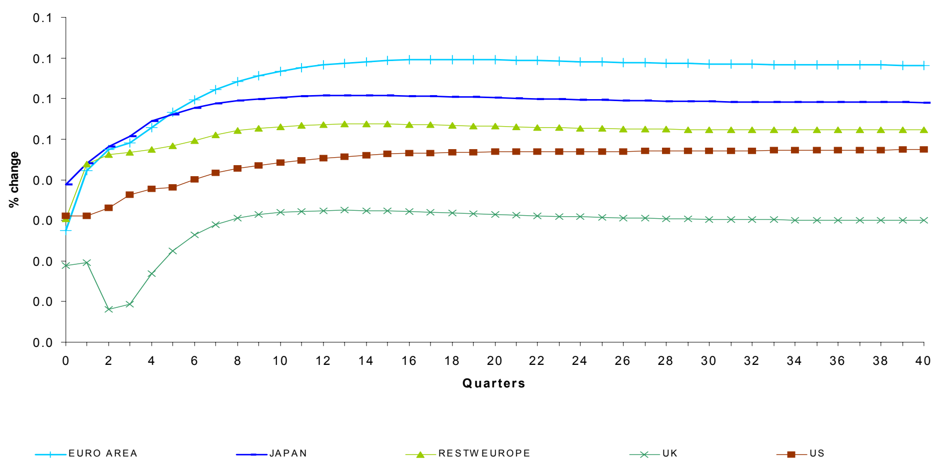


Figure B6: Generalized Impulse Responses of a Positive (+1 s.e.) Shock to Oil Prices on Long-Term Interest Rates Across Regions

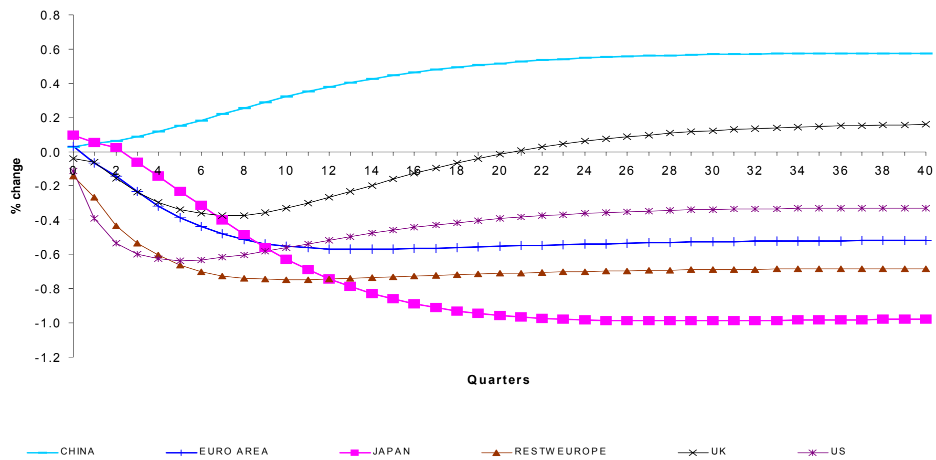


Figure B7: Generalized Impulse Responses of a Negative (-1 s.e.) Shock to US Real Equity Prices on Real Output Across Regions

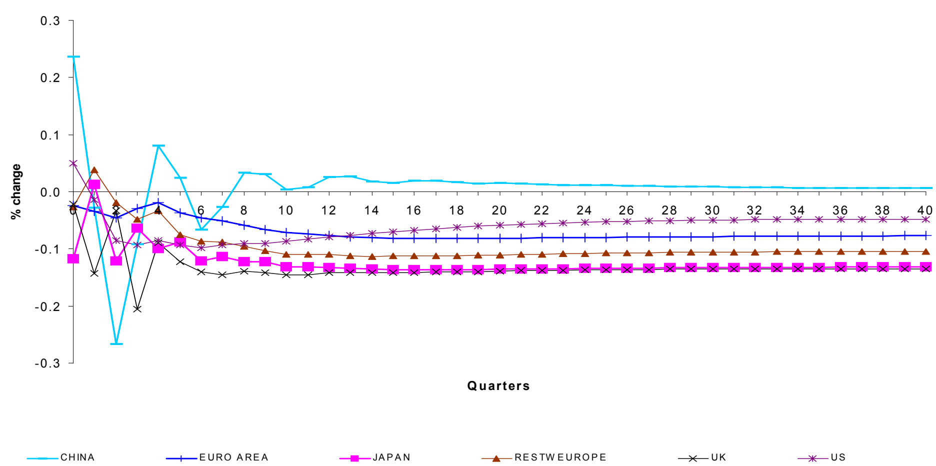


Figure B8: Generalized Impulse Responses of a Negative (-1 s.e.) Shock to US Real Equity Prices on Inflation Across Regions

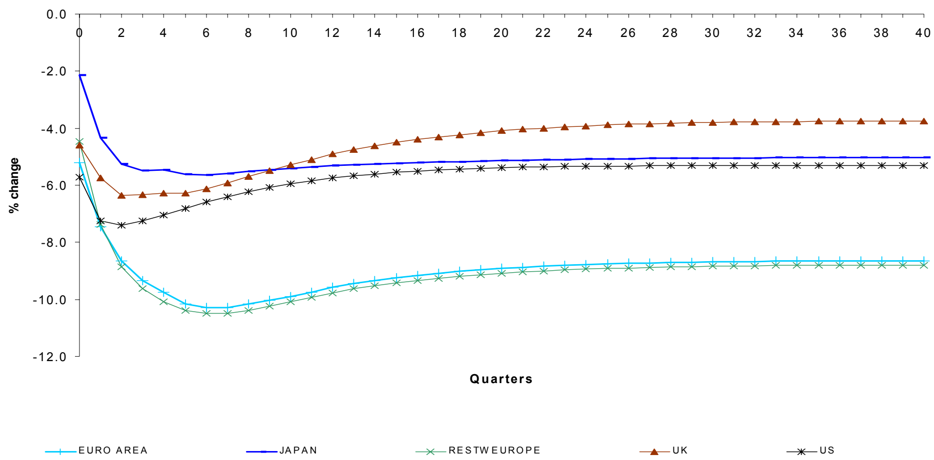


Figure B9: Generalized Impulse Responses of a Negative (-1 s.e.) Shock to US Real Equity Prices on Real Equity Prices Across Regions

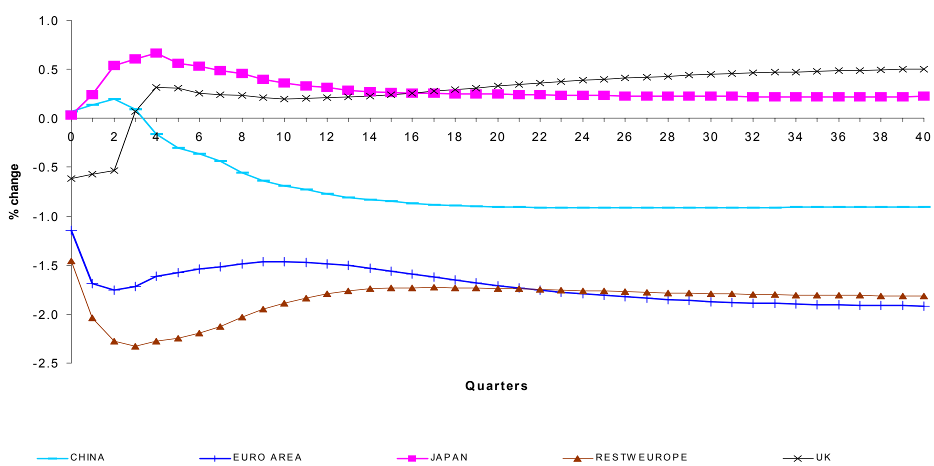


Figure B10: Generalized Impulse Responses of a Negative (-1 s.e.) Shock to US Real Equity Prices on Real Exchange Rates Across Regions

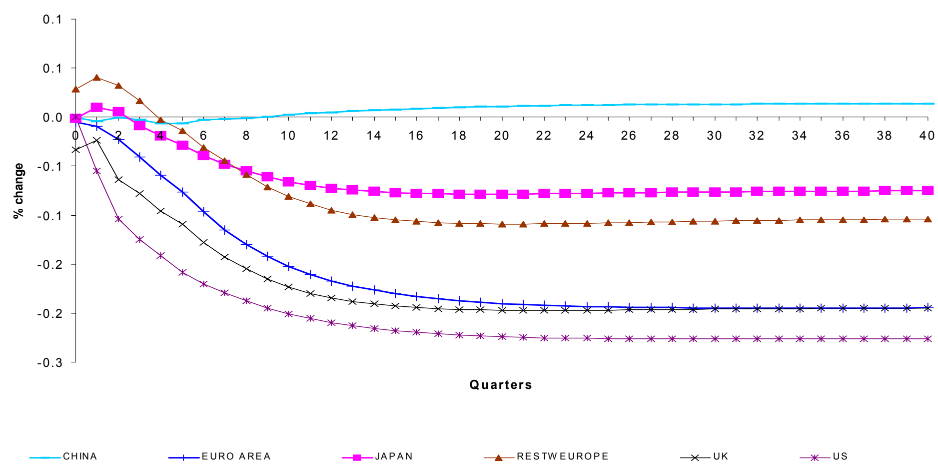


Figure B11: Generalized Impulse Responses of a Negative (-1 s.e.) Shock to US Real Equity Prices on Short-Term Interest Rates Across Regions

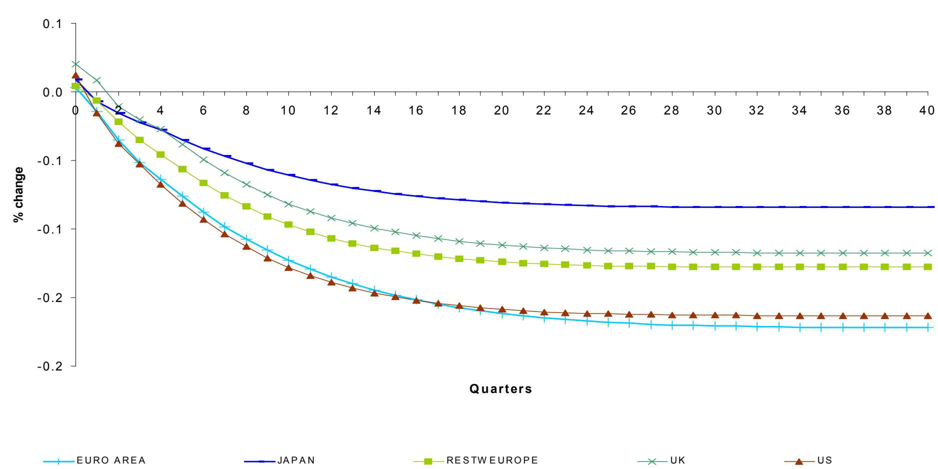


Figure B12: Generalized Impulse Responses of a Negative (-1 s.e.) Shock to US Real Equity Prices on Long-Term Interest Rates Across Regions

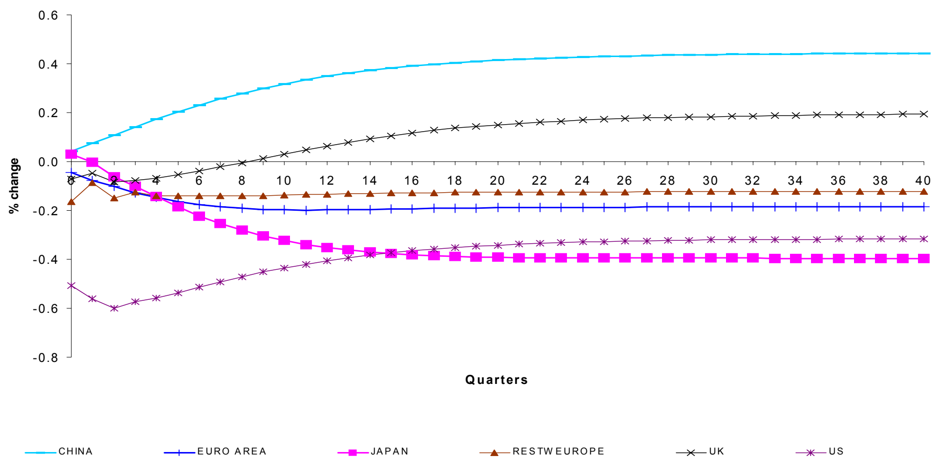


Figure B13: Generalized Impulse Responses of a Negative (-1 s.e.) Shock to US Real Output on Real Output Across Regions

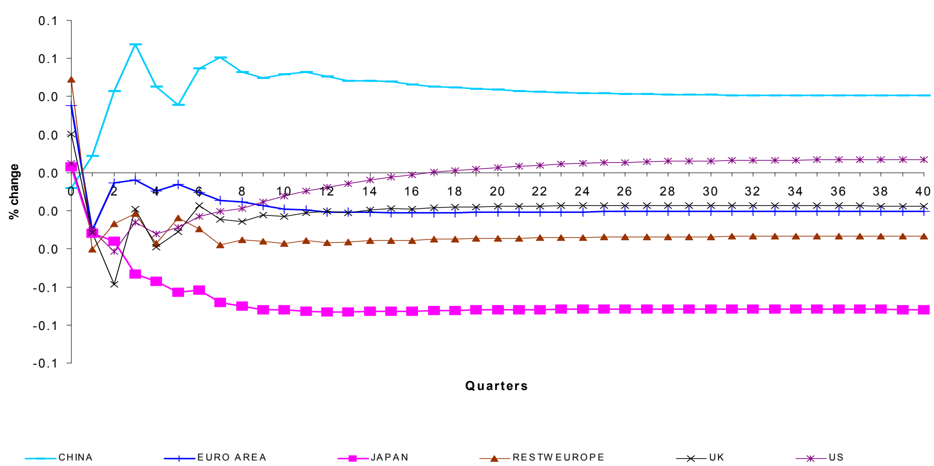


Figure B14: Generalized Impulse Responses of a Negative (-1 s.e.) Shock to US Real Output on Inflation Across Regions

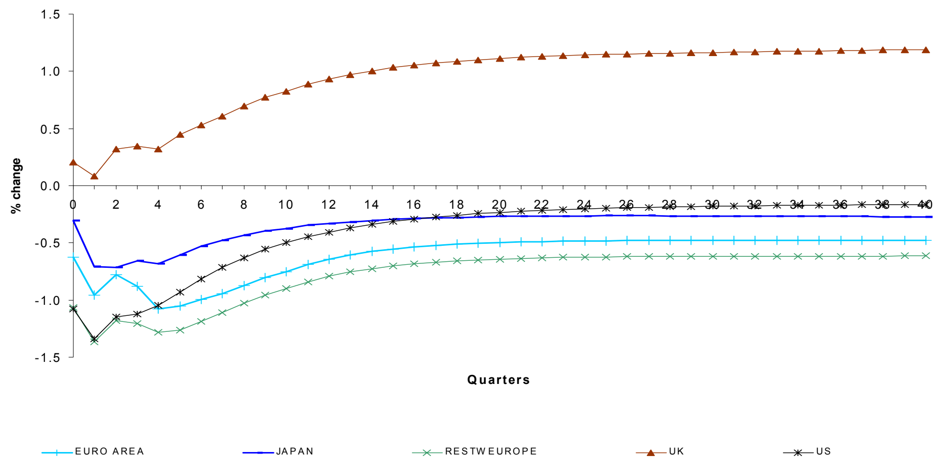


Figure B15: Generalized Impulse Responses of a Negative (-1 s.e.) Shock to US Real Output on Real Equity Prices Across Regions

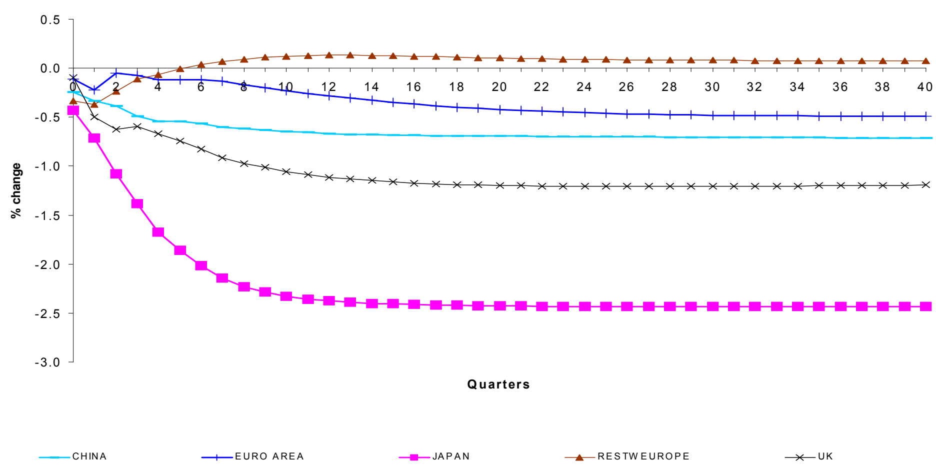


Figure B16: Generalized Impulse Responses of a Negative (-1 s.e.) Shock to US Real Output on Real Exchange Rates Across Regions

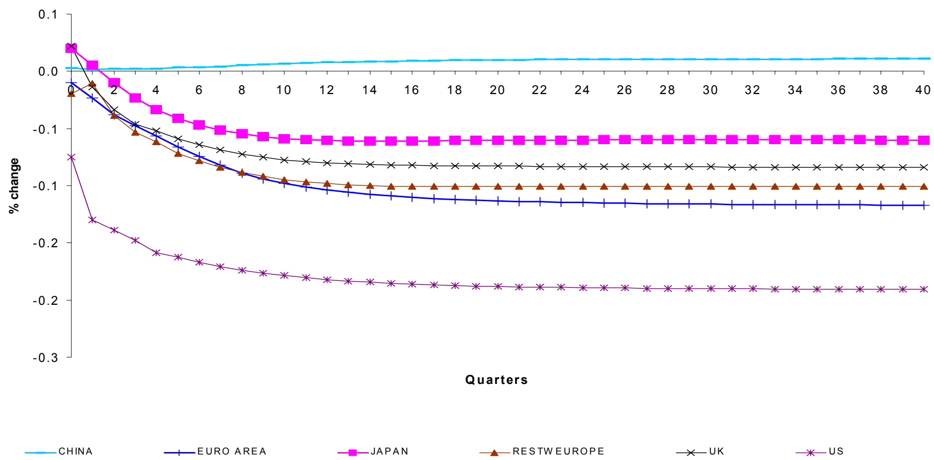


Figure B17: Generalized Impulse Responses of a Negative (-1 s.e.) Shock to US Real Output on Short-Term Interest Rates Across Regions

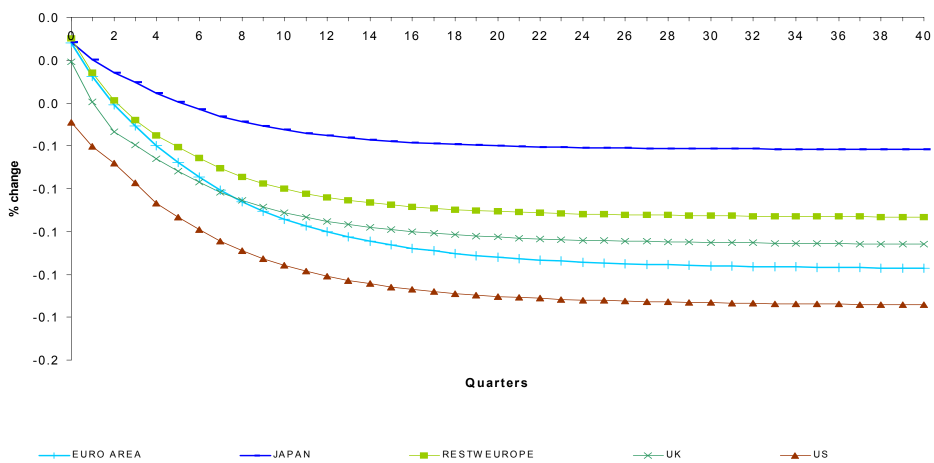


Figure B18: Generalized Impulse Responses of a Negative (-1 s.e.) Shock to US Real Output on Long-Term Interest Rates Across Regions

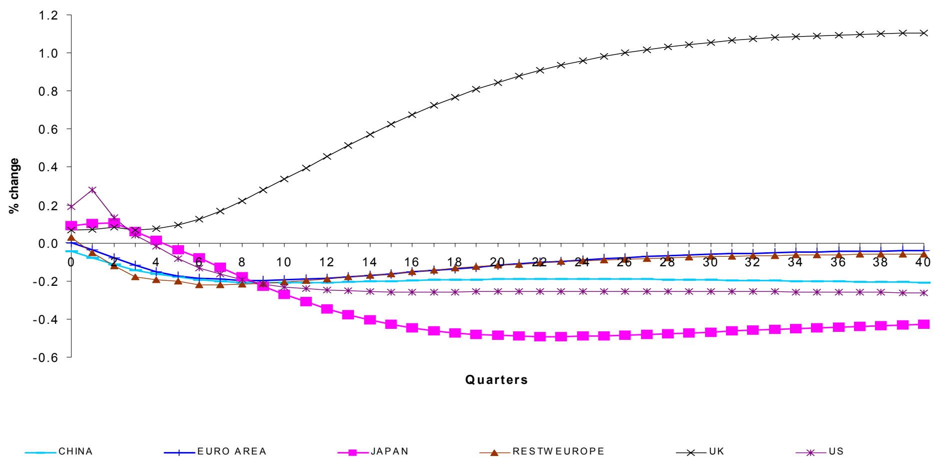


Figure B19: Generalized Impulse Responses of a Positive (+1 s.e.) Shock to US Short-Term Interest Rate on Real Output Across Regions

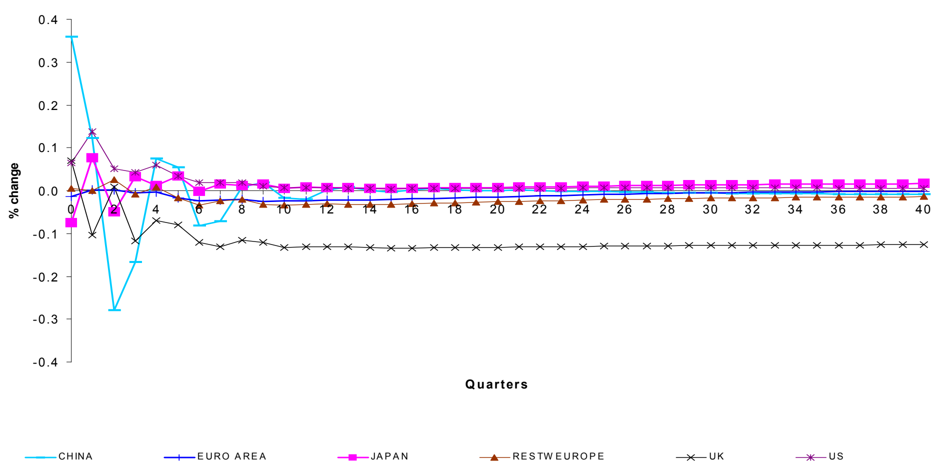


Figure B20: Generalized Impulse Responses of a Positive (+1 s.e.) Shock to US Short-Term Interest Rate on Inflation Across Regions

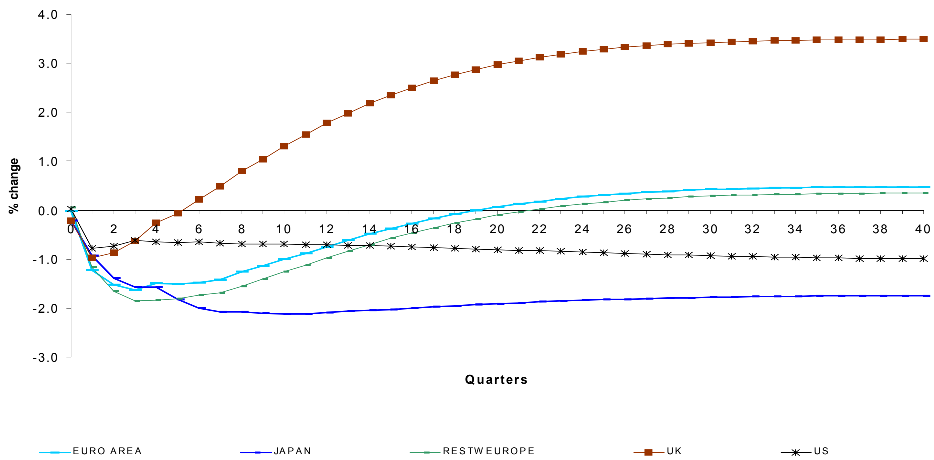


Figure B21: Generalized Impulse Responses of a Positive (+1 s.e.) Shock to US Short-Term Interest Rate on Real Equity Prices Across Regions

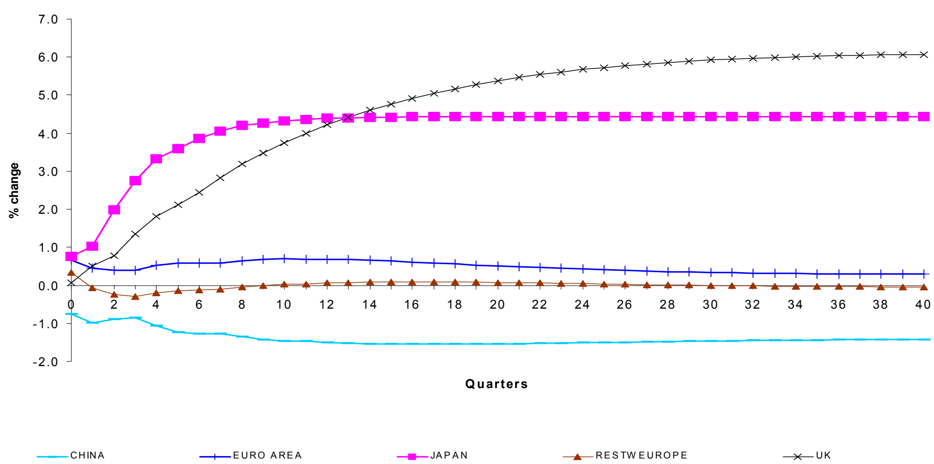


Figure B22: Generalized Impulse Responses of a Positive (+1 s.e.) Shock to US Short-Term Interest Rate on Real Exchange Rates Across Regions

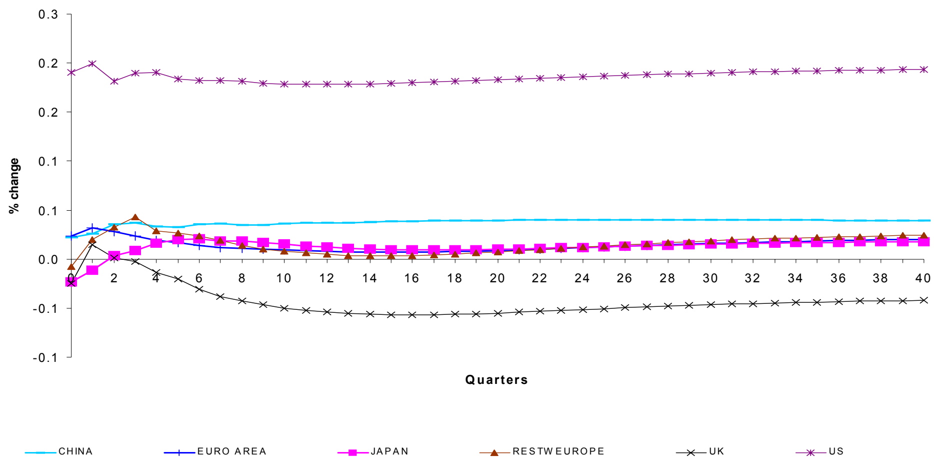


Figure B23: Generalized Impulse Responses of a Positive (+1 s.e.) Shock to US Short-Term Interest Rate on Short-Term Interest Rates Across Regions

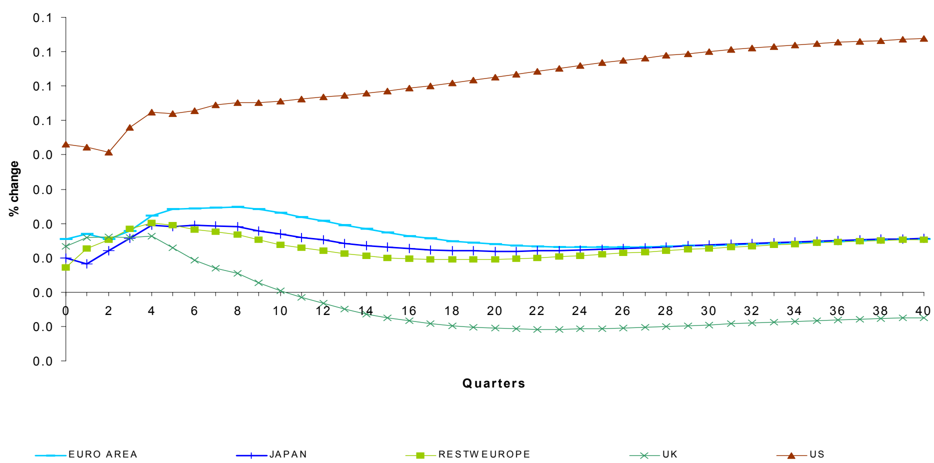


Figure B24: Generalized Impulse Responses of a Positive (+1 s.e.) Shock to US Short-Term Interest Rate on Long-Term Interest Rates Across Regions