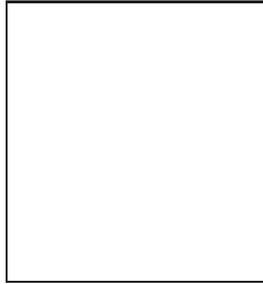


New possibilities in the study of NLL BFKL

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The high energy limit of scattering processes in QCD is, at least on the purely theoretical level, described by the BFKL equation. However, many phenomenological studies of BFKL fail miserably when confronted with data. In this talk we will briefly review the application of (LL) BFKL in phenomenology, and critically examine the application of LL eigenfunctions in the study of the NLL BFKL kernel. We then introduce a recently proposed iterative solution of the NLL BFKL equation that allows for a detailed study of physical properties of the BFKL evolution.

1 Introduction

The Balitsky–Fadin–Kuraev–Lipatov¹ (BFKL) framework systematically resums a class of logarithms dominant in the Regge limit of scattering amplitudes, where the centre of mass energy $\sqrt{\hat{s}}$ is large and the momentum transfer $\sqrt{-\hat{t}}$ is fixed. In this presentation we will focus on the use of BFKL in so-called *forward scattering*, which is applicable to the description of multi-jet production at large rapidities from a hard scattering (see e.g. Ref.² for an introduction), and we will often take gluon–gluon scattering as an example of a physical process. However, the BFKL exchange is applicable to the high energy limit of many processes, and can serve as background to many channels for new physics (e.g. W +jets). The BFKL exchange is also interesting on its own right, since it probes QCD in a region not described well by the standard fixed-order perturbative calculation.

When the gluon scattering results in jets spanning a large rapidity interval, one finds that at each order in the perturbative calculation, the matrix element is dominated by processes with a t -channel gluon exchange. Furthermore, the higher order corrections to the leading order $2 \rightarrow 2$ process have terms that grow logarithmically with the rapidity span. These terms come from

both real and virtual corrections arising from $2 \rightarrow 2 + n$ gluon processes, and dominate the full matrix element in the kinematical region where the transverse momenta of the gluons are similar ($k_a \approx k_b \approx k_i$) and the invariant mass of each gluon pair is large. These are the contributions resummed through the BFKL equation.

In the high energy limit of $\hat{s} \gg |\hat{t}|$, the partonic cross section factorises to the required logarithmic accuracy due to the dominance of the diagrams featuring a t -channel gluon exchange. The partonic cross section can be approximated by

$$\hat{\sigma}(\Delta) = \int \frac{d^2\mathbf{k}_a}{2\pi\mathbf{k}_a^2} \int \frac{d^2\mathbf{k}_b}{2\pi\mathbf{k}_b^2} \Phi_A(\mathbf{k}_a) f(\mathbf{k}_a, \mathbf{k}_b, \Delta) \Phi_B(\mathbf{k}_b), \quad (1)$$

where $\Phi_{A,B}$ are the impact factors characteristic of the particular scattering process, and $f(\mathbf{k}_a, \mathbf{k}_b, \Delta)$ is the gluon Green's function describing the interaction between two Reggeised gluons exchanged in the t -channel with transverse momenta $\mathbf{k}_{a,b}$ spanning a rapidity interval of length Δ . At leading order in α_s the gluon Green's function is a delta-functional, keeping the dijets back to back. The evolution of the Green's function by the (next-to^{3,4}) leading logarithmic corrections is governed by the BFKL equation, which is written using the Mellin transform (in Δ) of the gluon Green's function

$$\omega f_\omega(\mathbf{k}_a, \mathbf{k}_b) = \delta^{(2+2\epsilon)}(\mathbf{k}_a - \mathbf{k}_b) + \int d^{2+2\epsilon}\mathbf{k} \mathcal{K}(\mathbf{k}_a, \mathbf{k} + \mathbf{k}_a) f_\omega(\mathbf{k} + \mathbf{k}_a, \mathbf{k}_b), \quad (2)$$

with the kernel $\mathcal{K}(\mathbf{k}_a, \mathbf{k}) = 2\omega^{(\epsilon)}(\mathbf{k}_a) \delta^{(2+2\epsilon)}(\mathbf{k}_a - \mathbf{k}) + \mathcal{K}_r(\mathbf{k}_a, \mathbf{k})$ consisting of the gluon Regge trajectory, which includes the virtual contributions, and a real emission component. The delta functional in the driving term of the integral equation corresponds to the case of no emission from the Reggeised gluon exchange. Alternatively, the BFKL equation could have been written as a differential equation in Δ with the delta functional as the boundary condition at $\Delta = 0$, and the kernel describing the evolution in Δ .

2 Solutions of the BFKL equation

2.1 Leading Logarithmic Accuracy

At leading logarithmic accuracy the BFKL kernel is conformal invariant, since the running of the coupling only enters at higher logarithmic orders. The eigenfunctions of the angular averaged kernel are of the form $k^{2(\gamma-1)}$, which means that to this accuracy, the BFKL evolution can be solved analytically, with the transverse momentum of emitted gluons integrated to infinity, by analysing the Mellin transform of the kernel. One finds

$$\omega^{\text{LL}}(\gamma) \equiv \int d^{D-2}\mathbf{k} \mathcal{K}^{\text{LL}}(\mathbf{k}_a, \mathbf{k}) \left(\frac{\mathbf{k}^2}{\mathbf{k}_a^2} \right)^{\gamma-1} = \frac{\alpha_s(\mathbf{k}_a^2)N}{\pi} \chi^{\text{LL}}(\gamma), \quad (3)$$

with N being the number of colours and

$$\chi^{\text{LL}}(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1 - \gamma), \quad \psi(\gamma) = \Gamma'(\gamma)/\Gamma(\gamma). \quad (4)$$

At LL there coupling is formally fixed, and so the regularisation scale is completely arbitrary, but of course has to be physically motivated. Since both the eigenfunctions and eigenvalues are known, the angular averaged (over the angle between \mathbf{k}_a and \mathbf{k}_b) gluon Green's function can now be obtained as

$$\bar{f}(k_a, k_b, \Delta) = \frac{1}{\pi k_a k_b} \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} e^{\Delta\omega^{\text{LL}}(\gamma)} \left(\frac{k_b^2}{k_a^2} \right)^{\gamma-\frac{1}{2}}. \quad (5)$$

In this way, the BFKL evolution is known in terms of \mathbf{k}_a , \mathbf{k}_b and Δ only — there is no handle on the momentum of the gluons emitted from the BFKL evolution, since the phase space of these have been fully integrated over. In particular this means that the total energy of an event from the BFKL evolution can no longer be calculated, which means that when the partonic cross section has to be convoluted with the parton density functions to calculate a physical process, the Bjorken x 's will be underestimated, leading to an overestimate of the parton fluxes and BFKL cross sections. The contribution to the centre of mass energy from the gluons emitted from the BFKL evolution is indeed subleading compared to the leading scattered gluons. However, it was recently demonstrated⁵ that an estimate of the centre of mass energy based on the leading dijets alone on average underestimates the full partonic centre of mass energy of a BFKL event by roughly a factor 2.5. Whereas the asymptotic behaviour is unchanged, this will clearly have an effect for all BFKL phenomenology, and will indeed change the BFKL signatures by restricting the evolution^{6,7,8,9,10,11}. Once this is taken into account, the LL BFKL predictions are brought into much better agreement with data.

2.2 Next-to-Leading Logarithmic Accuracy

If one naïvely applies the analysis leading to Eq. (5) to the kernel at next-to-leading logarithmic accuracy one is immediately faced by a seemingly insurmountable problem, which would invalidate the whole approach: The “eigenvalue” $\omega(\gamma)$ has an imaginary part, which would result in oscillations with the rapidity. The result would become unphysical in the very limit it is supposed to describe well. However, it should be remembered that the conformal symmetry exhibited at LL accuracy is broken by the NLL corrections. Specifically this means that the NLL kernel is not diagonalised by the LL eigenfunctions, and therefore the solution to the NLL BFKL equation cannot be written on the form of Eq. (5). The use of a Fourier transform to solve linear differential equations can be used as an analogy to the use of a Mellin transform to solve the LL BFKL equation. The Fourier transform can only be applied straightforwardly as long as the exponential function is an eigenfunction of the differential operator. The same applies to the use of the Mellin transform in transverse momentum for the solution of the integral equation (2). This can only be applied straightforwardly if the eigenfunctions are of the LL form. In fact, it was noted already in Ref.³ that if instead of using the LL eigenfunctions one analyses the action of the NLL kernel on the set of LL eigenfunctions rescaled by the square root of the running coupling, the “eigenvalue” changes in a desirable way. It turns out that indeed, the term giving rise to the oscillations vanishes in this case! However, since this new set of functions still does not diagonalise the NLL BFKL kernel, this analysis still does not solve the NLL BFKL equation. Several analyses have recently dealt with the problem of the running of the coupling^{12,13,14} combined with resummation of additional terms^{15,16,17,18,19} within frameworks making use of the Mellin transform.

It should, however, be clear that it would be desirable to have an alternative approach to the solution of the BFKL equation at NLL that would treat the running coupling terms on equal footing with the scale-invariant NLL corrections at all intermediate steps. Such solution has recently been published^{20,21}. It generalises the iterative solution^{6,22} of the LL BFKL equation to NLL accuracy. The iterative solutions to the LL BFKL equation provide the tool to examine the radiation from the BFKL equation used in the study of the full energy and final state configurations of the BFKL evolution mentioned in Sec. 2.1. We hope to extend these studies to NLL and thereby start a program of NLL BFKL collider phenomenology, although much work remains to be done. The first step was recently taken in the study of the BFKL equation at NLL for $N=4$ Super Yang Mills^{23,24}. $N=4$ SYM respects conformal symmetry also at NLL, and so an analytic analysis along the lines of Sec. 2.1 solves the NLL BFKL equation exactly for this theory. It has been shown²⁵ that the recently proposed iterative solution to the NLL

BFKL equation indeed solves the BFKL equation exactly, including all information on higher conformal spins, which is necessary to reconstruct the correct angular dependence. So far, this has only been calculated analytically for $N = 4$ SYM and not for QCD, but in the iterative approach to the solution of the NLL BFKL equation this information is obtained for free.

The details of the iterative solution can be found in Ref.^{20,21,25}. Here we will just mention that the solution $f(\mathbf{k}_a, \mathbf{k}_b, \Delta)$ to the NLL BFKL equation is obtained as an explicit phase space integral, with regularised effective vertices connected with factors describing no-emission probabilities, as illustrated below for the case of multi-jet production in gluon-gluon scattering.

$$\begin{aligned}
f(\mathbf{k}_a, \mathbf{k}_b, \Delta) &= \exp(\omega_0(\mathbf{k}_a^2, \lambda^2, \mu) \Delta) \delta^{(2)}(\mathbf{k}_a - \mathbf{k}_b) \\
&+ \sum_{n=1}^{\infty} \prod_{i=1}^n \int d^2\mathbf{k}_i \int_0^{y_{i-1}} dy_i \left[V\left(\mathbf{k}_i, \mathbf{k}_a + \sum_{l=0}^{i-1} \mathbf{k}_l, \mu\right) \right] \\
&\times \exp\left[\omega_0\left(\left(\mathbf{k}_a + \sum_{l=1}^{i-1} \mathbf{k}_l\right)^2, \lambda^2, \mu\right) (y_{i-1} - y_i)\right] \\
&\times \exp\left[\omega_0\left(\left(\mathbf{k}_a + \sum_{l=1}^n \mathbf{k}_l\right)^2, \lambda^2, \mu\right) (y_n - 0)\right] \\
&\times \delta^{(2)}\left(\sum_{l=1}^n \mathbf{k}_l + \mathbf{k}_a - \mathbf{k}_b\right)
\end{aligned}$$

3 Conclusions

In this talk we have presented a solution to the NLL BFKL equation which promises well for the possibility of extending the detailed LL phenomenological studies of BFKL multi-jet events at colliders to next-to-leading logarithmic accuracy. Furthermore, the iterative solution will help in gaining a thorough understanding of the NLL corrections in QCD, and help separating true NLL effects from artifacts of the tools applied in the analyses of these. We are currently undertaking several studies within this framework, including an extension to the non-forward BFKL equation.

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