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## JOINT RESUMMATION FOR HEAVY QUARK PRODUCTION

ANDREA BANFI

*Cavendish Laboratory, University of Cambridge  
 Madingley Road, CB3 0HE Cambridge, UK*

ERIC LAENEN\*

*NIKHEF Theory Group  
 Kruislaan 409, 1098 SJ Amsterdam, The Netherlands*

We present the joint threshold and recoil resummed transverse momentum distributions for heavy quark hadroproduction, at next-to-leading logarithmic accuracy. We exhibit their dependence on the production channel and the color configurations, and compare these distributions to each other and to NLO.

*Keywords:* Resummation; heavy quark production.

### 1. Joint threshold and recoil resummation

The formalism<sup>1,2,3</sup> of hadronic cross sections for the joint resummation of distributions singular at partonic threshold and at zero recoil has so far been applied to  $Z/W$  production<sup>4</sup>, Higgs production<sup>5</sup>, and prompt photon hadroproduction<sup>6</sup>. For the latter  $2 \rightarrow 2$  process, the formalism implements the notion that, in the presence of QCD radiation, the actual transverse momentum produced by the hard collision is not  $\vec{p}_T$  but rather  $\vec{p}_T - \vec{Q}_T/2$ , with  $\vec{Q}_T$  the total transverse momentum of unobserved soft recoiling partons. The joint-resummed partonic  $p_T$  spectrum has the form of a hard scattering cross section as a function of  $p'_T \equiv |\vec{p}_T - \vec{Q}_T/2|$ , convoluted with a *perturbative*, albeit resummed  $\vec{Q}_T$  distribution. We have extended<sup>7</sup> the joint resummation formalism to the  $p_T$  distribution of heavy quarks produced in hadronic collisions. Key differences with the prompt-photon case are, first, the presence of the heavy quark mass  $m$ , preventing a singularity in the hard scattering function when  $Q_T = 2p_T$  and, second, the possibility of multiple colored states for the produced heavy quark pair.

### 2. Resummed heavy quark transverse momentum spectra

We consider the inclusive  $p_T$  distribution of a heavy quark produced via the strong interaction in a hadron-hadron collision at center of mass (cm) energy  $\sqrt{S}$ . Exact

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higher order corrections to the differential cross sections for these partonic processes have been computed to NLO <sup>8,9,10,11</sup>. Up to corrections  $\mathcal{O}(1/p_T^2)$ , the observable may at any order <sup>12</sup> be written in the following factorized form

$$\frac{d\sigma_{AB \rightarrow Q+X}}{dp_T} = \sum_{a,b} \int_0^1 d\xi_a d\xi_b \phi_{a/A}(\xi_a, \mu) \phi_{b/B}(\xi_b, \mu) \frac{d\hat{\sigma}_{ab \rightarrow Q+X}}{dp_T}(\xi_a, \xi_b, \alpha_s(\mu), p_T), \quad (1)$$

with  $d\hat{\sigma}_{ab \rightarrow Q+X}/dp_T$  the partonic differential cross-section,  $\phi_{a/A}$  and  $\phi_{b/B}$  parton densities, and  $\mu$  the factorization/renormalization scale.

Threshold enhancements essentially involve the energy of soft gluons. In the context of the factorization (1) we define hadronic and partonic threshold by the conditions  $S = 4m_T^2$  and  $\hat{s} = 4m_T^2$ , respectively, with  $m_T$  the transverse mass  $\sqrt{m^2 + p_T^2}$ . It is convenient to define the scaling variables

$$x_T^2 = \frac{4m_T^2}{S}, \quad \hat{x}_T^2 = \frac{4m_T^2}{\xi_a \xi_b S}, \quad (2)$$

so that hadronic (partonic) threshold is at  $x_T^2 = 1$  ( $\hat{x}_T^2 = 1$ ). The higher order corrections to the partonic cross section  $d\hat{\sigma}_{ab}/dp_T$  contain distributions that are singular at partonic threshold. Threshold resummation organizes such distributions to all orders.

There are also recoil effects, resulting from radiation of soft gluons from initial-state partons. We wish to treat these effects in the context of joint threshold and recoil resummation. We identify a hard scattering with reduced cm energy squared  $Q^2$  and at transverse momentum  $\vec{Q}_T$  with respect to the hadronic cm system. This hard scattering produces a heavy quark with transverse momentum

$$\vec{p}'_T \equiv \vec{p}_T - \frac{\vec{Q}_T}{2}. \quad (3)$$

The kinematically allowed range for the invariant mass  $Q$  of the heavy quark pair in this hard scattering is limited from below by  $2m'_T = 2\sqrt{m^2 + p_T'^2}$  so that threshold in the context of joint resummation is defined by

$$\tilde{x}_T^2 \equiv \frac{4m_T'^2}{Q^2} = 1. \quad (4)$$

A refactorization analysis <sup>2</sup> leads to the following expression for the observable in Eq. (1)

$$\frac{d\sigma_{AB \rightarrow Q+X}}{dp_T} = \int d^2Q_T \theta(\bar{\mu} - |\vec{Q}_T|) \frac{d\sigma_{AB \rightarrow Q+X}}{dp_T d^2\vec{Q}_T}, \quad (5)$$

where  $\bar{\mu}$  is a cut-off and

$$\begin{aligned} \frac{d\sigma_{AB\rightarrow Q+X}}{dp_T d^2\vec{Q}_T} &= \sum_{ab=q\bar{q}, gg} p_T \int \frac{d^2b}{(2\pi)^2} e^{i\vec{b}\cdot\vec{Q}_T} \int \frac{dN}{2\pi i} \phi_{a/A}(N, \mu) \phi_{b/B}(N, \mu) e^{E_{ab}(N, b)} \\ &= \frac{e^{-2 C_F t(N) (\text{Re}L_\beta + 1)}}{4\pi S^2} \left( \tilde{M}_{\mathbf{1}}^2(N) + \tilde{M}_{\mathbf{8}}^2(N) e^{C_A t(N) \left( \ln \frac{m_T^2}{m^2} + L_\beta \right)} \right) \\ &\quad \times \left( \frac{S}{4(m^2 + |\vec{p}_T - \vec{Q}_T/2|^2)} \right)^{N+1}. \end{aligned} \quad (6)$$

Notice in particular the last factor, which provides a kinematic link between recoil and threshold effects. The exponential functions  $E_{ab}$ <sup>2,6</sup> are to next-to-leading logarithmic (NLL) accuracy

$$E_{ab}(N, b) = \int_{\chi(N, b)}^Q \frac{d\mu'}{\mu'} [A_a(\alpha_s(\mu')) + A_b(\alpha_s(\mu'))] 2 \ln \frac{\bar{N}\mu'}{Q} - gb^2, \quad \bar{N} = N e^{\gamma_E}, \quad (7)$$

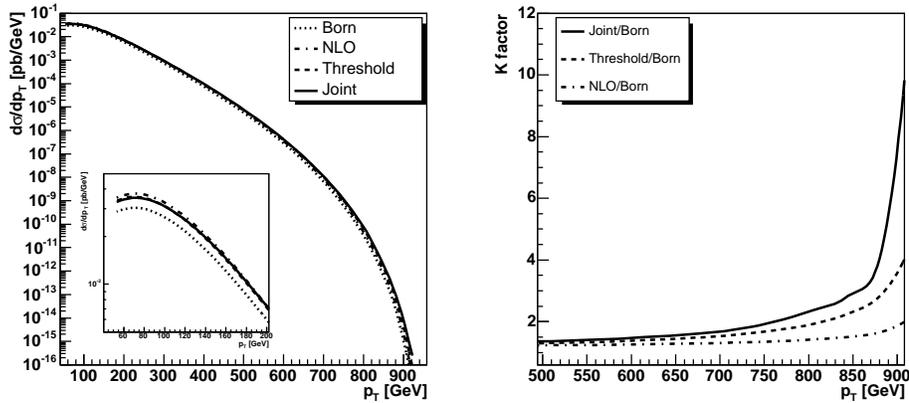
where the coefficients  $A_a$  and  $A_b$  can be found elsewhere<sup>2</sup>, and the function  $\chi(N, b)$  is chosen to reproduce either NLL resummed recoil or threshold distributions in the appropriate limits<sup>4</sup>. We also added to the perturbative exponent the non-perturbative (NP) Gaussian smearing term  $-gb^2$ , in terms of the impact parameter  $b$ . We have introduced the variables

$$t(N) = \int_Q^{Q/N} \frac{d\mu'}{\mu'} \frac{\alpha_s(\mu')}{\pi}, \quad \text{Re}L_\beta = \frac{1 + \beta^2}{2\beta} \left( \ln \frac{1 - \beta}{1 + \beta} \right), \quad \beta = \sqrt{1 - m^2/m_T^2}. \quad (8)$$

The functions  $\tilde{M}_{\mathbf{1}}^2(N)$ ,  $\tilde{M}_{\mathbf{8}}^2(N)$  are the Mellin moments of the lowest order heavy quark production matrix elements for either the  $q\bar{q}$  or  $gg$  channel, as appropriate, the index labeling the color-state of the heavy quark pair. Their explicit expressions can be found elsewhere<sup>7</sup>. The threshold-resummed result can now easily be derived, by substituting Eq. (6) into (5) and neglecting  $\vec{Q}_T$  in the last factor in Eq. (6). Then the  $\vec{Q}_T$  integral sets  $\vec{b}$  to zero everywhere, yielding the threshold-resummed result.

To illustrate these analytic results, we show for the case of top quark production at the Tevatron the  $p_T$  distribution for the dominant  $q\bar{q}$  channel in Fig. 1. We observe that, while the resummed and NLO curves are close for small and moderate  $p_T$  (the inset provides a somewhat better view of the low  $p_T$  region), for large  $p_T$  values the resummed curves depart significantly from the NLO curve. Of course, cross sections for top quark production at such large  $p_T$  at the Tevatron are far too small to be measured, so that our plots at large  $p_T$  have only theoretical interest. For such large  $p_T$  values, the hadronic threshold, defined in Eq. (2), approaches the partonic one, where larger  $N$  values dominate, a prerequisite for seeing significant effects for both resummations. The enhancements relative to the Born cross section are shown in the form of a K-factor. Threshold resummation produces an overall enhancement of the cross section that increases with increasing  $p_T$ , yielding e.g. an enhancement

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Fig. 1. Top quark  $p_T$  spectra and  $K$ -factors for the  $q\bar{q}$  channel.

over NLO at  $p_T = 800$  GeV. Joint resummation almost doubles that effect: the joint-resummed enhancement at large  $p_T$  effectively constitutes a smearing of the threshold-resummed  $p_T$  spectrum by a resummed recoil function.

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