Robust Incentives

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Abstract

In this paper we consider a moral hazard problem, in which the agent after receiving his wage contract but before undertaking the costly effort can borrow on his future wage earnings. The game between the agent and potential lenders is modelled as an infinite stochastic game with an exogenous stopping probability. We show that the principal cannot design a wage scheme that is robust to hedging by the agent. In particular, we show that, if the exogenous stopping probability is non zero, the principal's wage offer will be followed by several rounds of borrowing by the agent. This is compared to the recontracting-proofness equilibria which most of the literature has concentrated on, assuming that this stopping probability is zero. Furthermore, we show that the equilibrium of the model with a strictly positive stopping probability does not converge to the equilibrium of the model in which it is zero. We also find that the principal's profit is lower, the maximum wage payment can be higher and effort is lower.

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1 Introduction

Robust incentives are incentives that are immune to the possibility that the incentivized party engages in further contractual relationships to offset or partly undo those incentives.

Consider the case of a firm's manager. Traditional principal-agent theory predicts that a manager's wealth should be tied to firm performance to provide incentives to the manager to maximize shareholder value. Thus, his pay is indexed on firm profits or he is rewarded via shares or stock options. However, if profits are risky and the manager is risk-averse he will prefer to hedge some of the risk inherent in such a remuneration scheme. Selling some of his stock holdings, for instance, will reduce his exposure to stock price movements. Since such hedging activities break the link between firm performance and the manager's wealth, the use of stock awards or other incentive schemes to incentivize managers becomes questionable.

In this paper we analyze the optimal design of incentive schemes when recontracting opportunities are present and ask whether robust incentives, incentives that will not be 'contracted away', exist.

Recontracting is a real life phenomenon. Many intermediaries offer financial products for corporate insiders to hedge their large exposure to their firm's stock. Among such instruments are zero-cost collars, equity swaps and loans against stock holdings. Certainly, hedging or retrading incentive schemes is limited by, for instance, the use of vesting periods for stock options and there are legal limits to the possibility for executives to hedge their risk.[1] Still, Bettis, Bizjak, and Lemmon (2001) report that high ranking insiders, such as CEO's and members of the board of directors cover an average of 36% of their share holdings with costless collars. Ofek and Yermack (2000) show that managers with high ownership shares in their company tend to sell their existing stock holdings after option or stock awards. Similarly, borrowers can often enter into multilateral contracts which impact on their "global" incentives. For instance, no legal mechanism can completely eliminate the possibility that a debtor country contracts further loans. A current U.S. website (www.cardweb.com) reports that the average U.S. household with at least one credit card has 6 bank credit cards and 8.3 retail credit cards for a total of 14.3 credit cards. Likewise, insurance contracts are subject not only to moral hazard but also to a recontracting hazard. In fact, the possibility that an agent retrades after entering into an insurance contract is the rule rather than the exception.

In this paper, we investigate the impact of such recontracting opportunities on the optimal compensation package offered by a firm to its manager and on the efficiency of a manager's effort decision. We study a simple moral hazard

[1] This is not so for non-executives, see the arbitration claim filed on October 28, 2003 against CISCO, which seeks compensation damages directly related to the failure to recommend hedging strategies to employee stock option plan participants.

problem, similar to Holmstrom (1979) and Grossman and Hart (1983), in which a principal hires an agent to perform a task. After receiving his wage contract but before undertaking a costly effort, the agent can borrow on his future wage from a competitive market. He can contract with a potentially infinite number of lenders. We model this lending game as a stochastic game with an exogenous stopping probability, in which the agent meets lenders sequentially. With each lender he can sign a bilateral agreement after which the game either ends or moves to the next round of contracting. If the lending game ends the agent will undertake the effort. We characterize the essentially unique subgame perfect equilibrium in this game for all parameter values of the exogenous stopping probability. We distinguish between two situations, one in which this probability is zero and one in which it is positive.

We term the situation in which the stopping probability is zero as *perfect recontracting*. The following results obtain. The agent either borrows on his entire wage allocation and exerts no effort or borrows on only part of his wage, retaining some residual future wage (and thus some incentives), such that he is just indifferent between borrowing an additional amount and quitting the lending game. We call this residual wage the *recontracting-proof wage.* We show that without loss of generality the agent borrows from one lender only. Thus, there is no reason to expect multiple contracts in equilibrium. The principal's problem of choosing an optimal wage scheme simplifies to finding the recontracting-proof wage allocation and associated effort level. We confirm the findings of several related papers discussed below that in this model a wage that induces high effort is associated with higher risk and a higher expected payment to the agent than the conventional second best. Also, a flat wage that induces low effort is chosen more often by the principal than in the second best. Thus, this model predicts that the principal can indeed design robust incentives, albeit at higher costs.

Our key finding is that this is no longer true when the stopping probability is non zero, a situation which we call *imperfect recontracting.* Thus, the type of equilibrium in which the agent receives a risky wage but prefers not to recontract is not implementable. Instead, he borrows sequentially from a finite number of lenders and stops only if he is forced to (because the recontracting game ends for exogenous reasons) or if he has borrowed on his entire future wage. Thus, the agent always obtains multiple loans in equilibrium and no "recontracting-proof-principle" holds in a game of imperfect recontracting. This is important empirically. Since the analysis of recontracting games is motivated by the observed prevalence of recontracting, models where no recontracting takes place fail to explain the data.

The reason why the predictions of the model of perfect recontracting differ from the one of imperfect recontracting is the following. In the latter, in each recontracting round the agent and the current lender perceive that this could be the last opportunity for the agent to obtain a loan. Because future borrowing imposes a negative externality on existing borrowing agreements, being the last lender is valuable. Consequently, the agent will be able to secure more favorable loans. This makes borrowing more attractive and makes a recontracting-proof wage more difficult and, as we in fact show, impossible to implement.

We then compare the outcome of the perfect recontracting model to the one of an imperfect recontracting model that is arbitrarily close. Surprisingly, it turns out that for a certain range of parameters, the two equilibria are distinct even if the exogenous stopping probability tends to zero.

First, neither the agent's effort nor his wage payment in the equilibrium of the imperfect recontracting model approach the effort and the wage scheme in the recontracting-proof equilibrium under perfect recontracting. Second, the principal receives a lower payoff when recontracting is imperfect than when it is perfect. Because the principal's payoff from low effort is independent of the agent's recontracting opportunities the above result also implies that low effort will be more often implemented in a world of imperfect recontracting. Third, the number of loans obtained by the agent goes to infinity as the stopping probability in the imperfect recontracting game approaches zero, and the size of those loans becomes vanishingly small. Thus, there is a discontinuity in the amount of borrowing when the stopping probability is zero. Also, the wage corresponding to high output under imperfect recontracting is far above the second-best wage or the recontracting-proof wage under perfect recontracting, and it is infinitely costly for the principal to induce high effort with probability one. Thus, very high wage payments are more likely to be observed in a world of imperfect recontracting, which might explain some of the abnormally high salaries paid in recent years to CEO's and other top executives.

These last findings are puzzling because we would have expected the equilibria of the two models to converge. The reason for this seeming inconsistency is the following. To obtain the recontracting-proof equilibrium under perfect recontracting we need to make an additional assumption about equilibrium selection. This assumption is also made implicitly in all of the literature. Namely, we assume that, although there is a potentially infinite number of lenders, the agent only contracts with N of them. This assumption can be justified by the unmodelled possibility that there is a small cost of contracting, so that an infinite amount of contracting agreements is infinitely costly. That is, an equilibrium with an infinite number of active lenders is precluded *by assumption.* Thus, by assumption, the limiting equilibrium of the imperfect recontracting model with an infinite round of borrowing cannot coincide with the equilibrium of the perfect recontracting model.[2]

[2]In Reiche (2006) we study a model that contains both a positive cost of contracting and an exogenous stopping probability. We show that if both the contracting cost and the stopping probability go to zero, which of the two equilibria obtain depends on the speed with which the two parameters approach zero. That is, for fixed contracting cost, taking the stopping probability to zero results in the equilibrium of the perfect recontracting model described above. In contrast, for a fixed stopping probability, taking the contracting cost to zero results in the equilibrium of the imperfect recontracting model.

The paper is related to several papers in the literature on non-exclusive contracts, such as Bizer and DeMarzo (1992), (1999) and Kahn and Mookherjee (1998). The first two papers study a moral hazard problem, in which a wealth constraint entrepreneur can borrow sequentially from an infinite number of banks. In contrast to our paper, these two papers only consider standard debt contracts as they do not allow the borrowing contracts to be contingent on the output realization. Kahn and Mookherjee (1998) study an insurance problem, in which a risk-averse agent can buy insurance sequentially from an infinite number of insurers but they do not address the principal's problem. In contrast to our paper all three papers investigate the perfect recontracting case, in which they show a recontracting-proofness-principle similar to the one explained above. They also show that the agent can limit borrowing or buying insurance from one bank or insurance company only.

Parlour and Rajan (2001) study an unsecured loan market, in which borrower's can default strategically. In contrast to our paper and the ones above, the lenders make all the contracting offers. They show that equilibria exist, in which lenders make positive profits. Nevertheless, default never occurs in equilibrium and without loss of generality the agent borrows from only one lender. Bisin and Guaitoli (2004) model a problem similar to Kahn and Mookherjee (1998) but assume that insurers *simultaneously* offer insurance contracts to the agent. They show that positive profit equilibria can be sustained and that multiple contracts are offered in equilibrium. Some of these contracts are actively traded and some are latent. Nevertheless, they obtain allocations that are robust to further retrading, that is, equilibria in which high effort is sustained with probability one exist. In Bisin et al. (2006) the authors use this approach to study optimal compensation packages offered to managers when managers can hedge their financial positions but firms have an imperfect monitoring technology at their disposal to monitor and punish those hedging activities. They show that a manager's compensation is more incentivized when the monitoring technology is costly or when financial markets are more developed. In our paper, we do not consider monitoring of a manager's hedging activities directly, but the exogenous stopping probability in the lending game can be seen as a proxy for how difficult it is to prevent the agent from hedging his position or how well developed the financial market is. In our paper, the relationship between the quality of recontracting and incentive pay are not as clear-cut. A decrease in the stopping probability goes hand in hand with a decrease in the optimal wage in case of high output that induces a fixed number of borrowing rounds. However, because a decrease in the stopping probability increases the likelihood that low effort is induced in equilibrium the principal might now offer a more incentivized wage in order to induce a higher number of borrowing rounds (and consequently a higher probability of high effort). Also, in Bisin et al., as in the other papers, hedging is prevented by the optimal compensation package, so that their paper cannot explain the findings on insider hedging activities cited above.

The paper is structured as follows. The following section contains the model. Section 3 describes the agent's optimal effort choice. Section 4 solves for the optimal wage contract if borrowing can be prevented. Section 5 solves for the agent's and principal's equilibrium strategies when the exogenous stopping probability in the lending game is bounded away from zero. Section 6 solves for their stragegies when this probability is zero. Section 7 compares the two equilibria and shows that they are distinct even if the stopping probability in the first model tends to zero. Finally, conluding remarks are provided in section 8.

2 The Model

At date 0, a principal hires an agent to manage a production technology with uncertain profit stream $\tilde{x}$. The technology allows for two possible output realizations at date 2, $\overline{x} > \underline{x}$, which are verifiable and accrue directly to the principal.

At an intermediate date 1, the agent undertakes a costly and unobservable effort that affects the probability distribution over output levels. He can choose between two effort levels, high effort e_h and low effort e_l, where effort e_i results in high output with probability p_i and costs c_i, $i = h, l$.[3] We assume that high effort yields a larger expected output than low effort and that it is more costly, that is, $p_h > p_l$ and $c_h > c_l$. To compensate the agent for his effort cost the principal offers a compensation scheme $w \in \mathbb{R} \times \mathbb{R}^2$, that consist of a sign-up fee $w_1 \in \mathbb{R}$ and a wage $w_2 \in \mathbb{R}^2$. The sign-up fee is paid to the agent directly after he has been hired at date 1. The wage is paid at date 2 and can consequently be indexed on the output realization, $w_2 = (\overline{w}_2, \underline{w}_2)$.

The agent has a utility function that is separable in date 1 and date 2 consumption (money) and separable in effort. He discounts future payoffs by a discount factor $0 < \delta < 1$. For simplicity, we assume that he is risk-neutral and so his utility can be written as

$$U(w, e_i) = w_1 - c_i + \delta E_{p_i}[w_2], \tag{1}$$

where $E_{p_i}[w_2] = p_i \overline{w}_2 + (1 - p_i)\underline{w}_2$.[4] The principal is also risk-neutral and for simplicity, we assume that he does not discount future profits.[5] The principal's expected returns can then be written as

$$\Pi(w, e_i) = -w_1 + E_{p_i}[x - w_2]. \tag{2}$$

[3] We will also sometimes write $p(e_i) = p_i$ and $c(e_i) = c_i$.

[4] Some preliminary results were obtained for more general utility functions, in particular when the agent is risk-averse. They are available from the author. However, it is impossible to obtain closed form solutions with risk-aversion and it is therefore very difficult to obtain comparative static results. Note however, that, because the utility function is assumed to be continuous, the obtained results also hold for an agent with sufficiently small risk aversion.

[5] Our results could be established with the assumption that the principal discounts the future as well. All we need is that his discount factor is higher han the agent's.

To motivate the assumptions about the parties' differing time preferences consider the following examples. Assume, that at date 1 the agent has the possibility to invest I into a project that returns $R > I$ at date 2. Assume further that only he can make this investment. For instance, to be successful the project needs the agent's personal input as well as his capital. Then, the agent has preferences as in (1) with $\delta = \frac{I}{R} < 1$ and the principal has preferences as in (2). Alternatively, assume that the agent has some financial needs at date 1 that are not covered by the up-front fee received as part of his wage package. Assume in contrast that the principal has almost unlimited funds. If there are credit market imperfections, the rate r_1 at which the agent can borrow money is likely to be higher than the rate r_2 at which the principal can place his. Then, by setting $\delta = \frac{1+r_2}{1+r_1} < 1$, we obtain the above preferences.

We assume that the principal makes a take-it-or-leave-it wage offer to the agent. If the agent rejects the offer both parties receive their reservation utilities which we normalize to 0. We also assume that the agent has no initial wealth and is protected by limited liability. He can therefore not receive a negative wage. Furthermore, the wage cannot be made directly contingent on his effort, because effort is unobservable. In addition, it cannot include other prescriptions, for example it cannot forbid the agent to enter into another contract with a third party. Implicit in this assumption is the idea that the agent can secretly solicit financing from outsiders at date 1, that is, after he has signed the employment contract but before the underlying uncertainty is resolved. Importantly, he can do so *before* he undertakes the effort. This game is described in the next section.

2.1 The Lending Game

The fact that the agent discounts his future wage payments at a rate $\delta < 1$ suggests that there are gains from trade when he exchanges part of his future earnings against an up-front payment from a lender.

We assume that there is an infinite number of risk neutral lenders whom the agent meets sequentially. In contrast to the initial stage game in which the principal offers the wage contract, we assume that in the lending game the agent makes all contracting offers. This can be justified by assuming that both the financial market and the labor market are perfectly competitive. Thus, lenders earn zero profits and the agent, when being offered employment, is kept at his reservation utility.

A contract with a lender consists of an up-front payment $t_1 \in \mathbb{R}$ that is paid at date 1 by the lender to the agent, and a payment $t_2 \in \mathbb{R}^2$, also from the lender to the agent, that is contingent on the realization of uncertainty at date 2. Thus, $t_2 = (\bar{t}_2, \underline{t}_2)$, where $\bar{t}_2$ (respectively $\underline{t}_2$) is the payment made in case of high (respectively low) output. After each stage of the lending game, with an exogenously given probability $q < 1$ the agent will be able to contract with an additional lender, and with probability $1-q$ he will not meet another lender and has to undertake the effort.[6] The lending subgame is also terminated if the agent voluntarily decides to solicit no further contracts. This modelling assumption has two advantages. First, we do not need to worry about the possibility that the lending subgame does not end. In Kahn and Mookherjee (1998), the authors address this problem by assuming that the agent's utility is equal to negative infinity if he enters into an infinite number of contracts. They motivate this assumption with reference to contracting cost: If each contract involves a small cost an infinite round of contracting is infinitely costly. Second, the assumption of an exogenous end to the lending game allows us to study the dynamics of the recontracting game. In addition, it is reasonable to assume that the lending game will end due to some exogenous factors, which are outside the parties' control. Consider again the motivating example in the Introduction of a manager who wants to hedge the risk inherent in his remuneration package. It is plausible that the CEO of a large company cannot consecrate an unlimited amount of time to design an optimal portfolio, that is, he might be forced to stop searching for a further lender even if there are still gains from trade.

A lender's expected profits depend on the contract he has signed and the odds on output resulting from the agent's effort choice. We again assume for simplicity that each lender is risk neutral and does not discount future payoffs. Thus, we can write a lender's expected profits as

$$V(t, e_i) = -t_1 - E_{p_i}[t_2]. \tag{3}$$

Neither the initial employment contract nor any of the insurance or loan contracts depend on any consecutive contract that the agent may sign. In other words, we do not consider *universal mechanisms* as defined in Epstein and Peters (1999) and Peters (2001), where a principal's contract may depend on the contract that the agent writes with another principal. One way to justify this restriction is to assume that it is impossible for the principal to foresee all potential ways in which the agent can obtain financing after he has been employed. Furthermore, it is implausible that an employment contract can prohibit, or be made contingent on the possibility that a third party related to the agent obtains a financial position in the firm's stock. For example, assume that a CEO's remuneration package consists partly of stock options. He could instruct some member of his family to purchase put options on the firm's stock, which would undo the incentive effects of the remuneration scheme if the two parties' wealths are interdependent.

Finally, it is assumed that a lender observes the agent's wage scheme and all previously signed contracts and therefore knows the agent's total wage allocation when they meet. Thus, we abstract from all informational problems.

[6] This probability also applies to the first stage, that is, the probability that the agent enters into the lending game is q.

2.2 Timing

To summarize the timing:

– Date 0 : The principal offers a wage contract w to the agent at date 0. The agent accepts or rejects. If he rejects, the game ends and both parties receive a utility of 0. If he accepts we move to date 1.

– Date 1 : The principal pays the sign-up fee w_1 to the agent.

Lending Game: The agent meets a lender with probability q. With probability $1-q$ he does not meet a lender and the game moves to the effort subgame. If the agent meets a lender he can either decide not to request a loan and choose his effort directly or he can ask for a loan contract t. The lender can either accept or reject the demand. If he accepts he pays the up-front fee t_1. Then, the above described stage game is repeated.

Effort Subgame: The agent undertakes effort $e_i \in \{e_h, e_l\}$.

– Date 2: Output is realized. The agent is paid a wage $\overline{w}_2$ or $\underline{w}_2$ by the principal depending on the realization of output and receives $\overline{t}_2$ or $\underline{t}_2$ from each lender with whom he has signed a contract.[7]

2.3 Equilibria

We first describe the agent's strategy set in the lending game. Since a lender is a short-lived player who cares only about his payoff, we can neglect history dependent strategies for the agent and assume without loss of generality that the agent's strategy is stationary, that is, only depends on his current total allocation ω. For that, define $\Omega := \mathbb{R} \times \mathbb{R}^2$. An element $\omega = (\omega_1, \overline{\omega}_2, \underline{\omega}_2) \in \Omega$ consists of the agent's total wealth ω_1 at date 1, which includes the up-front payment made by the principal and all payments made by preceding lenders, and the promised date 2 payments $\omega_2 = (\overline{\omega}_2, \underline{\omega}_2)$, which include the promised wage by the principal and the promised repayments to and from lenders. To simplify on notation we can formulate the agent's decision not to request a loan as offering a contract $t = (0, 0, 0)$. Then, the function $\sigma : \Omega \longrightarrow \Omega$ fully describes the agent's strategy, where $\sigma(\omega)$ is the agent's 'new' wage, that is, $\sigma(\omega) - \omega$ is the contract offered to the lender. For instance, if $\sigma(\omega) = \omega$, the agent offers $t = (0, 0, 0)$ at ω.

The agent's strategy in the effort game is a function $e : \Omega \mapsto \{e_l, e_h\}$.

Finally, when the agent is offered an employment contract by the principal he can either accept or reject. He will always accept the initial wage contract if it provides him with an expected utility of at least zero. Without loss of generality it is therefore assumed that the principal's initial offer w fulfills this constraint.

Thus, the agent's overall strategy is $(\sigma(\cdot), e(\cdot))$. The agent's expected utility from an initial allocation ω, following strategy $(\sigma(\cdot), e(\cdot))$ can be written as

$$EU(\omega, \sigma, e) = (1-q) \sum\nolimits_{n=0}^{\infty} q^n U(\sigma^n(\omega), e(\sigma^n(\omega))),$$

[7] For each recontractor t might differ.

where $\sigma^n(\omega) = \underbrace{\sigma \circ \sigma ... \circ \ \sigma}_{n}(\omega)$ and $\sigma^0(\omega) = \omega$. Similarly, define the expected probability of high output as

$$\rho(\omega, \sigma, e) = (1-q) \sum_{n=0}^{\infty} q^n p(e(\sigma^n(\omega))).$$

To understand these two expressions, observe that with probability $(1-q)q^n$ the lending game is terminated at allocation $\sigma^n(\omega)$ and the agent chooses effort $e(\sigma^n(\omega))$.

A lender's strategy in the lending game is a decision $a \in \{0, 1\}$, where $a = 0$ ($a = 1$) signifies that the lender rejects (accepts) the contract $\sigma(\omega) - \omega$. Since he only cares about his one period payoff, he always accepts a contract that allows him to break-even given the agent's strategy in the lending game and the associated probabilities of high and low output. Thus, if the agent is at allocation ω and follows strategy $(\sigma(\cdot), e(\cdot))$, the lender will set $a = 1$ if and only if $V(\sigma(\omega) - \omega, \rho(\sigma(\omega), \sigma, e)) \geq 0$. Since one possible contract that satisfies this constraint is the null contract, we can without loss of generality assume that the agent offers a contract $\sigma(\omega) - \omega$ that allows the lender to break even.

We can now define the equilibrium concept of Subgame Perfection in this context.

Definition 1 *A Subgame Perfect Equilibrium is a tuple $(w^*, \sigma^*(\cdot), e^*(\cdot))$, where w^* maximizes*

$$-w_1 + E_{\rho^*(w)}[x - w_2] \quad s.t. \quad EU(w^*, \sigma^*, e^*) \geq 0, \tag{4}$$

$\sigma^*(\cdot)$ maximizes

$$\begin{aligned} EU\left(\sigma(\omega), \sigma^*, e^*\right)) \qquad & \forall \omega \in \Omega \\ s.t \qquad & \\ V(\sigma(\omega) - \omega, \rho^*(\sigma(\omega))) \geq 0 \qquad & \forall \omega \in \Omega \\ \sigma(\omega) \geq 0 \qquad & \forall \omega \in \Omega, \end{aligned} \tag{5}$$

and $e^*(\omega)$ maximizes $U(\omega, e)$ for all ω with $\rho^*(\omega) := \rho(\omega, \sigma^*, e^*)$.

Line (4) is the principal's maximization problem, where his expected profits are calculated using $\rho^*(w)$, that is, taking into account the agent's equilibrium strategy (σ^*, e^*), and where he has to guarantee the agent's participation. Line (5) describes the agent's maximization problem in the lending game. At every ω, $\sigma^*(\omega)$ must be optimal given that he employs his subgame perfect strategy $\sigma^*(\cdot)$ in every other state. Two constrains must be satisfied in this maximization problem. Lenders must break even and contracts must fulfill the agent's limited liability constraint.

Before providing a characterization of the subgame perfect equilibria we make some assumptions about the agent's behavior when he is indifferent between several strategies. As is standard in the principal-agent literature, it is assumed that the agent, when indifferent between two effort levels, will choose the effort level preferred by the principal. We expand this idea by assuming that whenever the agent is indifferent between two contract proposals to a lender he will offer the contract that induces an equilibrium path with maximum expected effort. Formally:

Assumption 1: *Ties will be broken as follows*

1. $e^*(\omega) = e_h$ if $U(\omega, e_h) = U(\omega, e_l)$
2. *Assume that both $\sigma(\cdot)$ and $\sigma'(\cdot)$ are part of a subgame perfect equilibrium. Then, if $\rho(\omega, \sigma, e^*) \geq \rho(\omega, \sigma', e^*)$ set $\sigma^*(\omega) := \sigma(\omega)$.*

3 Optimal Effort

The agent will choose $e^* = e_h$ if and only if $U(w, e_h) \geq U(w, e_l)$, or equivalently

$$\overline{\omega}_2 \geq \frac{\Delta c}{\delta \Delta p} + \underline{\omega}_2, \tag{6}$$

where $\Delta c = c_h - c_l$ and $\Delta p = p_h - p_l$. Thus, the agent's optimal effort choice at allocation ω is independent of ω_1. Set $IC := \left\{\omega \in \Omega \mid \overline{\omega}_2 \geq \frac{\Delta c}{\delta \Delta p} + \underline{\omega}_2\right\}$, and $\neg IC := \Omega \setminus IC$. With a slight abuse of notation, we will also sometimes set $IC := \left\{\omega_2 \in R^2 \mid \omega \in IC\right\}$ and $\neg IC := R^2 \setminus IC$. Then the agent's equilibrium strategy is

$$e^*(\omega) = \begin{cases} e_h & \iff \quad \omega \in IC \\ e_l & \iff \quad \omega \in \neg IC. \end{cases} \tag{7}$$

In the following section we solve the contacting problem between the principal and the agent when there are no borrowing possibilities. We show that if the principal offers an employment contract that induces the agent to exert high effort, the agent has an incentive to acquire liquidity from a lender. Thus, the possibility of the agent's borrowing constitutes a real constraint for the principal.

4 Benchmark: No Borrowing

Assume that the agent cannot borrow on his date 2 wage. To induce low effort, the principal pays an up-front payment just large enough to cover the effort cost and pays nothing at date 2. Thus, he sets $w_1 = c_l$ and $w_2 = (0, 0)$. This is optimal since it is costly to defer paying the agent ($\delta < 1$) and the date 2 wage is beneficial only for providing incentives.

In contrast, if the principal wants to induce high effort he should choose $w \in IC$. To maximize the principal's payoff (2) the inequality in (6) must hold with equality and therefore:

$$\overline{w}_2 = \frac{\Delta c}{\delta \Delta p} + \underline{w}_2. \tag{8}$$

Because deferring payment to the agent is costly, it is optimal to set

$$\underline{w}_2 = 0. \tag{9}$$

Finally, the agent's participation and limited liability constraints allow to solve for the up-front fee:

$$w_1 = \max\left\{\frac{p_h c_l - p_l c_h}{\Delta p}, 0\right\}.$$

Assume, that it is optimal for the principal to induce high effort in the second best and also assume that the agent's limited liability constraint is not binding.[8]

Assumption 2:

$$w^{sb} = \left(0, \frac{\Delta c}{\delta \Delta p}, 0\right) \tag{10}$$

We now show that with the second-best compensation scheme the agent strictly prefers to borrow against his date 2 wage. First, note that at the second-best wage the agent is indifferent between choosing high or low effort, that is, $U(w^{sb}, e_h) = U(w^{sb}, e_l)$. Second, if he borrows against his entire future wage earnings, the lender, foreseeing that the agent will put in low effort, is willing to pay up to $E_{p_l}[w_2^{sb}]$. Thus, the agent receives $E_{p_l}[w_2^{sb}] - c_l > U(w^{sb}, e_l)$, because $\delta < 1$, and we have shown the following.

Proposition 1 *The second-best wage contract is vulnerable to borrowing, that is, the agent strictly prefers to borrow on his entire date 2 wage in exchange for a date 1 payment. He then chooses low effort.*

5 Borrowing: The case $q < 1$

We now solve for the subgame perfect equilibrium when the agent can borrow from an infinite sequence of lenders. In all of the following we will use the agent's optimal effort choice $e^*(\cdot)$ as derived in section 3.

We first solve for the up-front payment that a lender pays to the agent as a function of the second period borrowing agreement. Obviously, it is optimal for the agent to keep a lender at his reservation utility. Thus, the up-front payment is going to be equal to the fraction of the expected second period wage that the agent promises to the lender. This expectation is calculated using the odds induced by the agent's equilibrium strategy. Formally,

[8] If it is optimal to induce low effort in the seond best, recontracting is not an issue. Assuming that the agent's limited liability constraint is not binding is without loss of generality. It saves on notations because we can set $w_1 = 0$ in what follows.

Lemma 1 *For all ω,*

$$\sigma^*(\omega)_1 - \omega_1 = E_{\rho^*(\sigma(\omega))}[\omega_2 - \sigma^*(\omega)_2].$$

Proof. see Appendix. ■

Lemma 1 fixes the first part of the agent's equilibrium strategy. We will now discuss an intuitive way of finding $\sigma^*(\cdot)_2$, the second part of his equilibrium strategy. Most formal statements and proofs are relegated to the appendix. Note, that with every strategy $\sigma(\cdot)_2$ is associated a unique second period wage path $\omega_2 \to \sigma(\omega)_2 \to \sigma(\sigma(\omega))_2 \to ...$ in $\mathbb{R}^2$. Therefore, we will sometimes use the expression that a strategy 'traces a wage path' or 'has an associated wage path' or equivalently that it is 'induced by a wage path'. We will call a wage path 'feasible' if the associated strategy is feasible, that is, if it fulfills the agent's limited liability and the lenders' break-even constraints.

5.1 The $\neg IC$ region

The equilibrium strategy within the $\neg IC$ region is straightforward. The agent optimally pledges his entire date 2 wage against an up-front payment from a single lender. We call this strategy $\sigma_1(\cdot)$. Thus, $\sigma_1(\cdot)$ has associated wage path $\omega_2 \to \omega_2^0$ with

$$\overline{\omega}_2^0 = 0 \text{ and } \underline{\omega}_2^0 = 0. \tag{11}$$

Its optimality in $\neg IC$ is proved formally in Lemma 3 in the Appendix. Here, we try to give some intuition and outline the steps of the proof. First, it is easy to see that the agent should never voluntarily quit the lending game with a non zero date 2 wage $\omega_2 \in \neg IC$. This is worth $\delta E_{p_l}[\omega_2]$, but he can obtain $E_{p_l}[\omega_2]$ as an up-front payment from any lender. Second, borrowing from more than one lender within the $\neg IC$ region is strictly dominated by borrowing from only one. This is because the agent obtains the same odds (p_l) from both lenders but risks not meeting the second one and being left with some positive residual date 2 wage. Finally, one can show that exchanging a residual wage inside the $\neg IC$ region against one inside the IC region is also not optimal: First, Lemma 2 in the Appendix shows that a strategy with an associated wage path that contains a jump from the $\neg IC$ into the IC region (or vice versa) must then have the agent quit the lending game. To see this, assume to the contrary that the associated path contains the sequence $\omega_2 \to \sigma(\omega)_2 \to \sigma(\sigma(\omega))_2$ with $\omega_2 \in \neg IC$, $\sigma(\omega)_2 \in IC$ and $\sigma(\omega)_2 \neq \sigma(\sigma(\omega))_2$. Then it is easy to show that substituting for this the sequence $\omega_2 \to \sigma(\sigma(\omega))_2$ is feasible and better for the agent. Using this result Lemma 3 in the Appendix shows that exchanging a wage allocation inside the $\neg IC$ region against one inside the IC region and then quitting the lending game to undertake high effort is dominated by borrowing against the entire wage allocation and undertaking low effort.

5.2 The IC region

Here we develop the equilibrium strategy within the IC region. Take an allocation $\omega_2 \in IC$. Obviously, the agent can follow the same strategy as in the $\neg IC$ region and exchange his entire date 2 wage against an up-front payment. But this is not always optimal. One other possible strategy, call it $\sigma_0(\cdot)$, is not to borrow at all and undertake high effort. This strategy yields a payoff:

$$\delta E_{p_h}[\omega_2] - c_h, \tag{12}$$

whereas $\sigma_1(\cdot)$ yields

$$(1-q)(\delta E_{p_h}[\omega_2] - c_h) + q(E_{p_l}[\omega_2] - c_l). \tag{13}$$

It is possible that (12) is larger than (13). A necessary condition for this is $\delta p_h > p_l$. A variant of $\sigma_0(\cdot)$, we call $\sigma_s(\cdot)$, can perform even better. With $\sigma_s(\cdot)$ the agent exchanges only part of his date 2 wage against an up-front payment, effectively retaining a residual wage $\omega_2^s \in IC$. He then quits the lending game. This strategy traces the path $\omega_2 \to \omega_2^s \to \omega_2^s \to ....$ It results in utility

$$(1-q)(\delta E_{p_h}[\omega_2] - c_h) + q(E_{p_h}[\omega_2 - \omega_2^s] + \delta E_{p_h}[\omega_2^s] - c_h). \tag{14}$$

Note that the lender uses p_h to calculate the up-front payment because the agent exerts high effort for sure.

What other strategies could be optimal? The agent can for example decide to borrow from two lenders. This strategy is called $\sigma_2(\cdot)$ and looks as follows. With the first lender the agent exchanges part of his date 2 wage against an up-front payment, keeping some residual wage $\omega_2^1 \in IC$. With the second lender he borrows against the entire remainder of his wage. Compared with the first strategy this strategy has a 'cost' and a 'benefit'. It is costly for the agent to use this strategy because with positive probability he will not meet the second lender and will be left with the residual wage ω_2^1, which is discounted at $\delta < 1$. The benefit is that since $\omega_2^1 \in IC$, the agent undertakes high effort with some probability (precisely in the case where he does not meet the second lender) and this in turn implies that he receives a better deal from the first lender. In fact, the first lender uses $\rho_2 = (1-q)p_h + qp_l$ to calculate the up-front payment (in Lemma 1) which is greater than p_l. Thus, the agent obtains utility

$$(1-q)(\delta E_{p_h}[\omega_2] - c_h) + q(E_{\rho_2}[\omega_2 - \omega_2^1] + (1-q)(\delta E_{p_h}[\omega_2^1] - c_h) + q(E_{p_l}[\omega_2^1] - c_l)) \tag{15}$$

Comparing (15) and (13), depending on ω_2 and ω_2^1, strategy $\sigma_2(\cdot)$ might perform better than strategy $\sigma_1(\cdot)$.

Finally, consider extensions to $n > 2$ lenders. We call such strategies $\sigma_n(\cdot)$, $n = 3, 4, ...$ With the first lender the agent exchanges part of his date 2 wage against an up-front payment and retains a residual wage $\omega_2^{n-1} \in IC$, with the second lender he exchanges another part of his wage and retains a residual wage

$\omega_2^{n-2} \in IC$ and so on until the last lender, from whom he borrows against his entire remaining wage. For a given set of allocations $\{\omega_2^i\}_{i=1,\dots,n-1} \in IC$, this strategy is uniquely defined by its associated wage path: $\omega_2 \to \omega_2^{n-1} \to \omega_2^{n-2} \to \dots \to \omega_2^1 \to \omega_2^0$. Borrowing from more lenders increases the risk of not meeting all of them and thus increases the cost of not being able to borrow against the entire date 2 wage. At the same time it increases the benefit of obtaining better borrowing deals from early lenders. For example, with n lenders, the first one will estimate the probability of high output at

$$\rho_n = (1 - q^{n-1})p_h + q^{n-1}p_l \tag{16}$$

and is consequently willing to pay a relatively high up-front transfer.

Thus, our candidate equilibrium strategies inside the IC region are the strategies $\{\sigma_n(\cdot)\}_n$ and $\sigma_s(\cdot)$ with correctly specified allocations $\{\omega_2^n\}_n \in IC$ and $\omega_2^s \in IC$.[9] Two properties of those allocations are relatively easy to establish.

Property 1

$$\underline{\omega}_2^n = 0 \ \forall n \text{ and } \underline{\omega}_2^s = 0.$$

Property 1 says that after each round of borrowing the residual date 2 wage must pay nothing in case of low output. Intuitively, retaining some positive date 2 wage is only valuable for the agent if it allows him to credibly promise high effort. This will in turn allow him to obtain better borrowing conditions from lenders, who will receive most of their repayments in the high output state. Put differently, without an incentive problem the agent would strictly prefer to receive all of his remuneration at date 1. Therefore, payments in case of low output, which dilute effort incentives and are costly to the agent, should optimally be set equal to zero.

Property 2

$$\overline{\omega}_2^n > \overline{\omega}_2^{n-1} \qquad \forall n.$$

Property 2 says that after each round of contracting the agent is left with a smaller residual date 2 wage, that is, he *borrows* in equilibrium. The intuition is as above: Because ω_2 is in IC the odds with which future borrowing risk will be evaluated are unaffected by a contract that *increases* the agent's residual wage in case of high output. Therefore, an increase in this wage only imposes a cost on the agent. The two properties are proved formally in Lemma 5 in the Appendix.

It remains to determine the exact locations of the allocations $\{\omega_2^n\}_{n>1}$ and ω_2^s on the vertical axis. Two conditions must be met. First, for strategy $\sigma_n(\cdot)$ to be credible it must be optimal for the agent to use strategies $\{\sigma_i(\cdot)\}_{i<n}$ at allocations $\{\omega_2^i\}_{i<n}$. Second, for strategy $\sigma_n(\cdot)$ to be optimal, the allocations $\{\omega_2^i\}_{i<n}$ must be the allocations with minimal expected value in IC that satisfy

[9] This is proved formally in Lemma 4 in the appendix.

the first condition. Similarly, for strategy $\sigma_s(\cdot)$ to be credible, strategy $\sigma_0(\cdot)$ must be optimal at ω_2^s and ω_2^s must be the allocation with minimal expected value in IC that satisfies this condition.

This suggests the following recursive method for defining the allocations $\{\omega_2^n\}_{n>1}$ and ω_2^s: ω_2^1 is found by setting $\underline{\omega}_2^1 = 0$ and solving

$$EU(\omega_1, \omega_2^1, \sigma_0, e_h) = EU(\omega_1, \omega_2^1, \sigma_0, e_l), \tag{17}$$

that is, ω_2^1 is the second best wage that induces high effort. As was shown in section 4, strategy $\sigma_1(\cdot)$ is optimal at ω_2^1 and it is also the allocation with the smallest expected value in the IC region.

Given ω_2^1, strategy $\sigma_2(\cdot)$ is defined as the strategy inducing wage path $\omega_2 \rightarrow \omega_2^1 \rightarrow \omega_2^0$. From ω_2^1 move upwards on the vertical axis, that is, increase $\overline{\omega}_2$. Using (15) we see that $\sigma_2(\cdot)$'s payoff increases by $(1-q)\delta p_h + q\rho_2$, whereas $\sigma_1(\cdot)$'s payoff increases by $(1-q)\delta p_h + qp_l$ (see (13)). Since $\rho_2 > p_l$, the marginal increase in strategy $\sigma_2(\cdot)$'s payoff from an increase in $\overline{\omega}_2$ is larger than the marginal increase in strategy $\sigma_1(\cdot)$'s payoff. Thus, there exists an allocation ω_2^2 on the vertical axis with $\overline{\omega}_2^2 > \overline{\omega}_2^1$, at which strategies $\sigma_2(\cdot)$ and $\sigma_1(\cdot)$ yield the same payoff. We can calculate this allocation explicitly by equating (15) and (13) and setting $\underline{\omega}_2^2 = 0$:

$$\overline{\omega}_2^2 = \frac{(1-\delta)p_l}{\delta \Delta p^2} \Delta c + \overline{\omega}_2^1. \tag{18}$$

Everywhere above ω_2^2 on the vertical axis strategy $\sigma_2(\cdot)$ dominates $\sigma_1(\cdot)$, and everywhere below $\sigma_1(\cdot)$ dominates $\sigma_2(\cdot)$. From Property 2 and its discussion it should be clear that on the latter section $\sigma_1(\cdot)$ also dominates all strategies $\sigma_n(\cdot)$ with $n > 2$.[10] It remains to evaluate strategy $\sigma_0(\cdot)$'s payoff to see whether it is possible to find an allocation ω_2^{1s} on the vertical axis between ω_2^1 and ω_2^2, where $\sigma_0(\cdot)$ is optimal. This would make strategy $\sigma_s(\cdot)$ credible. To do this we proceed as before. We know that at ω_2^1 strategy $\sigma_1(\cdot)$ yields a higher payoff than $\sigma_0(\cdot)$. Then, moving upwards from ω_2^1 on the vertical axis, $\sigma_0(\cdot)$'s payoff increases by δp_h (see (12)), whereas $\sigma_1(\cdot)$'s payoff increases by $(1-q)\delta p_h + qp_l$. If $\delta p_h \leq p_l$, there is never an allocation at which $\sigma_0(\cdot)$ dominates $\sigma_1(\cdot)$. Assume the contrary. Then, $\sigma_0(\cdot)$'s payoff increases faster than $\sigma_1(\cdot)$'s payoff and we can find an allocation, such that everywhere above it $\sigma_0(\cdot)$ dominates and everywhere below it $\sigma_1(\cdot)$ dominates. This allocation, call it ω_2^{1s}, can be calculated by equating (13) and (12) and setting $\underline{\omega}_2^{1s} = 0$, which yields

$$\overline{\omega}_2^{1s} = \frac{1}{\delta p_h - p_l} \Delta c. \tag{19}$$

Comparing (18) and (19), it is easy to see that $\overline{\omega}_2^{1s} > \overline{\omega}_2^2$ and so we have shown that $\sigma_1(\cdot)$ is the agent's subgame perfect equilibrium strategy on the vertical axis below allocation ω_2^2. We can extend this argument to find the entire subset of the IC region on which $\sigma_1(\cdot)$ is optimal: $\Omega_1 = \{\omega \in IC \mid \overline{\omega}_2^1 \leq \Delta\omega_2 < \overline{\omega}_2^2\}$.

[10] More precisely, this follows from the proof of Lemma 5 in the Appendix.

Using an inductive argument we can find the remaining allocations $\{\omega_2^n\}_{n>2}$, strategies $\{\sigma_n(\cdot)\}_{n>2}$ and the subsets of the IC region on which the latter constitute the subgame perfect equilibrium strategy. We can also establish that strategy $\sigma_s(\cdot)$ is never credible, that is, $\sigma_0(\cdot)$ never dominates the 'best' $\sigma_n(\cdot)$. Before stating the result formally, we provide the explicit solutions for the residual wage payments in case of high output for $n > 2$:

$$\overline{\omega}_2^n = \frac{(1-\delta)p_l\Delta c}{\delta\Delta^2 p}\sum_{i=1}^{n-2}\left(\frac{(1-\delta)p_h}{q^{(i+1)/2}\Delta p}\right)^i + \overline{\omega}_2^2. \tag{20}$$

The following proposition contains the formal description of the agent's subgame perfect equilibrium in the lending game:

Proposition 2 *The agent's subgame perfect equilibrium strategy in the lending game is as follows:*

$$\sigma^*(\omega) := \begin{cases} \sigma_1(\omega) & \text{for all } \omega \in \neg IC \\ \sigma_n(\omega) & \text{for all } \omega \in \Omega_n \end{cases} \tag{21}$$

with $\Omega_n = \{\omega \in IC \mid \overline{\omega}_2^n \leq \Delta\omega_2 < \overline{\omega}_2^{n+1}\}$, $1 \leq n$.

Proof. See Appendix. ■

Figure 1 in the Appendix illustrates the subgame perfect equilibrium. Several conclusions can be drawn from Proposition 2. First, recontracting is an equilibrium phenomenon. Contrary to most results in the literature (for example, Bizer and DeMarzo (1992), (1999) and Kahn and Mookherjee (1998)), the principal cannot prevent the agent from requesting and receiving subsequent financing by choosing an appropriate wage allocation. Instead, the agent will always choose to borrow from a finite number of lenders.

The intuition for this can be seen as follows. In order for $\sigma_0(\cdot)$ to be a credible strategy at some allocation ω_2, it must dominate the 'best' strategy $\sigma_n(\cdot)$. In particular, at this wage the agent must have the following 'preference' relation over borrowing strategies:

$$\sigma_0(\omega_2) \succ \sigma_n(\omega_2) \succ \sigma_{n+1}(\omega_2). \tag{22}$$

However, this implies that also a combination of the strategies $\sigma_0(\cdot)$ and $\sigma_n(\cdot)$, let's call this strategy $\hat{\sigma}_{n+1}(\cdot)$, must dominate $\sigma_{n+1}(\cdot)$. Playing $\hat{\sigma}_{n+1}(\cdot)$ means that the agent signs the null contract with the first lender, that is, he plays strategy $\sigma_0(\cdot)$ with the first lender, and plays according to strategy $\sigma_n(\cdot)$ with all subsequent lenders. We write with a slight abuse of notation

$$\hat{\sigma}_{n+1}(\omega_2) = (1-q)\sigma_0(\omega_2) + q\sigma_n(\omega_2). \tag{23}$$

From (22) and (23) follows

$$\hat{\sigma}_{n+1}(\omega_2) \succ \sigma_{n+1}(\omega_2). \tag{24}$$

It is easy to see that this cannot be true, since the two strategies are indentical starting with the second lender but $\sigma_{n+1}(\cdot)$ has the advantage that part of the payment in ω_2 is brought forward because the agent borrows on part of his wage from the first lender.

Second, the more incentivized the initial wage allocation the higher the incidence of borrowing in equilibrium and also, the higher the likelihood of high equilibrium effort. Thus, the principal when choosing the optimal incentive scheme trades off large rewards in case of high output against high equilibrium effort. We formally solve the principal's problem (4) in the next section. In section 7 we will compare the result on recontracting found here with the equilibria usually exhibited in other papers in the literature.

5.3 The Principal's Problem

Following the analysis in the preceding section, a wage $w \in \Omega_n$ induces the agent to play strategy $\sigma_n(\cdot)$, which leads to an expected effort $e_n = (1 - q^n)e_h + q^n e_l$, where $e_0 = e_l$ and $e_\infty = e_h$. Consequently, similar to the analysis in Grossman and Hart (1986) the principal's problem can be divided into two parts. First, for each effort level e_n the principal chooses a wage $w^n \in \Omega_n$ that induces this effort and minimizes his expected wage payments. Second, the principal maximizes his expected payoff over n.

Trivially, the wage that induces e_0 is $w^0 = (c_l, \omega_2^0)$, see Section 4. For $n \geq 1$, the principal solves

$$\min_{w \in \Omega_n} w_1 + E_{\rho_{n+1}}[w_2] \text{ s.t. } EU(w, \sigma_n, e^*) \geq 0. \tag{25}$$

It is very easy to show that the date 2 wage schemes solving (25) are the allocations $\{\omega_2^n\}_n$. Because of Assumption 2, $w_1^n = 0$ for all $n \geq 1$.

Then, the principal solves

$$\max_{\tilde{n}} E_{\rho_{\tilde{n}+1}}[x - \omega_2^{\tilde{n}}]. \tag{26}$$

and we have

Proposition 3 *For all $q < 1$, the principal will set a wage w^n, where n solves (26).*

6 Borrowing: The Case $q = 1$.

All earlier models of moral hazard with recontracting (see for instance, Kahn and Mookherjee (1998), Bizer and DeMarzo (1992)) only consider the special case of perfect recontracting, that is, the situation where the agent meets subsequent lenders with probability 1. Since we would like to compare our results with the ones derived in these papers, we need to solve for the equilibrium in our model when recontracting is perfect, that is, when q is equal to 1. Under this assumption it is possible for the agent to enter into an infinite number of lending agreements. As this is impracticable in reality, the literature usually focuses on equilibria that involve only a finite number of lenders.[11] One way to justify this restriction is to assume that the agent's utility from an infinite series of transactions is equal to $-\infty$. For instance, if each transaction imposes a small cost on the agent, signing an infinite number of contracts becomes prohibitively costly. Alternatively, one can assume that the agent has only a limited amount of time to allocate between different tasks. Therefore, spending all of it soliciting funds will leave him with no time to consecrate on the job for which he has been hired. Consequently, no output will be realized and the principal will pay a zero wage. Anticipating this outcome, lenders are only willing to sign the null contract and this is a trivial equilibrium. In the following discussion we therefore follow the literature and exclude infinite subgame perfect equilibria. This assumption will turn out to be less innocent than previously suggested.

Turning to the analysis of the lending game for $q = 1$, it is easy to see that in contrast to the setting with $q < 1$, the agent does not need to borrow from more than one lender in equilibrium. Assume to the contrary that he borrows sequentially from two sources. Because the agent meets the second lender with probability 1, the first lender will offer exactly the same loan as the second lender. Thus, the agent could achieve the same allocation by borrowing the total amount from only one lender.

Then, there are only two situations to consider. The agent either immediately borrows against his entire wage (strategy $\sigma_1(\cdot)$) and exerts low effort, or he borrows against only part of his wage, retaining an amount $\omega_2^s \in IC$ (strategy $\sigma_s(\cdot)$), and then undertakes high effort. The first strategy is optimal in $\neg IC$ and in $\Omega_1 := \{\omega \in IC \mid \overline{\omega}_2^1 \leq \Delta\omega_2 < \overline{\omega}_2^{1s}\}$ and the latter is optimal in $\Omega_s := \{\omega \in IC \mid \overline{\omega}_2^{1s} \leq \Delta\omega_2\}$, where $\overline{\omega}_2^{1s}$ is as in (19). For this wage to be well defined we need

Assumption 3:

$$\delta p_h > p_l.$$

For the remainder of the paper we will assume that Assumption 3 holds. Figure 2 in the Appendix illustrates this discussion.

The first strategy leads to low effort in equilibrium and the principal implements this effort optimally by setting the date 2 wage equal to ω_2^0. The agent does not borrow in equilibrium. If the principal wants to induce high effort he needs to set a wage in Ω_s. The wage in Ω_s that minimizes his expected wage payments is ω_2^s. Remark, that again at this wage *no borrowing occurs in equilibrium.* Thus, the principal's overall problem reduces to a third-best problem, in which to the usual incentive and participation constraints is added a *recontracting proofness* constraint. To ease comparison with the limiting equilibrium

[11] Kahn and Mookherjee (1998) acknowledge this restriction, but most papers neglect to do so.

that will be derived in the following section we will call the equilibrium wage and effort if $q = 1$, $w_2(1)$ and $e(1)$ respectively. Summarizing, we have:

Proposition 4 *The equilibrium is characterized by either an incentivized wage and high effort or by a flat wage and low effort, that is, either $w_2(1) = \omega_2^s$ and $e(1) = e_h$, or $w_2(1) = \omega_2^0$ and $e(1) = e_l$. There is no borrowing in equilibrium.*

7 Comparative Statics

The aim of this section is to investigate whether the equilibrium described in Propositions 2 and 3 for q smaller than 1 converges to the equilibrium described in Proposition 4. Surprisingly, it turns out not to be the case.

7.1 The limiting equilibrium

To denote the dependency of the equilibrium described in Propositions 2 and 3 on q, let $n(q)$ be the optimal amount of lenders solved for in (26) and $w_2(q) = \omega_2^{n(q)}$ the corresponding date 2 wage. Call the effort induced by this wage $e(q) = (1 - q^{n(q)})e_h + q^{n(q)}e_l$.

The first step is to see whether the number of active lenders $n(q)$ remains finite when q approaches 1. From the definition of $e(q)$ it is immediate that if $n(q)$ remains finite, equilibrium effort will approach e_l. But then, the principal is better off inducing no borrowing at all by setting the date 2 wage equal to ω_2^0.

The next step is to see what happens if the number of active lenders goes to infinity when q approaches 1. If $n(q)$ converges more quickly to infinity than q goes to 1, $\lim_{q\to 1} q^{n(q)} = 0$ and consequently $\lim_{q\to 1} e(q) = e_h$. But then consider the expression derived for the high wage payment $\overline{\omega}_2^n$ in (20)

$$\overline{\omega}_2^n = \frac{(1-\delta)p_l\Delta c}{\delta\Delta^2 p}\sum_{i=1}^{n-2}\left(\frac{(1-\delta)p_h}{q^{(i+1)/2}\Delta p}\right)^i + \overline{\omega}_2^2. \tag{27}$$

From (27), if $\lim_{q\to 1} q^{n(q)} = 0$ then $\overline{w}_2(q) = \overline{\omega}_2^{n(q)}$ must tend to infinity. Surely, this cannot be optimal for the principal. In fact, we must have for q sufficiently close to 1

$$\frac{(1-\delta)p_h}{q^{(n(q)-1)/2}\Delta p} \leq 1,$$

which puts a strictly positive lower bound on $\lim_{q\to 1} q^{n(q)}$:

$$\lim_{q\to 1} q^{n(q)} \geq \frac{(1-\delta)^2 p_h^2}{\Delta^2 p}.$$

We can show that this inequality must be satisfied with equality (see proof of the following proposition), which implies that the wage for high output approaches

$$\overline{\omega}_2^{\infty} = \frac{(1-\delta)p_l \Delta c}{\delta \Delta^2 p} \lim_{q\to 1} \sum_{i=1}^{\log_q\left(\frac{(1-\delta)^2 p_h^2}{\Delta^2 p}\right)-2} \left(\frac{(1-\delta)p_h}{q^{(i+1)/2}\Delta p}\right)^i + \overline{\omega}_2^2. \tag{28}$$

Set $\omega_2^{\infty} = (\overline{\omega}_2^{\infty}, 0)$. This discussion is summarized in the following Proposition:

Proposition 5 *The limiting equilibrium for q tending to 1 is characterized by either an incentivized wage and an intermediate effort level or by a flat wage and a low effort level, that is, either $\lim_{q\to 1} w_2(q) = \omega_2^{\infty}$ and $\lim_{q\to 1} e(q) = (1-\kappa)e_h + \kappa e_l$ with $\kappa = \frac{(1-\delta)^2 p_h^2}{\Delta^2 p}$, or $\lim_{q\to 1} w_2(q) = \omega_2^0$ and $\lim_{q\to 1} e(q) = e_l$. In the first type of equilibrium the number of potential recontracting rounds approaches infinity at a rate $\log_q\left(\frac{(1-\delta)^2 p_h^2}{\Delta^2 p}\right) - 2$. In the second type of equilibrium there is no recontracting.*

Proof. See Appendix. ■

We now turn to a comparison between the equilibrium for $q = 1$ and the limiting equilibrium for q tending to 1.

7.2 Comparing the two equilibria

Assume first that $w_2(1) = \omega_2^s$ and $e(1) = e_h$. Remark that we can write (19) as

$$\overline{\omega}_2^s = \frac{(1-\delta)p_l \Delta c}{\delta \Delta^2 p} \sum_{i=1}^{\infty} \left(\frac{(1-\delta)p_h}{\Delta p}\right)^i + \overline{\omega}_2^2. \tag{29}$$

Therefore, since the last element in the series in (28) converges to 1, whereas the last element in (29) is $\frac{(1-\delta)p_h}{\Delta p} < 1$, we have

$$\overline{\omega}_2^{\infty} > \overline{\omega}_2^s.$$

From this we obtain two results.

First, if $w_2(1) = \omega_2^s$ and $e(1) = e_h$ the limiting equilibrium of the model with imperfect recontracting can be of the first type described in Proposition 5. Then, the limit wage is higher and more incentivized, effort is lower and total surplus and the principal's payoff are lower than in the equilibrium with perfect recontracting. Moreover, the agent is better off with imperfect recontracting. This is so, because he is paid a higher wage and can always mimic his equilibrium behavior under perfect recontracting, namely recontract with no-one and undertake high effort. He must therefore obtain a higher payoff by following his actual equilibrium strategy.

Second, since the principal's payoff is lower when he pays an incentivized wage in a world of imperfect recontracting, it is possible that he foregoes incentives altogether and pays a flat wage. Thus, even if in the model with perfect recontracting the equilibrium is characterized by an incentivized wage, the limiting equilibrium of the model with imperfect recontracting can contain a flat wage as in the second type of equilibrium described in Proposition 5. Consequently, the limit wage is lower and less incentivized, effort, total surplus and the principal's and the agent's payoff are lower. Summarizing, in neither case does the equilibrium for $q < 1$ converge to the one for $q = 1$.

Assume second that $w_2(1) = \omega_2^0$ and $e(1) = e_l$. Then, it is easy to verify that the limiting equilibrium of the model with imperfect recontracting coincides with this equilibrium. We summarize our discussion in the following proposition:

Proposition 6 *Assume that the equilibrium of the model with perfect recontracting contains an incentivized wage. Then this equilibrium will differ from the limiting equilibrium of the model with imperfect recontracting. The latter will either contain a higher powered incentive scheme, lower effort and a non-negligible amount of borrowing or a flat wage, the minimum effort and no borrowing. If the equilibrium of the model of perfect recontracting is characterized by a flat wage, it will coincide with the limiting equilibrium of the model with imperfect recontracting.*

8 Conclusion

In this paper we have developed a model in which sequential borrowing impacts on an agent's incentive to undertake a costly effort. We have studied the principal's problem of designing an optimal incentive scheme for the agent if such borrowing opportunities are present and have shown that in contrast to existing results in the literature, the principal cannot design an incentive scheme that is immune to borrowing. This model is therefore able to explain the large evidence of recontracting (borrowing from multiple lenders, hedging of financial positions by corporate insiders etc.) in the real world. We have also shown that the perfect ($q = 1$) and imperfect ($q < 1$) recontracting model deliver distinct results even if we take the parameter measuring the imperfection to 1. Furthermore, we have shown that the incidence of borrowing and the bonus payments for high output are higher, the better developed are capital markets (the higher is q).

There are two ways in which this research can be extended. First, it is interesting to study a model of recontracting with transaction costs that nests the two models of perfect and imperfect recontracting studied in this paper, see Reiche (2006). It is possible to reconcile the two equilibrium results in such a bigger model and derive additional comparative statics results. In particular, the new model's framework can be used to study the impact of both a change in the quality of recontracting and the size of transaction costs on payoffs and overall welfare. This can then be used to answer normative questions on the optimal quality of recontracting that parties would like to see in place. It is possible to derive (potentially) testable conclusions about the link between the quality of recontracting and the occurrence of recontracting and its effect on wages. Such questions on the optimal amount of recontracting can not be asked meaningfully in any of the earlier models of perfect recontracting since as shown in Proposition 4 recontracting does not happen in equilibrium.

Second, we make many simplifying assumptions on the way in which the agent interacts with the credit market. First, we assume that all lenders observe earlier contracts. Second, lending agreements are bilateral and are priced optimally, that is, taking into account all subsequent borrowing by the agent. This way of modelling is quite distinct from the conditions in real world financial markets in which the agent together with a large number of other investors anonymously trades financial contracts and in which market makers price these contracts keeping in mind that there are insiders among their customers.

9 Appendix

Proof. (Lemma 1) Trivially, if $\sigma(\omega)_1 - \omega_1 < E_{\rho(\omega,\sigma,e)}[\omega_2 - \sigma(\omega)_2]$, the entrepreneur can ask for a higher up-front payment, which is accepted by the lender, and otherwise follow the same strategy. If $\sigma(\omega)_1 - \omega_1 > E_{\rho(\omega,\sigma,e)}[\omega_2 - \sigma(\omega)_2]$, the contractor will reject. The agent is as well off if he offers the null contract. ■

Lemma 2 *If $\sigma(\cdot)$ is part of a subgame perfect equilibrium strategy and either (i) $\omega \in IC$ and $\sigma(\omega) \in \neg IC$, or (ii) $\omega \in \neg IC$ and $\sigma(\omega) \in IC$, then $\sigma^i(\omega) \equiv \sigma(\omega)$ for all $i \geq 1$.*

Proof. We use several steps.

1. Assume that strategy $\sigma(\cdot)$ leads to the following wage path in $\mathbb{R}^2 ... \to \omega_2 \to \omega_2' \to \omega_2'' \to ...$, where $\omega_2 \neq \omega_2' \neq \omega_2''$. Then, either of the following must be true:

a. $\omega_2' \in \neg IC$ and $\overline{\omega}_2 - \overline{\omega}_2' \leq \underline{\omega}_2 - \underline{\omega}_2'$

b. $\omega_2' \in IC$ and $\overline{\omega}_2 - \overline{\omega}_2' \geq \underline{\omega}_2 - \underline{\omega}_2'$.

Proof by contradiction. Assume otherwise. We show that the path $... \to \omega_2 \to \omega_2'' \to ...$ is feasible and better than $... \to \omega_2 \to \omega_2' \to \omega_2'' \to ....$ To save on notation, set $\rho = \rho((\omega_1, \omega_2), \sigma, e^*)$. Define ρ' and ρ'' similarly. From Lemma 1 we know that up-front payments for the moves from ω_2 to ω_2' and ω_2' to ω_2'' are $t_1' = E_{\rho'}[\omega_2 - \omega_2']$ and $t_1'' = E_{\rho''}[\omega_2' - \omega_2'']$ respectively. Then, path $... \to \omega_2 \to \omega_2' \to \omega_2'' \to ...$ feasible requires

$$\begin{aligned} \omega_1 + t_1' &\geq 0 \quad (30) \\ \omega_1 + t_1' + t_1'' &\geq 0, \quad (31) \end{aligned}$$

where ω_1 is the total amount of money that the agent has available at allocation ω. Feasibility of the path $... \to \omega_2 \to \omega_2'' \to ...$ requires

$$\omega_1 + E_{\rho''}[\omega_2 - \omega_2''] \geq 0.$$

It is easy to see that if $\omega' \in \neg IC$ ($\omega' \in IC$), then $\rho'' \geq \rho'$ ($\rho'' \leq \rho'$). Therefore if neither (a) nor (b) hold,

$$(\rho'' - \rho')(\overline{\omega}_2 - \overline{\omega}_2' - (\underline{\omega}_2 - \underline{\omega}_2')) \geq 0.$$

This is equivalent to

$$E_{\rho''}[\omega_2 - \omega_2'] \geq E_{\rho'}[\omega_2 - \omega_2'], \quad (32)$$

and therefore

$$\begin{aligned} \omega_1 + E_{\rho''}[\omega_2 - \omega_2''] &= \omega_1 + E_{\rho''}[\omega_2 - \omega_2'] + E_{\rho''}[\omega_2' - \omega_2''] \\ &\geq \omega_1 + t_1' + t_1'' \quad (33) \\ &\geq 0, \quad (34) \end{aligned}$$

where (33) follows from (32) and (34) follows from (31). So, the path ... $\to \omega_2 \to \omega_2'' \to$ ... is feasible as well.

We now show that it is actually preferred. Denote by $\bar{U}$ the utility of remaining at ω_2 (define $\bar{U}'$ and $\bar{U}''$ similarly), that is,

$$\bar{U} = \omega_1 + \delta E_{p(e^*(\omega))}[\omega_2] - c(e^*(\omega)) \tag{35}$$

Denote by U the utility of the path starting at ω_2 (define U' and U'' similarly), that is,

$$U = EU(\omega_1, \omega_2, \sigma, e^*) \tag{36}$$

Then, we can write the first path's payoff as

$$\omega_1 + (1-q)\bar{U} + q(t_1' + (1-q)\bar{U}' + q(t_1'' + U''))$$

and the second path's payoff as

$$\omega_1 + (1-q)\bar{U} + q(E_{\rho''}[\omega_2 - \omega_2''] + U'').$$

So, the second path is preferred to the first if

$$E_{\rho''}[\omega_2 - \omega_2''] + U'' \geq t_1' + (1-q)\bar{U}' + q(t_1'' + U'').$$

Since ... $\to \omega_2 \to \omega_2' \to \omega_2'' \to$ ... is assumed to be part of an equilibrium the move $\omega_2' \to \omega_2''$ must be weakly better than staying at ω_2', that is,

$$t_1'' + U'' \geq \bar{U}'. \tag{37}$$

Therefore,

$$\begin{aligned} E_{\rho''}[\omega_2 - \omega_2''] + U'' &\geq t_1' + t_1'' + U'' && (38) \\ &\geq t_1' + (1-q)\bar{U}' + q(t_1'' + U''). && (39) \end{aligned}$$

2. Assume that strategy $\sigma(\cdot)$ leads to the wage path ... $\to \omega_2 \to \omega_2' \to \omega_2'' \to$ ..., where either $\omega_2 \in IC$ and $\omega_2' \in \neg IC$ or $\omega_2 \in \neg IC$ and $\omega_2' \in IC$. That is, either $\overline{\omega}_2 - \overline{\omega}_2' > \underline{\omega}_2 - \underline{\omega}_2'$ or $\overline{\omega}_2 - \overline{\omega}_2' < \underline{\omega}_2 - \underline{\omega}_2'$. Then, 1a. or 1b. imply that $\omega_2'' = \omega_2'''$. By reiterating the argument in 1. it follows that also all the subsequent allocations must be equal to ω_2''. ■

Lemma 3 *At $\omega \in \neg IC$ the subgame perfect equilibrium strategy traces the wage path $\omega_2 \to \omega_2^0$.*

Proof. Assume $\omega \in \neg IC$. First, take any strategy $\sigma(\cdot)$ that satisfies Lemma 1 and for which $\sigma^i(\omega) \in \neg IC$ for all i. The expected utility from such a strategy is

$$\omega_1 - c_l + E_{p_l}[\omega_2] - (1-q)(1-\delta)\sum_{i=0}^{\infty} q^i E_{p_l}[\sigma^i(\omega)_2],$$

Clearly, the strategy $\sigma^*(\cdot)$ that traces the wage path $\omega_2 \to \omega_2^0$ (i.e. $\sigma^{*i}(\omega) \equiv \omega_2^0$ for all i) is optimal among those strategies.

We finalize the argument by showing that $\sigma(\omega) \in IC$ cannot be optimal. From Lemma 2 it follows that then $\sigma^i(\omega)_2 \equiv \omega_2^s \in IC$ for all i. For this to be an equilibrium strategy staying at ω_2^s must be credible. In particular, staying at ω_2^s must be at least as good as moving to ω_2^0, that is,

$$\delta E_{p_h}[\omega_2^s] - c_h \geq E_{p_l}[\omega_2^s] - c_l. \tag{40}$$

But then at ω, $\sigma^*(\cdot)$ is (weakly) preferred by the agent over $\sigma(\cdot)$, that is,

$$E_{p_l}[\omega_2] - c_l \geq E_{p_h}[\omega_2 - \omega_2^s] + \delta E_{p_h}[\omega_2^s] - c_h. \tag{41}$$

Inequality (41) follows from the following two observations. Since $\omega \in \neg IC$,

$$\delta E_{p_h}[\omega_2] - c_h < \delta E_{p_l}[\omega_2] - c_l. \tag{42}$$

This implies that (41) is true if

$$E_{p_h}[\omega_2] - E_{p_l}[\omega_2] \leq E_{p_h}[\omega_2^s].$$

But (40) and (42) together imply that

$$\delta(E_{p_h}[\omega_2] - E_{p_l}[\omega_2]) < \delta E_{p_h}[\omega_2^s] - E_{p_l}[\omega_2^s],$$

which proves the result. ■

Lemma 4 *At $\omega \in IC$ the subgame perfect equilibrium strategy traces either of the following wage paths:*

1. $\omega_2 \to \omega_2^{n-1} \to \omega_2^{n-2} \to \ldots \to \omega_2^1 \to \omega_2^0$ *for some $n \in \mathbb{N}$ and appropriately defined allocations $\{\omega_2^i\}_{i=1,\ldots,n-1} \in IC$.*

2. $\omega_2 \to \omega_2^s$ *for some appropriately defined $\omega_2^s \in IC$.*

Proof.

1. This follows trivially from combining Lemma 2 and Lemma 3.

2. Take a strategy that satisfies Lemma 1 and for which $\sigma^i(\omega) \in IC$ for all i. The expected utility from this strategy is

$$\omega_1 - c_h + E_{p_h}[\omega_2] - (1-q)(1-\delta)\sum_{i=0}^{\infty} q^i E_{p_h}[\sigma^i(\omega)_2].$$

If this strategy is to be part of an equilibrium the allocations $\sigma^i(\omega)_2$ must all have the same expected value $E_{p_h}[\sigma^i(\omega)_2]$. Assume otherwise, that is, assume that there exists a k with $E_{p_h}[\sigma^k(\omega)_2] > E_{p_h}[\sigma^i(\omega)_2]$ for all $i \neq k$. Then, it is easy to construct an argument as in Lemma 2, that shows that an equilibrium with a path that leaves out $\sigma^k(\omega)_2$, that is, $... \to \sigma^{k-1}(\omega)_2 \to \sigma^{k+1}(\omega)_2 \to ...$, is both feasible and preferred by the agent. It then follows that, since at each $\sigma^i(\omega)_2$ it is (weakly) optimal to move to $\sigma^{i+1}(\omega)_2$, at each $\sigma^i(\omega)_2$ it is also (weakly) optimal to stay at $\sigma^i(\omega)_2$, since the path $\sigma^i(\omega)_2 \to \sigma^{i+1}(\omega)_2 \to \sigma^{i+2}(\omega)_2 \to$ ...yields utility

$$\begin{aligned}
& \omega_1^i - c_h + E_{p_h}[\sigma^i(\omega)_2] - (1-q)(1-\delta)\sum_{j=i}^{\infty} q^{j-i} E_{p_h}[\sigma^j(\omega)_2] \\
= \; & \omega_1^i - c_h + E_{p_h}[\sigma^i(\omega)_2] - E_{p_h}[\sigma^i(\omega)_2](1-q)(1-\delta)\sum_{j=0}^{\infty} q^j \\
= \; & \omega_1^i - c_h + \delta E_{p_h}[\sigma^i(\omega)_2],
\end{aligned}$$

where ω_1^i is the accumulated date 1 payments at $\sigma^i(\omega)$ and the final line represents the utility from staying at $\sigma^i(\omega)_2$.

■

Lemma 5 *The allocations $\{\omega_2^n\}_n$ must be such that $\underline{\omega}_2^n = 0$ and $\overline{\omega}_2^n \geq \overline{\omega}_2^{n-1}$ for all n. Similarly, $\underline{\omega}_2^s = 0$.*

Proof. First, we show that $\underline{\omega}_2^n = 0$ for all n. Assume that there is an allocation ω_2^n such that $\underline{\omega}_2^n > 0$. Then we will show that the wage path $... \to \omega_2^{n+1} \to \omega_2^n \to \omega_2^{n-1} \to ... \to \omega_2^1 \to \omega_2^0$ is dominated by the path $... \to \omega_2'^{n+1} \to \omega_2'^n \to \omega_2'^{n-1} \to ... \to \omega_2'^1 \to \omega_2'^0$, where $\overline{\omega}_2'^n = \overline{\omega}_2^n - \underline{\omega}_2^n$, $\underline{\omega}_2'^n = 0$ and $\omega_2'^k = \omega_2^k$ for all $k \neq n$. A similar argument can be used to show that $\underline{\omega}_2^s = 0$.

We first need to show that the latter path is feasible.

A lender who takes the agent from ω_2^{n+1} to ω_2^n (or $\omega_2'^{n+1}$ to $\omega_2'^n$) evaluates the probability of high output at $\rho_{n+1} = (1-q^n)p_h + q^n p_l$. The lender who takes the agent from ω_2^n to ω_2^{n-1} (or $\omega_2'^n$ to $\omega_2'^{n-1}$) evaluates it at ρ_n. The corresponding up-front payments (see also Lemma 1) are

$$\begin{aligned}
t_1^n &= E_{\rho_{n+1}}[\omega_2^{n+1} - \omega_2^n] \\
t_1'^n &= E_{\rho_{n+1}}[\omega_2'^{n+1} - \omega_2'^n] = t_1^n + \underline{\omega}_2^n \\
t_1^{n-1} &= E_{\rho_n}[\omega_2^n - \omega_2^{n-1}] \\
t_1'^{n-1} &= E_{\rho_n}[\omega_2'^n - \omega_2'^{n-1}] = t_1^{n-1} - \underline{\omega}_2^n.
\end{aligned}$$

The path $... \to \omega_2^{n+1} \to \omega_2^n \to \omega_2^{n-1} \to ....$ feasible requires

$$\omega_1^{n+1} + t_1^n \geq 0 \tag{43}$$

$$\omega_1^{n+1} + t_1^n + t_1^{n-1} \geq 0, \tag{44}$$

where ω_1^{n+1} is the total amount of money that the agent has available at allocation ω^{n+1}. Feasibility of the path ... $\to \omega_2'^{n+1} \to \omega_2'^{n} \to \omega_2'^{n-1} \to$ ... requires

$$\begin{aligned} \omega_1^{n+1} + t_1'^{n} &\geq 0 && (45)\\ \omega_1^{n+1} + t_1'^{n} + t_1'^{n-1} &\geq 0. && (46) \end{aligned}$$

(45) follows from (43) and the expression derived above for $t_1'^{n}$, and (46) follows from (44) and the expressions derived for $t_1'^{n}$ and $t_1'^{n-1}$.

We now show that path ... $\to \omega_2'^{n+1} \to \omega_2'^{n} \to \omega_2'^{n-1} \to$ ... is preferred.

Denote by $\bar{U}^{n+1}$ the utility of remaining at ω_2^{n+1} (define $\bar{U}^{n}$ and $\bar{U}^{n-1}$ similarly) and denote by U^{n+1} the utility of the path starting at ω_2^{n+1} when following the assumed wage path (define U^{n} and U^{n-1} similarly).[12] The first path yields utility

$$\omega_1^{n+1} + (1-q)\bar{U}^{n+1} + q(t_1^{n} + (1-q)\bar{U}^{n} + q(t_1^{n-1} + U^{n-1})). \qquad (47)$$

The second path yields utility

$$\omega_1^{n+1} + (1-q)\bar{U}^{n+1} + q(t_1'^{n} + (1-q)\bar{U}'^{n} + q(t_1'^{n-1} + U^{n-1}). \qquad (48)$$

So, the second path is preferred to the first if

$$t_1'^{n} + (1-q)\bar{U}'^{n} + qt_1'^{n-1} \geq t_1^{n} + (1-q)\bar{U}^{n} + qt_1^{n-1}. \qquad (49)$$

or equivalently if

$$t_1'^{n} - t_1^{n} + q(t_1'^{n-1} - t_1^{n-1}) \geq (1-q)\delta\underline{\omega}_2^{n}.$$

Substituting the expressions we have derived for the up-front payments this inequality becomes

$$(1-q)\underline{\omega}_2^{n} \geq (1-q)\delta\underline{\omega}_2^{n},$$

which is satisfied for all $\delta \leq 1$.

Second, we show that for a wage path $\omega_2 \to \omega_2^{n-1} \to \omega_2^{n-2} \to \ldots \to \omega_2^{1} \to \omega_2^{0}$ to be optimal, we must have $\overline{\omega}_2 \geq \overline{\omega}_2^{n-1}$. Setting $\overline{\omega}_2 = \overline{\omega}_2^{n}$ proves the second part of the Lemma's statement. Assume the contrary. Then we show that the wage path $\omega_2 \to \omega_2'^{n-1} \to \omega_2'^{n-2} \to \ldots \to \omega_2'^{1} \to \omega_2'^{0}$ dominates the first path, where $\omega_2'^{n-1} = \omega_2$ and $\omega_2'^{k} = \omega_2^{k}$ for all $k \neq n-1$

From the first part of this lemma, we know that $\underline{\omega}_2^{n} = 0$ for all n. The up-front payments for the first and second lender respectively in the two wage paths can be calculated as in the first part of this proof using Lemma 1:

$$\begin{aligned} t_1^{n-1} &= E_{\rho_n}[\overline{\omega}_2 - \overline{\omega}_2^{n-1}]\\ t_1'^{n-1} &= 0\\ t_1^{n-2} &= E_{\rho_{n-1}}[\overline{\omega}_2^{n-1} - \overline{\omega}_2^{n-2}]\\ t_1'^{n-2} &= E_{\rho_{n-1}}[\overline{\omega}_2 - \overline{\omega}_2^{n-2}] \end{aligned}$$

[12] See also the formal definitions in (35) and (36).

The path $\omega_2 \to \omega_2^{n-1} \to \omega_2^{n-2} \to ....$ feasible requires

$$\omega_1 + t_1^{n-1} + t_1^{n-2} \geq 0. \tag{50}$$

The path $\omega_2 \to \omega_2'^{n-1} \to \omega_2'^{n-2} \to ...$ feasible requires

$$\omega_1 + t_1'^{n-1} + t_1'^{n-2} \geq 0. \tag{51}$$

By using the expressions derived above for the up-front payments (51) will follow from (50) if

$$\rho_{n-1}(\Delta\omega_2 - \overline{\omega}_2^{n-1}) \geq \rho_n(\Delta\omega_2 - \overline{\omega}_2^{n-1}), \tag{52}$$

But (52) is true since $\rho_{n-1} \leq \rho_n$ and $\Delta\omega_2 \leq \overline{\omega}_2 < \overline{\omega}_2^{n-1}$ by assumption.

We now show that the path $\omega_2 \to \omega_2'^{n-1} \to \omega_2'^{n-2} \to ...$ is preferred.

This is true if the expected utility derived from the second path is larger than the expected utility derived from the first part, i.e. if

$$(1-q)\delta(p_h\overline{\omega}_2+(1-p_h)\underline{\omega}_2)+q(t_1'^{n-2}+U^{n-2}) \geq t_1^{n-1}+(1-q)\delta p_h\overline{\omega}_2^{n-1}+q(t_1^{n-2}+U^{n-2}).$$

Again, by replacing the expressions for the up-front payments, this is equivalent to:

$$(1-q)\delta(p_h(\Delta\overline{\omega}_2 - \overline{\omega}_2^{n-1}) + \underline{\omega}_2) \geq (1-q)(p_h(\Delta\omega_2 - \overline{\omega}_2^{n-1}) + \underline{\omega}_2),$$

which is satisfied for all $\delta \leq 1$. ∎

Before proving Proposition 2 we derive a useful simplification of $EU(\omega, \sigma_n, e^*)$, for $n \geq 1$ and $\omega \in IC$. To save on notation, we set $\omega_1 = 0$. For a given set of allocations $\{\omega_2^i\}_{i=1,\ldots,n-1}$ and assuming that the agent follows strategy $\sigma_n(\cdot)$, we can, compute the cumulative up-front payments that the agent will have received after i rounds of recontracting, $1 \leq i \leq n$, using Lemma 1:

$$\omega_1^{n-i}(\omega, \sigma_n) = E_{\rho_n}[\omega_2 - \omega_2^{n-1}] + \sum_{j=1}^{i-1} E_{\rho_{n-j-1}}[\omega_2^{n-j} - \omega_2^{n-j-1}]. \tag{53}$$

In what follows, we will simplify notation by setting $\omega^{n-i} := (\omega_1^{n-i}(\omega, \sigma_n), \omega_2^{n-i})$. Then,

$$\begin{aligned}
EU(\omega, \sigma_n, e^*) &= (1-q)U\left(\omega, e^*(\omega)\right) + (1-q)\sum_{i=1}^{n-1} q^i U\left(\omega^{n-i}, e^*(\omega^{n-i})\right) \qquad (54)\\
&\qquad + q^n U\left(\omega^0, e^*(\omega^0)\right)\\
&= qE_{\rho_n}[\omega_2 - \omega_2^{n-1}] + \sum_{i=1}^{n-1} q^{i+1} E_{\rho_{n-i}}[\omega_2^{n-i} - \omega_2^{n-i-1}] \qquad (55)\\
&\qquad +(1-q)(\delta E_{p_h}[\omega_2] - c_h)) + (1-q)\sum_{i=1}^{n-1} q^i(\delta E_{p_h}[\omega_2^{n-i}] - c_h)) - q^n c_l\\
&= qE_{\rho_n}[\omega_2] + (1-q)\delta E_{p_h}[\omega_2] - (1-q)(1-\delta)\sum_{i=1}^{n-1} q^i E_{p_h}[\omega_2^{n-i}] \qquad (56)\\
&\qquad -((1-q^n)c_h + q^n c_l),
\end{aligned}$$

where (55) follows from (53) and (56) follows from

$$qE_{\rho_{n-i}}[\omega_2^{n-i}]-E_{\rho_{n-i+1}}[\omega_2^{n-i}]+(1-q)\delta E_{p_h}[\omega_2^{n-i}] = -(1-q)(1-\delta)E_{p_h}[\omega_2^{n-i}]. \quad (57)$$

We now provide the proof of Proposition 2:

Proof. (Proposition 2) The proof is via induction. We show that for any n we have

1_n The allocations $\{\omega_2^i\}_{i=0,\ldots,n}$ are defined by

$$EU((\omega_1,\omega_2^i),\sigma_i,e^*) = EU((\omega_1,\omega_2^i),\sigma_{i-1},e^*),$$

and for $2 \leq i \leq n$

$$\overline{\omega}_2^i = \frac{(1-\delta)^{i-1}p_h^{i-2}p_l}{\delta q^{(i-1)(i-2)/2}\Delta p^i}\Delta c + \overline{\omega}_2^{i-1}. \quad (58)$$

2_n Strategy σ_i, $1 \leq i \leq n-1$, is optimal on the vertical axis between ω_2^i and ω_2^{i+1}.

3_n Strategy σ_n is the optimal strategy at ω_2^n.

4_n Strategy σ_n dominates σ_i, $1 \leq i \leq n-1$, on the vertical axis above ω_2^n.

5_n If

$$\theta_n := \delta p_h - \rho_n > 0,$$

the allocations $\{\omega_2^{is}\}_{i=1,\ldots,n}$ defined by $\underline{\omega}_2^{is} = 0$ and

$$EU((\omega_1,\omega_2^{is}),\sigma_i,e^*) = EU((\omega_1,\omega_2^{is}),\sigma_0,e^*)$$

are well defined and for $2 \leq i \leq n$

$$\overline{\omega}_2^{is} = \frac{q^{i-1}\Delta p}{\theta_i}\frac{(1-\delta)^{i-1}p_h^{i-2}p_l}{\delta q^{(i-1)(i-2)/2}\Delta p^i}\Delta c + \overline{\omega}_2^{i-1}. \quad (59)$$

We have shown most of the Inductive Hypothesis for $n = 2$ in the discussion preceding Proposition 2. It remains to prove 5_n for $n = 2$. The argument proceeds similarly to the one that shows how to obtain ω_2^{1s}. Since, it will be provided below for general n, we won't repeat it here and just note that by equating (12) and (15) we obtain,

$$\begin{aligned}
\delta E_{p_h}[\omega_2^{2s}] - c_h &= E_{\rho_2}[\omega_2^{2s} - \omega_2^1] + (1-q)(\delta E_{p_h}[\omega_2^1] - c_h) + q(E_{p_l}[\omega_2^1] - c_l) \\
&\Leftrightarrow \\
\theta_2\overline{\omega}_2^{2s} &= ((1-q)\delta p_h - \rho_2 + qp_l)\overline{\omega}_2^1 + q\Delta c
\end{aligned}$$

Therefore, only if $\theta_2 > 0$ is ω_2^{2s} well defined and is given by

$$\overline{\omega}_2^{2s} = \frac{q(1-\delta)p_l}{\theta_2 \delta \Delta p}\Delta c + \overline{\omega}_2^1. \tag{60}$$

We now show the Inductive Step, namely that if $1_n - 5_n$ are true for some n, then $1_{n+1} - 5_{n+1}$ are also true. We proceed exactly as in the discussion preceding Proposition 2:

Given ω_2^n, strategy $\sigma_{n+1}(\cdot)$ is defined as the strategy inducing wage path $\omega_2 \to \omega_2^n \to ... \to \omega_2^1 \to \omega_2^0$. Then, since $\sigma_n(\cdot)$ is optimal at ω_2^n we know that it must yield a higher payoff than $\sigma_{n+1}(\cdot)$. Then, from ω_2^n move upwards on the vertical axis, i.e. increase $\overline{\omega}_2$. Using (56) we see that $\sigma_{n+1}(\cdot)$'s payoff increases by $(1-q)\delta p_h + q\rho_{n+1}$, whereas $\sigma_n(\cdot)$'s payoff increases by $(1-q)\delta p_h + q\rho_n$. Since $\rho_{n+1} > \rho_n$, the marginal increase in strategy $\sigma_{n+1}(\cdot)$'s payoff from an increase in $\overline{\omega}_2$ is larger than the marginal increase in strategy $\sigma_n(\cdot)$'s payoff. Thus, there exists an allocation ω_2^{n+1} on the vertical axis with $\overline{\omega}_2^{n+1} > \overline{\omega}_2^n$, at which strategies $\sigma_{n+1}(\cdot)$ and $\sigma_n(\cdot)$ yield the same payoff. Everywhere above ω_2^{n+1} on the vertical axis strategy $\sigma_{n+1}(\cdot)$ dominates $\sigma_n(\cdot)$. This proves 4_{n+1} since by the Inductive Hypothesis 4_n, $\sigma_n(\cdot)$ dominates all $\sigma_i(\cdot)$ with $1 \leq i \leq n-1$.

We now write (58) for $2 \leq i \leq n$ as

$$\overline{\omega}_2^i = \frac{(1-\delta)p_l \Delta c}{\delta \Delta^2 p} \sum_{j=1}^{i-2} \left(\frac{(1-\delta)p_h}{q^{(j+1)/2}\Delta p} \right)^j + \overline{\omega}_2^2.$$

and show that (58) also holds for $i = n+1$. Using (56), $\overline{\omega}_2^{n+1}$ is defined by

$$\begin{aligned}
q(\rho_{n+1} - \rho_n)\overline{\omega}_2^{n+1} &= (1-q)(1-\delta)p_h \sum_{i=1}^{n} q^i (\overline{\omega}_2^{n+1-i} - \overline{\omega}_2^{n-i}) + (1-q)q^n \Delta c \\
&\Longleftrightarrow \\
\overline{\omega}_2^{n+1} &= \tfrac{(1-\delta)p_h}{q^n \Delta p} \textstyle\sum_{i=1}^{n-1} q^i \left(\tfrac{(1-\delta)^{n-i} p_h^{n-i-1} p_l}{\delta q^{(n-i)(n-i-1)/2} \Delta p^{n+1-i}} \Delta c \right) \\
&\qquad + \tfrac{(1-\delta)p_l}{\delta \Delta p^2}\Delta c + \overline{\omega}_2^1 &(61)\\
&= \textstyle\sum_{i=1}^{n-1} \tfrac{(1-\delta)^{n+1-i} p_h^{n-i} p_l}{\delta q^{(n+1-i)(n-i)/2} \Delta p^{n+2-i}} \Delta c \\
&\qquad + \tfrac{(1-\delta)p_l}{\delta \Delta p^2}\Delta c + \overline{\omega}_2^1 &(62)\\
&= \frac{(1-\delta)^n p_h^{n-1} p_l}{\delta q^{n)(n-1)/2} \Delta p^{n+1}} \Delta c + \overline{\omega}_2^n &(63)
\end{aligned}$$

where (61) follows from the definition of $\overline{\omega}_2^{n-i}$ for $i = 2, ..., n$ and

$$q(\rho_{n+1} - \rho_n) = (1-q)q^n \Delta p.$$

The second line (62) follows from

$$\frac{q^i}{q^n q^{(n-i)(n-i-1)/2}} = \frac{1}{q^{(n+1-i)(n-i)/2}}$$

and (63) again uses the definition of $\overline{\omega}_2^{n-i}$ for $i = 2, ..., n$. This concludes 1_{n+1}.

Consider now the section of the vertical axis between ω_2^n and ω_2^{n+1}. From the above discussion we know that $\sigma_n(\cdot)$ dominates $\sigma_{n+1}(\cdot)$ on this section of the vertical axis. By the Inductive Hypothesis 4_n, $\sigma_n(\cdot)$ dominates all $\sigma_i(\cdot)$, $i < n$ and Property 2 and the proof of Lemma 5 imply that it also dominates all strategies $\sigma_i(\cdot)$, $i > n+1$. To show that it is indeed the optimal strategy on this section we need to show that ω_2^{ns}, the allocation at which strategies $\sigma_0(\cdot)$ and $\sigma_n(\cdot)$ yield the same payoff, lies above ω_2^{n+1}, i.e. we need to show that $\overline{\omega}_2^{n+1} < \overline{\omega}_2^{ns}$. This would imply that strategy $\sigma_0(\cdot)$ is not optimal on the vertical axis between ω_2^n and ω_2^{n+1} and that consequently strategy $\sigma_s(\cdot)$ is not credible on this section. Therefore, $\sigma_n(\cdot)$ is the subgame perfect strategy on the vertical axis between ω_2^n and ω_2^{n+1}.

We obtain the expression for $\overline{\omega}_2^{ns}$ from (59) for $i = n$, which is true by the Inductive Hypothesis. Then, using (63), $\overline{\omega}_2^{n+1} < \overline{\omega}_2^{ns}$ is equivalent to

$$\frac{(1-\delta)^n p_h^{n-1} p_l}{\delta q^{n(n-1)/2} \Delta p^{n+1}} \Delta c + \overline{\omega}_2^n < \frac{q^{n-1} \Delta p}{\theta_n} \frac{(1-\delta)^{n-1} p_h^{n-2} p_l}{\delta q^{(n-1)(n-2)/2} \Delta p^n} \Delta c + \overline{\omega}_2^{n-1}.$$

Using (58) for $i = n$, is in turn equivalent to

$$\frac{(1-\delta)^n p_h^{n-1} p_l}{\delta q^{n(n-1)/2} \Delta p^{n+1}} \Delta c + \frac{(1-\delta)^{n-1} p_h^{n-2} p_l}{\delta q^{(n-1)(n-2)/2} \Delta p^n} \Delta c < \frac{q^{n-1} \Delta p}{\theta_n} \frac{(1-\delta)^{n-1} p_h^{n-2} p_l}{\delta q^{(n-1)(n-2)/2} \Delta p^n} \Delta c.$$

Since $\theta_n = \delta p_h - \rho_n$, this is equivalent to

$$-(1-\delta)^2 p_h^2 < 0,$$

which is trivially true. This concludes 2_{n+1}.

3_{n+1} now follows easily since from 2_{n+1} we know that $\sigma_n(\cdot)$ is optimal on the vertical axis between ω_2^n and ω_2^{n+1}, and ω_2^{n+1} is defined as the allocation at which $\sigma_n(\cdot)$ and $\sigma_{n+1}(\cdot)$ yield the same payoff. Therefore, $\sigma_{n+1}(\cdot)$ is optimal at ω_2^{n+1}.

The last step is to show 5_{n+1}. We proceed as before. We know that at ω_2^{n+1} strategy $\sigma_{n+1}(\cdot)$ is optimal and therefore yields a higher payoff than $\sigma_0(\cdot)$. Then, moving upwards from ω_2^{n+1} on the vertical axis, $\sigma_0(\cdot)$'s payoff increases by δp_h (see (12)), whereas $\sigma_{n+1}(\cdot)$'s payoff increases by $(1-q)\delta p_h + q\rho_{n+1}$. If $\delta p_h \leq \rho_{n+1}$, i.e. $\theta_{n+1} \leq 0$, there is never an allocation at which $\sigma_0(\cdot)$ dominates $\sigma_{n+1}(\cdot)$. If on the other hand $\theta_{n+1} > 0$, $\sigma_0(\cdot)$'s payoff increases faster than $\sigma_{n+1}(\cdot)$'s payoff, and we can find an allocation ω_2^{n+1s}, such that everywhere above it $\sigma_0(\cdot)$ dominates and everywhere below it $\sigma_{n+1}(\cdot)$ dominates. Note, that since $\theta_1 > \theta_2 > ... > \theta_{n+1}$ we know that if $\theta_{n+1} > 0$, also $\theta_i > 0$ for all $i = 1, ..., n$. Therefore, if $\theta_{n+1} > 0$, by the Inductive Hypothesis, (59) holds for all $i = 1, ..., n$.

We now want to show that it also holds for $n+1$. For this, it is first useful to note that by equating (56) and (12) we get for all j:

$$\theta_j \overline{\omega}_2^{js} = -(1-q)(1-\delta) p_h \sum_{k=0}^{j-2} q^k \overline{\omega}_2^{j-k-1} + q^{j-1} \Delta c. \tag{64}$$

Therefore,

$$\begin{aligned}
\theta_{n+1}\overline{\omega}_2^{n+1s} &= -(1-q)(1-\delta)p_h\sum_{k=0}^{n-1}q^k\overline{\omega}_2^{n-k-1}+q^n\Delta c & (65)\\
&= -(1-q)(1-\delta)p_h\overline{\omega}_2^n+q\theta_n\overline{\omega}_2^{ns} & (66)\\
&= -(1-q)(1-\delta)p_h\overline{\omega}_2^n & (67)\\
&\qquad +q\left(\theta_n\overline{\omega}_2^{n-1}+q^{n-1}\Delta p\frac{(1-\delta)^{n-1}p_h^{n-2}p_l}{\delta q^{(n-1)(n-2)/2}\Delta p^n}\right) & \\
&= -(1-q)(1-\delta)p_h\overline{\omega}_2^n+q^n\Delta p\overline{\omega}_2^n-q(1-\delta)p_h\overline{\omega}_2^{n-1} & (68)\\
&= \theta_{n+1}\overline{\omega}_2^n+q\frac{(1-\delta)^n p_h^{n-1}p_l}{\delta q^{(n-1)(n-2)/2}\Delta p^n} & (69)
\end{aligned}$$

where (65) and (66) follow from (64) for $j = n+1, n$, (67) follows from the Inductive Hypothesis, and (68) and (69) both follow from (58) for $i = n, n-1$ and $\theta_{n+1} = \delta p_h - \rho_{n+1}$. The expression (59) for $n+1$ then follows because

$$\frac{q}{q^{(n-1)(n-2)/2}} = \frac{q^n}{q^{n(n-2)/2}}.$$

This proves 5_{n+1}.

Claim 2_n for all n describes the subgame perfect equilibrium strategy on the vertical axis in IC. To extend this to the entire IC region, the same proof is repeated (including the discussion preceding Proposition 2, which proves the Inductive Hypothesis) by fixing $\underline{\omega}_2$ and then starting the proof at the boundary with the $\neg IC$ region, i.e. at $(\overline{\omega}_2^1 + \underline{\omega}_2, \underline{\omega}_2)$. ■

Proof. (Proposition 5)

We need to prove the remaining claim, namely that $\kappa := \lim_{q\to 1} q^{n(q)} = \frac{(1-\delta)^2 p_h^2}{\Delta^2 p}$. Assume to the contrary that the equilibrium for all $q < 1$ is characterized by a wage $w(q) = (0, \overline{w}(q), 0)$ and a corresponding number of lenders $n(q)$ (i.e. $\overline{w}(q) = \overline{\omega}_2^{n(q)}(q)$), such that $\kappa = \lim_{q\to 1} q^{n(q)} > \frac{(1-\delta)^2 p_h^2}{\Delta^2 p}$. Then, we want to show that for all q sufficiently close to 1, the principal can profitably deviate from this equilibrium by offering a more incentivized wage $w'(q)$ with $\overline{w}'(q) = \overline{\omega}_2^{n'(q)}(q) > \overline{w}(q)$ and by inducing a higher number of lenders $n'(q) > n(q)$. For this choose $n'(q)$ such that $\kappa' := \lim_{q\to 1} q^{n'(q)} \geq \frac{(1-\delta)^2 p_h^2}{\Delta^2 p}$. Since $n'(q) > n(q)$ we know that $q^{n'(q)} < q^{n(q)}$ and consequently $\kappa' = \lim_{q\to 1} q^{n'(q)} < \kappa$.

To show that this is indeed a profitable deviation for the principal we show that it raises his payoff. Reconsider the principal's maximization problem in (26). By making the simplifying assumption[13] $w_1(q) = 0$, we can write his payoff for

[13] This does not affect the results.

all $q < 1$ as a function of the chosen high wage $\overline{\omega}_2^n(q)$:

$$\begin{aligned}\Pi^n(q) &= \rho_{n+1}(\overline{x} - \overline{\omega}_2^n(q)) + (1 - \rho_{n+1})\underline{x} \\ &= \underline{x} + \rho_{n+1}(\Delta x - \overline{\omega}_2^n(q)) \\ &= \underline{x} + (p_h - q^n \Delta p)(\Delta x - \overline{\omega}_2^n(q)). \end{aligned} \tag{70}$$

The deviation increases the first bracket in (70) since it increases the probability of high effort and thus the probability of high output and it decreases the second bracket in (70) since the principal pays a higher wage in case of high output. We argue that the second effect can be made arbitrarily small while keeping the first effect bounded away from 0 for all q sufficiently close to 1.

We can write the difference in the principal's payoff when inducing $n'(q)$ as opposed to $n(q)$ recontracting agreements as

$$\begin{aligned}\Pi^{n'(q)}(q) - \Pi^{n(q)}(q) &= (q^{n'(q)}\Delta p - p_h)(\overline{\omega}_2^{n'(q)}(q) - \overline{\omega}_2^{n(q)}(q)) \\ &\quad + (q^{n(q)} - q^{n'(q)})\Delta p(\Delta x - \overline{\omega}_2^{n(q)}(q)).\end{aligned}$$

The second term on the right-hand-side of this expression is positive and stays bounded away from zero when q approaches 1, since $\Delta x - \overline{\omega}_2^{n(q)}(q) > 0$ for all q (a necessary condition for $\overline{\omega}_2^{n(q)}(q)$ to be the equilibrium wage) and $q^{n(q)} - q^{n'(q)}$ approaches $\kappa - \kappa' > 0$. The first term on the right-hand-side is negative, since $\overline{\omega}_2^{n'(q)}(q) > \overline{\omega}_2^{n(q)}(q)$ and $q^{n'(q)}\Delta p - p_h < 0$ (necessary for $\Pi^{n'(q)}(q)$ to be a profitable deviation, see (70)). We show that it can be made arbitrarily small. To see this, note first that the first bracket remains bounded by $\kappa'\Delta p - p_h$. The second bracket can be written as follows (see (27)):

$$\overline{\omega}_2^{n'(q)}(q) - \overline{\omega}_2^{n(q)}(q) = \frac{(1-\delta)p_l\Delta c}{\delta\Delta^2 p} \sum_{i=n(q)-1}^{n'(q)-2} \left(\frac{(1-\delta)p_h}{q^{(i+1)/2}\Delta p}\right)^i. \tag{71}$$

Since by definition $\lim_{q\to1} q^{n'(q)} = \kappa'$, there must exist a neighborhood around $q = 1$, such that for all $q < 1$ in this neighborhood the sum in (71) is bounded from above by

$$\begin{aligned}&\frac{(1-\delta)p_l\Delta c}{\delta\Delta^2 p} \sum_{i=n(q)-1}^{n'(q)-2} \left(\frac{(1-\delta)p_h}{\kappa'\Delta p}\right)^i \\ = &\frac{(1-\delta)p_l\Delta c}{\delta\Delta^2 p} \frac{\left(\frac{(1-\delta)p_h}{\kappa'\Delta p}\right)^{n(q)-1} - \left(\frac{(1-\delta)p_h}{\kappa'\Delta p}\right)^{n'(q)-1}}{1 - \frac{(1-\delta)p_h}{\kappa'\Delta p}}\end{aligned}$$

For q tending to 1, the denominator of this expression tends to 0, whereas the numerator is fixed and lies between 0 and 1, which proves that (71) can be made arbitrarily small.

Finally, since this argument can be applied to any $n(q)$ with $\lim_{q\to1} q^{n(q)} > \frac{(1-\delta)^2 p_h^2}{\Delta^2 p}$, we conclude that in equilibrium $\kappa = \lim_{q\to1} q^{n(q)} = \frac{(1-\delta)^2 p_h^2}{\Delta^2 p}$. ■

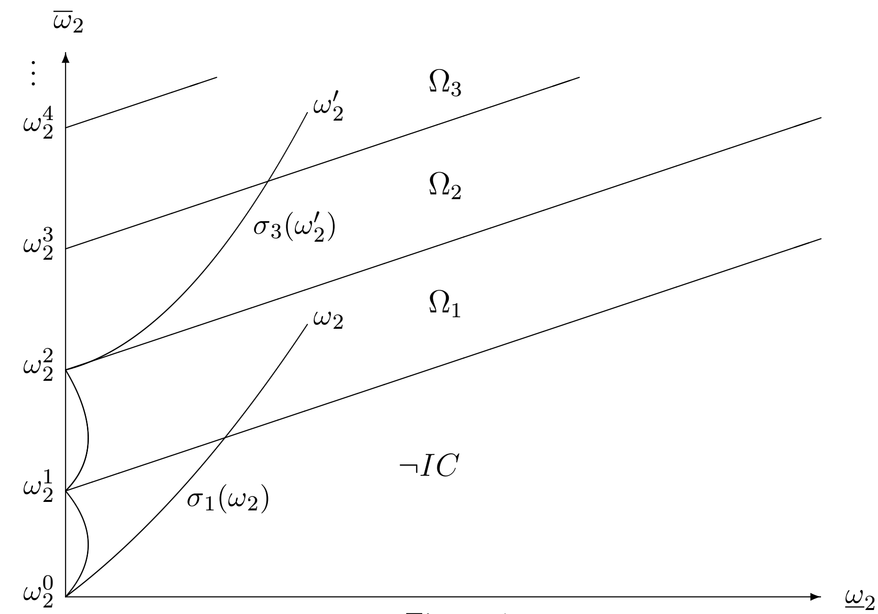

Figure 1

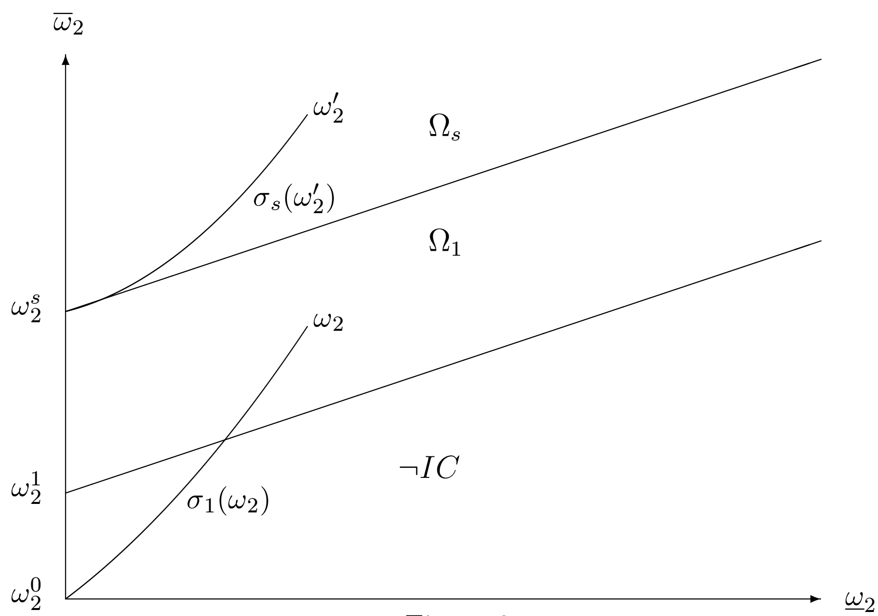

Figure 2

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