

The Principle of Moderate Differentiation

Susanna E. Sällström Matthews

Faculty of Economics University of Cambridge,
and St John's College, Cambridge, CB2 1TP, UK.

May 3, 2007

Abstract

What would happen if firms could collusively choose cost of transport (inconvenience) in Hotelling's spatial model? This paper endogenises inconvenience in a three stage game, where firms choose locations, the inconvenience, and finally compete in price, on the assumption of a common reservation price. The equilibrium of the game reveals a novel mechanism which induces firms to differentiate their products in moderation by locating halfway to the center and choosing inconvenience such that the market remains covered in equilibrium. Furthermore, using Launhardt's model with differential freight rate, it is shown that the collusive inconvenience is a Nash equilibrium.

Keywords: spatial differentiation, location, market structure, cost of transport, inconvenience, freight rate, business strategies

JEL Classification: L11, L13, D43, D21

*I am grateful to seminar participants at Cambridge, the Institute for Industrial Economics in Stockholm, European Association for Research in Industrial Economics Annual Meeting at Helsinki 2003, Swedish Competition Authority in Stockholm, the University of Keele and the Scottish Economic Society Meeting in Perth 2004. The paper has been greatly enhanced after comments from two anonymous referees and the Editor.

1 Introduction

This paper investigates the implications of the following famous quote:

‘These particular merchants would do well, instead of organizing improvement clubs and booster associations to better the roads, to make transportation as difficult as possible.’¹ [p. 50 Hotelling (1929)]

Whilst being a very old insight (see e.g. Cantillon (1755)² few attempts have been made to endogenise the inconvenience parameter in spatial models of product differentiation.³ Given that inconvenience can be influenced when the parameter represents for example cost of customising the good, or the probability of a having to return a faulty item, the question remains, what if firms were to follow Hotelling’s advice? How difficult should the merchants collectively make transportation? Furthermore, will it influence their choice of locations, and thereby possibly resolve the existence problem? Finally, how stable is the collusive inconvenience?

I answer these questions in a three stage version of Hotelling’s linear cost of transport model with the addition of a common reservation price. In the first period the firms independently choose locations which are fixed forever, they then jointly choose the inconvenience parameter⁴, and finally compete in price. To avoid singularity problems associated with the uniform distribution I solve the model for a class of distribution functions which in the

¹This is a property of both linear and quadratic cost of transport as was formally derived in d’Aspremont, Gabszewicz and Thisse [1979].

²Cantillon is the oldest reference I found describing the mechanism with a tailor pushing up prices ‘till the Villagers find it to their advantage to have their cloaths made in another Village, Town or City losing the time spent going and returning’ [p.21].’ See Hébert (1981).

³The only examples are models of differential freight rate based on Launhardt’s (1885) and Cheysson (1887) shipping models. See Dos Santos Ferreira (1998).

⁴This might involve lobbying for less money being spent on public transport, or in the case of customizing the good, a technology for customising the good in a research joint venture.

limit are the uniform. I find that the firms will locate halfway to the center, and choose the inconvenience such that the market remains covered in equilibrium. Furthermore, using Launhardt's (1885) model with differential freight rate I show that the collusive inconvenience is a unique non-cooperative Nash equilibrium in pure strategies for the equilibrium locations.

The collusive choice of inconvenience, in a setting where firms compete in price, is an important topic since it analyses the effects of collusive behaviour in markets where active anti-trust prevents firms from colluding in price. Instead firms will choose product characteristics in such a way that they can credibly commit to not being too aggressive in their pricing strategies. This is done by locating halfway to the center which minimises the distance that marginal customers have to travel and therefore allows the firms to maximise the inconvenience, and thus prices, subject to keeping the market covered. It should be noted that since these locations are determined by the marginal rather than the average distance to travel they only coincide with the socially efficient ones for a uniform distribution of consumers.⁵ The benefits are due to higher inconvenience being both a strategic commitment to a firm itself to be less aggressive, as well as inducing its competitor to be less aggressive.⁶ Furthermore, since these strategic benefits require that the firms are indeed competing for the customers, there is no incentive to make inconvenience so high that the firms become local monopolists. The key assumption behind the principle of moderate differentiation is thus the combination of elastic demand, endogenous inconvenience, and endogenous locations. Other contributions in the field have identified existence problems and solutions at the extremes when they exist, i.e. minimum or maximum differentiation, in scenarios where at least one of these assumptions do not apply.

⁵This should be contrasted with Lederer and Hurter (1986) who have a model of spatial price discrimination where the incentive to choose efficient locations comes from minimising the average cost of travelling.

⁶Which can be seen in the analysis of differential inconvenience.

The existence problem has been neatly dealt with by amongst others Dasgupta and Maskin (1986), Caplin and Nalebuff (1991), Graitson (1982), and Shaked (1975, 1982), with the general conclusion that there indeed exists no pure strategy equilibrium, but that a mixed strategy equilibrium can be derived. The classic example of maximum differentiation is due to D'Aspremont, Gabszewicz, and Thisse (1979) who showed that the existence problem in Hotelling's model would be resolved by assuming quadratic cost of transport and result in firms locating at the end points.⁷ It is, however, interesting to note that the principle of moderate differentiation would apply to the quadratic case as well.⁸ Hotelling (1929) constructed his model to explain the phenomenon of excessive sameness. Examples of contributions confirming his intuition in different settings include Eaton and Lipsey (1979) (for several firms), de Palma et al. (1985) (in probabilistic logit model), Stahl (1982) (with consumer search), Friedman and Thisse (1993) (with semi-collusion) and Aguirre and Espinoza (2003) (with endogenous point of delivery), Christou and Vettas (2004) (with quality uncertainty). Gal-Or (1982) has an interesting interpretation of the model as a model of sales. Another classic contribution is Rotschild (1979) who considered sequential entry and existence problems.

An example of endogenous freight rate appears in Dos Santos Ferreira and Thisse (1996) who find that firms that are located at the end points will choose the maximum freight rate, in Launhardt's (1885) shipping model on the assumption of two fixed locations and an upper limit to the freight rate. The implication of these assumptions is that there is no link between location and the maximum freight rate that can be charged and therefore no incentive to differentiate in moderation.

Lerner and Singer (1937) in their reply to Hotelling's contribution identified the problem of assuming inelastic demand and suggested that consumers would have a finite reservation

⁷Another example of excessive differentiation can be found in Devetoglou (1965), who considered the case of two dimensions.

⁸See the discussion.

price. Since they assumed inconvenience to be exogenously given they and more recent contributions by Economides (1984,1986), Hinloopen and van Marrewijk (1999) and Hinloopen (2002), have derived existence in settings where the market may be uncovered. Once inconvenience is endogenised those outcomes can be ruled out since in equilibrium the firms will choose to keep the market covered as is shown in this paper.

Outline of the paper. Section 2 presents the model and derives price equilibria for various combinations of locations and inconvenience. Section 3 derives collusive inconvenience and shows that it is a unique Nash equilibrium for the equilibrium locations. Section 4 finally solves for the location, and shows that there exists an equilibrium for the three stage game characterised by the principle of moderate differentiation. The paper concludes with a discussion of model assumptions and the interpretation of the results. More extensive proofs can be found in the appendix.

2 The Model

Let the length of the market be 1, a and b be the distance from the end points of firm A and B respectively, and $r = (1 - a - b)$ the distance between the firms. Let p_a and p_b denote the prices charged by firm A and B respectively. The production technology of the firms is constant returns to scale with a marginal cost $c \geq 0$.

Consumers are distributed on $x \in [0, 1]$ according to the distribution function

$$F(x) = \begin{cases} x \frac{1 - h(1 - 2z)}{2z} & \text{if } x \in [0, z) \\ \frac{1 - h(1 - 2x)}{2} & \text{if } x \in [z, 1 - z] \\ 1 - (1 - x) \frac{1 - h(1 - 2z)}{2z} & \text{if } x \in (1 - z, 1] \end{cases} \quad (1)$$

with density

$$f(x) = \begin{cases} \frac{1 - h(1 - 2z)}{2z} & \text{if } x \in [0, z) \\ h & \text{if } x \in [z, 1 - z] \\ \frac{1 - h(1 - 2z)}{2z} & \text{if } x \in (1 - z, 1], \end{cases} \quad (2)$$

where $h > 0$ and $z \in (\frac{h-1}{2h}, \frac{1}{2})$. This is a symmetric distribution which becomes the uniform distribution for $h = 1$. For $h > 1$, it has two flat tails in the regions $x < z$ and $x > 1 - z$ with less density than the flat middle section. This function enables me to study the system for distributions epsilon close to the uniform distribution thereby avoiding singularity problems associated with the uniform distribution in deriving the equilibrium of the game.⁹

The consumers have a common reservation price V for a perfect match. For a less than perfect match there is a cost t per unit of distance the consumer has to travel. A consumer with location $x \in [0, 1]$ buys one unit from firm A if the participation constraint (PC)

$$V - t |x - a| - p_a \geq 0, \quad (3)$$

and the incentive compatibility constraint (IC)

$$V - t |x - a| - p_a \geq V - t |1 - b - x| - p_b, \quad (4)$$

are both satisfied. From (IC) follows that a consumer with location

$$\tilde{x} = \frac{p_b - p_a + t(1 + a - b)}{2t}, \quad (5)$$

will be indifferent between the two firms.

The effect of the participation constraints is that the demand will be discontinuous in price. If the price is low, the market will be covered and the demand will be determined by the location of the indifferent consumer. As the price gets higher, the participation constraint will either bind in the hinterland, if the hinterland is closer than the location of the indifferent consumer, or for the indifferent consumer. In the latter case the firm becomes a local monopoly. For even higher prices the constraint will be binding in both hinterland and for the indifferent consumer, making the firm a local monopoly with elastic demand in both directions. Hence, there are three conditions which in different combinations determine

⁹This distribution can be seen as a stylized city versus country side version, where z is the location of the boundary between the city and the countryside where the population density suddenly drops.

where the demand curve kinks.¹⁰

First, the participation constraint of the indifferent consumer which is binding if $V - t(\tilde{x} - a) - p_a \leq 0$. Substitution of \tilde{x} gives,

$$tr \geq 2V - p_a - p_b. \quad (6)$$

If this constraint is binding there will be a marginal consumer

$$x^m = \frac{V + ta - p_a}{t} < \tilde{x}. \quad (7)$$

Second, the participation constraint in the hinterland which is binding if $V - ta - p_a \leq 0$, i.e. if it is too costly to travel due to distance a and/or cost per unit of distance t ,

$$ta \geq V - p_a. \quad (8)$$

If it is binding there will be a marginal consumer

$$x^h = \frac{ta - V + p_a}{t} > 0. \quad (9)$$

Third, the condition that determine which constraint will be binding first when the firm increases its price. Firm A is located closer to the hinterland than the indifferent consumer if $\tilde{x} - a > a$. Substituting for \tilde{x} and simplifying gives

$$t(1 - 3a - b) > p_a - p_b. \quad (10)$$

Hence, at equal prices, the constraint will bind for the indifferent consumer before it binds in the hinterland if $1 > 3a + b$.

The demand to firm A can thus be summarised as follows

$$d_a = \begin{cases} F(\tilde{x}) & \text{if no participation constraint is binding;} \\ F(\tilde{x}) - F(x^h) & \text{if (8) holds and (10) does not;} \\ F(x^m) & \text{if (6) and (10) hold;} \\ F(x^m) - F(x^h) & \text{if (6) and (8) hold.} \end{cases} \quad (11)$$

¹⁰Note that symmetry implies that it is sufficient to give a complete characterisation of the problem for firm A .

The timing of the game is as follows. In the first period the firms independently choose their respective locations which are fixed forever. In the second period their choice of locations are revealed and the firms collusively decide on the technology for customising the good, i.e. the inconvenience. In the third period the firms compete in price. This game is solved using backward induction. Hence we start in the final period by characterising the price equilibria.

In the final period each firm chooses price taking locations and inconvenience as given. Hence, the objective of firm A is to choose price p_a to maximise the profit $\pi_a = (p_a - c)d_a$. The first order condition for an interior solution can be found by setting the first derivative equal to zero. However, since the demand curve is kinked in several places, so will the first derivative be,

$$\frac{\partial \pi_a}{\partial p_a} = \begin{cases} F(\tilde{x}) - \frac{(p_a - c)}{2t} f(\tilde{x}) & \text{if no participation constraint is binding;} \\ F(\tilde{x}) - F(x^h) - (p_a - c) \left(\frac{f(\tilde{x})}{2t} + \frac{f(x^h)}{t} \right) & \text{if (8) holds and (10) does not;} \\ F(x^m) - \frac{(p_a - c)}{t} f(x^m) & \text{if (6) and (10) hold;} \\ F(x^m) - F(x^h) - \frac{p_a - c}{t} [f(x^m) + f(x^h)] & \text{if (6) and (8) hold.} \end{cases} \quad (12)$$

The same applies to firm B . Hence, the equilibrium when it exists will either occur at a kink or be interior depending on parameter values.

Definition 1 Critical values on t for an interior solution are

$$t^{max} = \frac{2h(V - c)}{2 + hr}, \quad (13)$$

$$t_a^h = \frac{3h(V - c)}{3 + h(4a - b)}, \quad (14)$$

$$t_b^h = \frac{3h(V - c)}{3 + h(4b - a)}. \quad (15)$$

These are derived from the binding participation constraints in equilibrium for the three marginal consumers with locations $\{0, \tilde{x}, 1\}$.

The existence of an interior solution furthermore depends on locations of the firms and parameters of the distribution function.

Proposition 1 (Classical Hotelling) Let $t \leq \min\{t^{max}, t_a^h, t_b^h\}$, and a, b be such that

$$z < \frac{1}{2} + \frac{a-b}{6} \quad (16)$$

and

$$\min \left\{ \frac{2h-1}{2h} + \frac{h}{2} \left(\frac{a-b}{3} \right)^2 - \frac{a+5b}{3}, \frac{2h-1}{2h} + \frac{h}{2} \left(\frac{b-a}{3} \right)^2 - \frac{5a+b}{3} \right\} \geq 0 \quad (17)$$

then the interior solution is a price equilibrium with

$$p_a^H = c + t \left(\frac{1}{h} + \frac{a-b}{3} \right) \quad (18)$$

$$p_b^H = c + t \left(\frac{1}{h} + \frac{b-a}{3} \right) \quad (19)$$

and profit

$$\pi_a^H = \frac{th}{2} \left(\frac{1}{h} + \frac{a-b}{3} \right)^2 \quad (20)$$

$$\pi_b^H = \frac{th}{2} \left(\frac{1}{h} + \frac{b-a}{3} \right)^2 \quad (21)$$

Proof. in the appendix.

The main implication from $h > 1$ is that there will exist an equilibrium in price even when firms are located closer than halfway to the center, i.e. $a > 1/4$. This can be illustrated for the symmetric case, i.e. for $a = b$. The condition for under cut proof equilibrium (17) then becomes

$$r > \frac{1}{2h}. \quad (22)$$

Hence, the higher the relative proportion of consumers in the middle segment, the smaller the distance between the firms r that is compatible with an equilibrium. This is because a higher h means more elastic demand in the middle and thus lower equilibrium prices and profits, which makes undercutting relatively less profitable since the firm has to undercut by a fixed amount tr .

Since the profit is increasing in t and a and b respectively firms have incentives to move closer to the center and to increase inconvenience. However, if they do so, the classical case will no longer prevail due to binding participation constraints. If $t > t^{max}$ the firms become local monopolists, whereas if $t > \min\{t_a^h, t_b^h\}$ at least one firm will be faced with elastic hinterland. The monopoly outcome plays an important role in deriving the equilibrium of the game, as well as showing that the collusive inconvenience also is a Nash equilibrium. It has the following properties.

Proposition 2 (Monopoly) Let $a, b \leq 1/4$ and $t > t^{max}$. The monopoly solution is a corner solution with price and profit

$$p_a^M = V - \frac{t}{6}(3 - b - 5a), \quad (23)$$

$$\pi_a^M = \left(V - c - \frac{t}{6}(3 - b - 5a) \right) \left(\frac{3 + h(a - b)}{6} \right). \quad (24)$$

Proof: in the appendix.

The reason for this outcome is that a firm who is a monopoly would maximise profits by serving more than half of the market. Hence, if the firms were to implement the monopoly prices, they would be competing for the indifferent consumer and thus no longer be in a monopoly position.

Corollary 1 (Covered market) A firm can not raise profits by becoming a local monopolist through increasing inconvenience.

When $t > \min\{t_a^h, t_b^h\}$, the participation constraint in the hinterland under Hotelling prices will be binding for at least one firm. This has the implication that unless t is very high, at least one firm will choose a corner solution for the price. For example if one firm, say firm A moves closer to the center when the two firms are located halfway, i.e when $a > b = 1/4$, there exists an asymmetric equilibrium in the immediate neighbourhood of t_a^h , where firm A chooses a corner solution and B chooses an interior solution for their respective prices. This equilibrium is characterised in the following proposition.

Proposition 3 (Asymmetric Equilibrium) Let $a > b$ be such that (8) is binding for firm A but not for firm B and

$$h > \frac{3}{(b-a)^2} \left[-(3-5a-b) + \sqrt{(3-5a-b)^2 + (b-a)^2} \right]. \quad (25)$$

Then there exists an equilibrium in the neighbourhood of t_a^h with prices

$$p_a^A = V - ta \quad (26)$$

$$p_b^A = \frac{t[1 + h(b-2a)] + h(V+c)}{2h} \quad (27)$$

and profits

$$\pi_a^A = (V - ta - c) \frac{t[3 + h(2a-b)] - h(V-c)}{4t} \quad (28)$$

$$\pi_b^A = \frac{(t(1 + h(b-2a)) + h(V-c))^2}{8ht}, \quad (29)$$

Proof. In the appendix.

Once B also faces a binding incentive constraint from the hinterland, due to either a higher t , or being located at $b > 1/4$ there exists an equilibrium in which both firms choose corner solutions with $p_a = V - ta$ and $p_b = V - tb$, which results in profits

$$\pi_a = (V - c - ta) \frac{1 + 2h(a-b)}{2} \quad (30)$$

$$\pi_b = (V - c - tb) \frac{1 + 2h(b-a)}{2}. \quad (31)$$

Necessary conditions for such an equilibrium to exist is that $h > 1$ and $t > \frac{(V-c)[1-2h(a-b)]}{2-a[3+2h(a-b)]}$.¹¹

When the corner solution no longer applies, there exists an interior solution in price with uncovered hinterland. There are two possibilities in this case. First the outcome where the marginal consumer in the hinterland is in the tails, which results in prices and profits that are definitely lower than for the Hotelling outcome.¹² Second, the case where the marginal

¹¹The first to guarantee existence for locations closer to the center, and the latter to make it unprofitable to undercut the delivered price of the competitor to get the entire market demand.

¹²Complete derivations of this case are very lengthy and are available from the author upon request.

consumer is on the city boundary and the countryside does not get served. This case is only potentially relevant when the tails of the distribution are very thin, whereas we are more interested in distributions close to the uniform. It is therefore not included here.¹³

3 Optimal inconvenience

In the second period firms observe their respective locations prior to jointly selecting the inconvenience that maximises their joint profits.

The joint profit will be maximised for a covered market even when firms are asymmetrically located as long as h belongs to a critical interval which is defined by,

$$h_1 = \frac{3 \left[11a - 2b - \sqrt{89a^2 + 8ab - 16b^2} \right]}{2(8a^2 - 13ab + 5b^2)} \quad (32)$$

$$h_2 = \frac{3 \left[11a - 2b + \sqrt{89a^2 + 8ab - 16b^2} \right]}{2(8a^2 - 13ab + 5b^2)}. \quad (33)$$

Hence,

Proposition 4 (Collusive inconvenience) The collusive inconvenience is given by

$$t^C = \begin{cases} t^{max} & \text{if } a, b \leq 1/4 \text{ and } h \geq 1 \\ t_a^h & \text{if } a > \max\{1/4, b\} \text{ and } h \in (h_1, h_2) \\ t_b^h & \text{if } b > \max\{1/4, a\} \text{ and } h \in (h_1, h_2). \end{cases} \quad (34)$$

Proof: in the appendix.

The intuition for this result is as follows. If firms are located relatively further away from the center or exactly midway, joint profits can be increased by increasing t as long as the indifferent consumer gets a non-negative surplus. Beyond that, the firms become monopolists with a profit which is decreasing in t . Similarly for locations closer to the center, profits can be raised through higher inconvenience up until one of the participation constraints starts to bind from the hinterland.

¹³The conclusions in this case are straightforward. Derivations are available from the author upon request.

Would either firm have an incentive to deviate from t^C if they had the chance prior to competing in price?

When firms can choose different technologies for customising the good the inconvenience will be indexed t_i , $i = a, b$. This implies that the participation constraint becomes $V - t_i | x - a | - p_i \geq 0$ for $i = a, b$, and similarly for the (IC) constraint. Solving for the indifferent consumer gives

$$\tilde{x}(t_a, t_b) = \frac{p_b - p_a + t_a a + t_b(1 - b)}{t_a + t_b}. \quad (35)$$

For t_a, t_b and a, b in the neighbourhood of t^C and $a = b = 1/4$ the demand to firm A is $F[\tilde{x}(t_a, t_b)]$ ¹⁴. When there is an interior solution to the problem the firm thus maximises

$$\max_{p_a} (p_a - c) \frac{t_a[1 - h(1 - 2a)] + t_b[1 + h(1 - 2b)] + 2h(p_b - p_a)}{2(t_a + t_b)}. \quad (36)$$

From first order conditions we can derive best response prices

$$p_a = \frac{t_a[1 - h(1 - 2a)] + t_b[1 + h(1 - 2b)]}{4h} + \frac{p_b + c}{2}. \quad (37)$$

Differential inconvenience allows us to decouple the strategic effects from inconvenience on price. First, the effect on oneself which is decreasing in h and increasing in a . If $h(1 - 2a) < 1$ this effect is positive, i.e. it induces a firm to be less aggressive in the terminology of Fudenberg and Tirole (1984). The second effect is to induce the competitor to be less aggressive, which is increasing in h and decreasing in b . Hence, when there is more competition in the middle of the market, i.e. $h > 1$, the effect of increasing inconvenience will have a weaker commitment effect on oneself, but a stronger effect on the competitor to be less aggressive. There is also a general effect on price levels when the market gets more competitive in the middle, which is to reduce those prices. The overall effect is negative

$$\frac{\partial p_a}{\partial h} = -\frac{t_a + t_b}{2h^2}. \quad (38)$$

¹⁴In the neighbourhood of parameter values for which there is an interior solution in the Hotelling case, there will also be an interior solution in the Launhardt case. See Dos Santos Ferreira and Thisse (1996) for a complete characterisation of demand patterns including offering a much lower freight rate such that customers in the competitors hinterland gets served.

How negative is a function of the average inconvenience.

The equilibrium prices in the neighbourhood of t^C can be derived from the intersection of the best response functions.

$$p_a^S = c + \frac{t_a[3 - h(1 - 2a)] + t_b[3 + h(1 - 2b)]}{6h}, \quad (39)$$

$$p_b^S = c + \frac{t_a[3 + h(1 - 2a)] + t_b[3 - h(1 - 2b)]}{6h}, \quad (40)$$

which results in profits

$$\pi_a^S = \frac{(t_a[3 - h(1 - 2a)] + t_b[3 + h(1 - 2b)])^2}{36h(t_a + t_b)}, \quad (41)$$

$$\pi_b^S = \frac{(t_a[3 + h(1 - 2a)] + t_b[3 - h(1 - 2b)])^2}{36h(t_a + t_b)}. \quad (42)$$

Note that $\pi_i^S = \pi_i^H$ for $t_a = t_b$. These profits can therefore be used to prove that the collusive inconvenience is indeed a Nash equilibrium.

Proposition 5 (Strategic Inconvenience) Let $h \in [1, 3]$. The cooperative inconvenience t^C is the unique Nash equilibrium of the sub-game where two firms with locations $a = b = 1/4$ independently choose their own inconvenience t_a, t_b prior to competing in price.

Proof. Symmetry implies that it is sufficient to show for one firm that there does not exist a profitable deviation. This is true if the profit is increasing in t_a for $t_a < t^C$ and decreasing for $t_a > t^C$. Taking the first derivative of the profit function with respect to t_a gives

$$\frac{\partial \pi_a}{\partial t_a} = \frac{(t_a[3 - h(1 - 2a)] + t_b[3 + h(1 - 2b)])}{36h(t_a + t_b)^2} (t_a[3 - h(1 - 2a)] + t_b[3 - h(3 - 4a - 2b)])$$

which is positive for the equilibrium locations $a = b = 1/4$ and $t_a = t_b$ if $h < 3$.

For $t_a > t^C$, Proposition 2 applies, since for a monopoly it makes no difference whether t is the same or different. Hence the profit is decreasing in t_a .

The equilibrium is unique since at $a = b = 1/4$, t_a, t_b is the unique solution to a system of two linear equations: $V - t_a a - p_a^S = 0$ and $V - t_b b - p_b^S = 0$, which are the participation constraints of the consumers with locations 0 and 1. Q.E.D.

For other locations the Nash equilibrium may no longer be unique. For example if firms are located at $a, b < 1/4$, all combinations of t_a, t_b such that the participation constraint of the indifferent consumer is exactly binding will be a Nash equilibrium.

Finally, how will firms strategically position themselves if they know they are going to choose inconvenience collusively, and that there will be no incentive to deviate once it has been implemented prior to competing in price. This is the topic of the next section.

4 Location and the degree of differentiation

In the first period the firms independently choose locations which are fixed forever. Before presenting the equilibrium of the three stage game, two useful benchmarks for the locations and the degree of differentiation will be derived.

Lemma 1 The degree of differentiation, subject to the market being covered, i.e. $t^C r$ is maximised for $r = 1$.

Proof. The degree of differentiation is

$$t^C r = \begin{cases} t^{max} r & \text{if } a, b \leq 1/4 \\ t_a^h r & \text{if } a > \max\{1/4, b\} \\ t_b^h r & \text{if } b > \max\{1/4, a\}. \end{cases} \quad (43)$$

It is clear that a maximum cannot happen for $t_i^h, i = a, b$, since the inconvenience and the distance are decreasing if firms move closer to the center. Whereas for locations such that t^{max} applies, moving closer implies a smaller r but a higher t^{max} , hence there will be a trade-off. To see this take the first derivative of

$$t^{max} r = \frac{2h(V - c)}{2 + hr} r \quad (44)$$

with respect to the distance between the firms r , which gives

$$\frac{\partial t^{max} r}{\partial r} = \frac{2h(V - c)}{2 + hr} - \frac{2h(V - c)}{(2 + hr)^2} hr \quad (45)$$

$$= \frac{4h(V-c)}{(2+hr)^2} > 0. \quad (46)$$

It is clear that the location effect dominates the inconvenience effect. Q.E.D.

Thus, even though the inconvenience will be lower when firms locate at the end points, these are the locations which maximise the total degree of differentiation. Let us next consider the locations that minimise the average time to travel in the market.

Lemma 2 The efficient locations are given by

$$a^e = \frac{2h-1}{4h} \quad (47)$$

Proof. Due to symmetry it is sufficient to minimise the average distance to travel for consumers with locations $x \in [0, \frac{1}{2}]$. For $a > z$ the average distance to travel is given by

$$\int_0^z (a-s) \frac{1-h(1-2z)}{2z} ds + \int_z^a (a-s) h ds + \int_a^{\frac{1}{2}} (s-a) h ds = \left(a - \frac{z}{2}\right) \frac{1-2h}{2} + a^2 h + \frac{h}{8}$$

Minimising over a gives $\frac{1-2h}{2} + 2ha = 0$. Q.E.D.

For a uniform distribution the efficient locations will be $a = b = 1/4$. However, for $h > 1$ a location slightly closer to the center will be optimal.

Having derived these two benchmarks for the locations and the degree of differentiation, it is time turn to the first period choices in the three stage game.

What location maximises the firm's profit taking into account the effect it will have on t and p_a and p_b in consecutive periods?

Proposition 6 (The Principle of Moderate Differentiation) Let $1 < h < \min\{\frac{2}{b}, \frac{2}{a}\}$.

Firms locating at $a^* = b^* = 1/4$, choosing inconvenience $t^* = \frac{4h(V-c)}{4+h}$, and prices $p_a^* = p_b^* = c + \frac{4(V-c)}{4+h}$, with payoff $\pi_a^* = \pi_b^* = \frac{2(V-c)}{4+h}$ constitute a Nash equilibrium in a sequential game where firms independently choose locations, collusively the inconvenience, and finally compete price.

Proof: Symmetry implies that it is sufficient to check that $a = 1/4$ is indeed a Nash equilibrium when $b = 1/4$.

Firm A has no incentive to move closer if the first derivative with respect to a is positive for $a < a^*$ and negative for $a \geq a^*$. The first derivative has two effects

$$\frac{d\pi_a}{da} = \frac{\partial \pi_a}{\partial a} + \frac{\partial \pi_a}{\partial t} \frac{\partial t}{\partial a}. \quad (48)$$

In Proposition 1 it was shown that the profit function in the neighbourhood of a^*, b^* will be given by π_a^H if $h > 1$. Hence, whilst the functional form of the optimal inconvenience will change at a^* , the profit function is unchanged in the neighbourhood of the equilibrium. Substitution of t^C gives

$$\pi_a^* = \begin{cases} \frac{h^2(V-c)}{2+hr} \left(\frac{1}{h} + \frac{a-b}{3} \right)^2 & \text{if } a < a^* \\ \frac{3h^2(V-c)}{2(3+h(4a-b))} \left(\frac{1}{h} + \frac{a-b}{3} \right)^2 & \text{if } a \geq a^* \end{cases} \quad (49)$$

Note that for $a < a^*$ both effects are positive

$$\frac{\partial \pi_a^*}{\partial a} = \frac{2(V-c)h^2}{3(2+hr)} \left[1 + \frac{a-b}{3} \right] + \frac{(V-c)h^3}{(2+hr)^2} \left[1 + \frac{a-b}{3} \right]^2 \quad (50)$$

whereas for $a \geq a^*$ the second effect is negative and dominates,

$$\frac{\partial \pi_a^*}{\partial a} = -(2-hb) \frac{3h^2(V-c)}{2[3+h(4a-b)]} \left(\frac{1}{h} + \frac{a-b}{3} \right) < 0, \quad (51)$$

if $h < 2/b$. Symmetry implies that firm B has no incentive to move closer to the center for $h < 2/a$.

The equilibrium inconvenience, prices and profits are derived by substituting for $a = b = 1/4$ in t^C , p_i^H , and π_i^H , $i = a, b$. Q.E.D.

The equilibrium locations are independent of h . However, the inconvenience is increasing in h whereas the profit is decreasing. When there is more agglomeration of taste in the middle, competition for customers will be more fierce, to which the firms respond by making cost of customising higher to relax competition, rather than moving further afield. However, the negative effect on profit from more elastic demand will still dominate the positive effect from higher inconvenience. This provides an interesting prediction from the model.

Corollary 2 There is a negative correlation between degree of differentiation and profitability.

Hence, markets with a greater concentration of taste will have more differentiated products, and lower profits. Furthermore the firms will choose locations that are further apart than the efficient ones the higher is h , $a^e > a^*$ but never so much that they maximise the degree of differentiation.

5 Discussion

The contribution of this paper is threefold. First, it has shown that collusive choice of inconvenience can resolve the existence problem in the Hotelling linear city model. Second, it has shown that the collusive inconvenience is a Nash equilibrium. Third, it has revealed that firms will not change the location of the standard variety when there is more concentration of consumers in the middle relative to the tails of the taste distribution. The locations will therefore be too far apart relative to social optimum. Instead the firms will respond by increasing the cost of customising the good, thereby increasing the degree of product differentiation in markets with a higher concentration of taste in the middle.

These results were derived under a specific set of assumptions, which I will discuss.

The uniform distribution is often used in models of spatial differentiation due to its simplicity. However, in the Hotelling model it does give rise to singularity problems such as the derivative of the profit function not being defined at the equilibrium point. The class of distribution functions I have introduced in this paper are in the limit the uniform distribution, and therefore allow me to show that I can avoid the singularity problems using a distribution function epsilon close the uniform. These distributions also have attractive economic features since the flat tails and higher flat middle section make them a cubist¹⁵ version of an S-shaped distribution function, capturing the essence without potential problems of

¹⁵It looks like the way Picasso would have painted the Gaussian distribution.

non-linearity. They can therefore be used for the purpose of comparative statics, which I have done in this paper to see how the degree of differentiation is affected by changes in the distribution of taste. The interpretation of this distribution as representing the differences in population density between the city and the countryside might also be of use in other applications.

There are three assumptions relating to inconvenience, the linearity, the collusive choice and the zero cost, which are worth commenting on. The linearity assumption is the case where there will be existence problems when inconvenience is not endogenous. However, the principle of moderate differentiation is not specific to linear inconvenience. Preliminary calculations of the quadratic case reveals that the principle of moderate differentiation would apply. Though in that case, because of a binding participation constraint from the indifferent consumer, rather than the consumer in the hinterland, which prevents firms from wishing to move too far out.

Collusive inconvenience has several advantages. It is a benchmark for the highest profit obtainable. It captures both the Hotelling and the Launhardt models, since it is shown to be a Nash equilibrium. Furthermore it captures what firms who are inclined to collude would do when they are prevented from colluding in price by active anti-trust. It also overcomes the problem of equilibrium selection when there exists several Nash equilibria in inconvenience, which happens for some locations. However, as was shown in this paper, at the equilibrium of the three stage game, the collusive inconvenience is not only a Nash equilibrium, it is also unique. Hence, it is stable and does not require repeated interaction to be implemented. This could be contrasted with semi-collusion derived in Friedman and Thisse (1993) who consider collusion in price which is supported by a trigger strategy, and non-cooperative choice of locations.

The analysis in this paper was made under the simplifying assumption that there is no cost of neither location nor inconvenience, thereby entirely focussing on the strategic role

of product characteristics. If the analysis were to be extended to include costs, additional trade-offs would enter the picture. For example if locations are more expensive at the center, the existence problem could be resolved by simply adding a differential cost of location. In the case of inconvenience, a cost saving associated with higher inconvenience, would give the firms an additional incentive to increase inconvenience. However, unless the marginal cost saving would be higher than the marginal reduction in price which happens in the monopoly regime, the firms would still not have incentives to become local monopolists, and hence we would still get the result that inconvenience is chosen as high as it can be to allow the firms to strategically benefit from it when competing in price.

Inconvenience has deliberately been chosen in favour of the more commonly used term *cost of transport*, as a generic term in instances where it can be chosen by the firms either collusively by choice or force of circumstance, or independently, rather than being exogenous to the model. When inconvenience, does represent the inconvenience associated with the service, it is a variable that could be chosen independently by the firms, however, there may be an implicit agreement in for example banking to collude on the level of inconvenience in opening bank accounts *et cetera*. And as have been shown in this paper, there would be no incentives to deviate from such collusion. Hence, no trigger strategy would be needed to support such an equilibrium. If, on the other hand, inconvenience represents a cost which can only be indirectly influenced by the firms through collective action, such as lobbying for just the right amount being spent on roads and public transport, it can only be chosen collusively. In the interpretation that it represents the cost of customising the good, the model indicates that the incentives to develop technology for customising goods in a research joint venture might be biased and lead to a more costly technology the more concentrated the taste is in the market. Whereas the firms would have incentives to develop a cheap technology for customising the goods when the distribution of taste is dispersed with very little concentration in the middle.

An alternative approach to endogenising inconvenience would be to have a monopoly supplier of transport who sets the freight rate. This would result in solving a three player game. Spulber (1981) looked at this problem in the case of a monopoly supplier of a good and a monopoly supplier of transport. What happens in the duopoly case is left for future research.

A Proofs

Proofs of Propositions 1, 2, 3 and 4 follow.

Proof of Proposition 1. The Hotelling outcome requires that the market is covered in equilibrium, hence the participation constraint cannot be binding for either of the marginal consumers. Substitution of p_a^H and p_b^H in the participation constraints of the consumers with locations 0, \tilde{x} and 1 respectively gives

$$V - ta - c - t \left(\frac{1}{h} + \frac{a-b}{3} \right) \geq 0, \quad (52)$$

$$V - t \left(\frac{1}{2} + \frac{a-b}{6} - a \right) - c - t \left(\frac{1}{h} + \frac{a-b}{3} \right) \geq 0, \quad (53)$$

$$V - tb - c - t \left(\frac{1}{h} - \frac{a-b}{3} \right) \geq 0. \quad (54)$$

Solving for t gives t_a^h, t^{max} and t_b^h respectively.

First order conditions when the market is covered will be given by

$$\frac{t[1 - h(b-a)] + hp_b}{2t} - \frac{2p_a - c}{2t}h = 0, \quad (55)$$

$$\frac{t[1 + h(b-a)] + hp_a}{2t} - \frac{2p_b - c}{2t}h = 0, \quad (56)$$

for $z < \tilde{x}$. Note that p_a^H, p_b^H solves this system of equations, hence $\tilde{x}^H(p_a^H, p_b^H) = \frac{1}{2} + \frac{a-b}{6}$. Substitution of equilibrium prices, $\pi_a = (p_a^H - c)F(\tilde{x}^H)$ gives the profit function.

Finally we have to confirm that it does not pay for either of the firms to undercut the delivered price of the competitor to get the entire market demand, i.e. firm A would not prefer to charge $p_b - tr - \varepsilon$ and get the entire market to p_a^H . This requires that

$$c + t \left(\frac{1}{h} + \frac{b-a}{3} \right) - t(1-a-b) - c < \frac{th}{2} \left(\frac{1}{h} + \frac{a-b}{3} \right)^2. \quad (57)$$

Collecting terms and simplifying

$$\frac{a+5b}{3} < \frac{2h-1}{2h} + \frac{h}{2} \left(\frac{a-b}{3} \right)^2 \quad (58)$$

By symmetry one can infer that firm B similarly has no incentive to undercut if the left hand side is $\frac{5a+b}{3}$. Since the condition has to be satisfied for both firms, the most constraining of the two, sets the requirement on a , and b . Q.E.D.

Proof of Proposition 2. An interior solution requires that the candidate monopoly price is at least as high as the hypothetical Hotelling price would be, i.e. $p^M \geq p_a^H$. From the first derivative of the profit function one can see that there are two potential candidate prices for interior solutions depending on whether one or two participation constraints are binding.

The first order condition if only the constraint is binding from the center is

$$\frac{1-h \left(1 - 2 \frac{V+ta-p_a}{t} \right)}{2} - \frac{p_a-c}{t} h = 0. \quad (59)$$

Solving for the price gives

$$p_a^{M1} = \frac{t(1-h(1-2a)) + 2h(V+c)}{4h}. \quad (60)$$

For this to be higher than the Hotelling price the following must be true

$$\frac{V+c}{2} + \frac{t(1-h(1-2a))}{4h} \geq c + t \left(\frac{1}{h} + \frac{a-b}{3} \right) \quad (61)$$

which is equivalent to

$$t \leq \frac{6h(V-c)}{9+h(3-2a-4b)}. \quad (62)$$

However, we know that $t > t^{max}$ for the monopoly case to apply. Hence,

$$\frac{2h(V-c)}{2+hr} < t \leq \frac{6h(V-c)}{9+h(3-2a-4b)}. \quad (63)$$

This is only possible if $3 \leq h(b-a)$. For $b \leq a$ it is therefore impossible. For $b > a$, we can derive the smallest h that may apply. Note that $b < \frac{1}{3}$ and $a \geq 0$ for the monopoly situation to arise. Substitution of the boundaries of a and b gives $h > 9$. We can find the upperbound to h , by noticing that the density for $x < z$ has to be positive and that $z < \tilde{x} = \frac{3+a-b}{6} = \frac{4}{9}$. Substituting for $z = \frac{4}{9}$ and noticing the density has to be positive gives

$$1 - h \left(1 - 2 \frac{4}{9} \right) > 0, \quad (64)$$

i.e. $h < 9$. Hence we arrive at two mutually exclusive conditions on h .

The other monopoly price is derived from the first order condition when the participation constraint is binding from both hinterland and the center. It is given by

$$\frac{1 - h(1 - 2x^m)}{2} - \frac{1 - h(1 - 2z)}{2z}x^h - \frac{p_a - c}{t} \left(h + \frac{1 - h(1 - 2z)}{2z} \right) = 0 \quad (65)$$

Simplifying

$$\frac{t}{1 - h(1 - 4z)} [(z - x^h)(1 - h) + 2hz(x^m - x^h)] - (p_a - c) = 0 \quad (66)$$

substitution of x^m and x^h and collecting terms gives

$$\frac{t(z - a)(1 - h)}{1 - h(1 - 4z)} - 2p_a + V + c = 0 \quad (67)$$

Thus

$$p_a^{M2} = \frac{V + c}{2} + \frac{t(z - a)(1 - h)}{2(1 - h(1 - 4z))} \quad (68)$$

Note that $p_a^{M2} < p_a^{M1}$ can be shown to be equivalent to $(1 - h(1 - 2z))(1 - h(1 - 4a)) > 0$, which is satisfied since $a < 1/4$. Given that $p_a^{M1} < p_a^H$ it thus follows that $p_a^{M2} < p_a^H$.

With a profit that is increasing over the entire range, the solution therefore occurs at the kink where $V - t(\tilde{x} - a) - p^M = 0$. Substitution of $\tilde{x} = \frac{3+a-b}{6}$ and solving for p^M gives the price. Q.E.D.

Proof of Proposition 3. The proof involves showing that these prices indeed satisfy first order conditions for a maximum, and that it does not pay to undercut these prices for the relevant range of parameter values.

When the constraint is binding in the hinterland firm A's best response price to p_b will occur at a kink, $p_a = V - ta$, if the first derivative of the profit function is increasing before and decreasing after the kink,

$$\frac{\partial \pi_a}{\partial p_a} = \begin{cases} \frac{1 - h(1 - 2\tilde{x})}{2} - \frac{p_a - c}{2t}h > 0 & \text{if } p_a < V - ta \\ \frac{(z - x^h)(1 - h) + 2hz(\tilde{x} - x^h)}{2z} - (p_a - c) \left(\frac{1 - h(1 - 3z)}{2zt} \right) < 0 & \text{otherwise.} \end{cases} \quad (69)$$

To derive the relevant p_b one has to consider B's best response to $p_a = V - ta$ which is

$$p_b^H(p_a) = \frac{t[1 + h(b - 2a)] + h(V + c)}{2h} \quad (70)$$

as long as the participation constraint is not binding in the hinterland, i.e.

$$V - tb - \frac{t[1 + h(b - 2a)] + h(V + c)}{2h} \geq 0, \quad (71)$$

which holds if $t \leq \frac{h(V-c)}{1+h(3b-2a)}$. For an equilibrium to exist it is required that $\frac{h(V-c)}{1+h(3b-2a)} > t_a^h$, which holds if $a > b$. Hence it is satisfied.

Substitution of p_b^H in (69) and simplifying shows that the first derivative is increasing before $p_a = V - ta$,

$$\frac{t(3 + h(4a - b)) - 3h(V - c)}{4t} > 0, \quad (72)$$

i.e. if $t > t_a^h$, and that it is decreasing after the kink

$$\frac{z(3 - h(4a + b)) - 2a(1 - h)}{4z} - \frac{(V - c)(2 - h(2 - 7z))}{4zt} < 0 \quad (73)$$

i.e. if

$$t < \frac{(V - c)(2 - h(2 - 7z))}{z(3 - h(4a + b)) + 2a(h - 1)} \quad (74)$$

For an equilibrium to exist, this upperbound has to be higher than t_a^h , thus

$$\frac{2(V - c)(2 - h(2 - 7z))}{z(1 + h(8a - b)) - 2a(h - 1)} > \frac{3h(V - c)}{3 + h(4a - b)}. \quad (75)$$

rearranging and simplifying gives

$$3 > h(3 - 7a + b - 6z) + h^2[a(7 - 20z) - b(1 - 2z)], \quad (76)$$

which is satisfied. To see this consider $z = 1/4$, in which case the condition is satisfied for all h in the relevant range $h \in (1, 2)$.¹⁶

Firm A has no incentive to undercut B by charging $p_b - tr - \varepsilon$ and get the entire market if

$$\frac{t[1 - h(2 - 3b)] + h(V - c)}{2h} \leq (V - ta - c) \frac{t[3 + h(2a - b)] - h(V - c)}{4t} \quad (77)$$

Substitution of t_a^h gives

$$6 + h(3 - 10b + 4a) + h^2(a - b)^2 \geq 0, \quad (78)$$

which holds with strict inequality since all three terms are positive. Hence, there is no incentive for A to undercut B in the neighbourhood of t_a^h .

Similarly firm B has no incentive to undercut A by charging $p_a - tr - \varepsilon$ and get the entire market if

$$[V - c - t(1 - b)] \leq \frac{(t(1 + h(b - 2a)) + h(V - c))^2}{8ht} \quad (79)$$

¹⁶ $h > 2$ implies the tails have negative mass which is impossible.

For t_a^h this becomes

$$9 \leq 6h(3 - 5a - b) + h^2(b - a)^2. \quad (80)$$

Calculating the positive root, this condition becomes

$$h > \frac{3}{(b - a)^2} \left[-(3 - 5a - b) + \sqrt{(3 - 5a - b)^2 + (b - a)^2} \right]. \quad (81)$$

The more asymmetrically located the firms, the higher the density required in the middle for B to abstain from undercutting. Note that in the immediate neighbourhood of $a = b = 1/4$, the requirement is $h > 1$. Q.E.D.

Proof of Proposition 4. To prove this we need to show that the joint profit is increasing for $t \leq t^C$, and that it is decreasing for $t \geq t^C$. Let $\Pi = \pi_a + \pi_b$. For $t \leq t^C$ Hotelling prices apply with the joint profit

$$\Pi^H(t) = \frac{th}{2} \left[\left(\frac{l}{h} + \frac{a-b}{3} \right)^2 + \left(\frac{l}{h} + \frac{b-a}{3} \right)^2 \right] \quad (82)$$

$$= th \left[\frac{1}{h^2} + \frac{(a-b)^2}{9} \right] \quad (83)$$

This profit is clearly increasing in t .

For $t > t^C$ there are two cases depending on the locations of the firms.

When $a, b \leq 1/4$, Π^H is maximised for t^{max} . For $t > t^{max}$ the monopoly outcome applies. It follows directly from Proposition 2 that the monopoly profit is decreasing in t .

Otherwise Π^H will be maximised for t_a^h , for locations closer to the center that are compatible with the condition for equilibrium existence as defined by (17).

In these instances the firms will either both choose a corner solution, in which case it follows directly from (30) that the joint profit is decreasing in t , or if only one of them faces a binding constraint from the hinterland the asymmetric case applies in the immediate neighbourhood of t_a^h .

Let $\Pi^A = \pi_a^A, \pi_b^A$. Then we need to show that $\frac{\partial \Pi^A}{\partial t} < 0$ at t_a^h , since we know that at t_a^h $\Pi^H = \Pi^A$ by definition.

$$\frac{\partial \Pi^A}{\partial t} = -\frac{t^2 a(3 + h(2a - b)) - h(V - c)^2}{4t^2} + \frac{t^2(1 + h(b - 2a))^2 - h^2(V - c)^2}{8ht^2} \quad (84)$$

$$= \frac{h^2(V - c)^2 - t^2(10ha - 2hb - 1 + h^2(2ab - b^2))}{8ht^2}, \quad (85)$$

evaluated at t_a^h gives

$$\frac{\partial \Pi^A}{\partial t} = \frac{9 - 3h(11a - 2b) + h^2(8a^2 - 13ab + 5b^2)}{36h} \quad (86)$$

this derivative is negative if $h \in (h_1, h_2)$, which are the roots of the numerator

$$h_1 = \frac{3 [11a - 2b - \sqrt{89a^2 + 8ab - 16b^2}]}{2(8a^2 - 13ab + 5b^2)} \quad (87)$$

$$h_2 = \frac{3 [11a - 2b + \sqrt{89a^2 + 8ab - 16b^2}]}{2(8a^2 - 13ab + 5b^2)} \quad (88)$$

To see that the derivative is indeed negative for h in this range, let us calculate the boundaries for $a = \frac{3}{8}$ and $b = 1/4$. This gives $h_1 \approx \frac{6}{7}$ and $h_2 \approx 48.85$, hence the derivative should be negative at $h = 1$. Evaluating (86) at $h = 1$ for $a = 3/8$ and $b = 1/4$ gives $-53/32 < 0$. Q.E.D.

References

- [1] Aguirre, I. and M. Espinoza (2003), ‘Product Differentiation with consumer arbitrage’, *International Journal of Industrial Organization*, *forthcoming*.
- [2] Cantillon, R. (1755), *Essai sur la nature du commerce général* (H Higgs ed and trans) London MacMillan 1931.
- [3] Caplin, Andrew and Barry Nalebuff (1991): “Aggregation and Imperfect Competition: On the Existence of Equilibrium,” *Econometrica*, 59, 25-59.
- [4] Cheysson, E. (1887): *La Statistique géométrique*
- [5] Christou D. and Vettas, N. (2004)L ‘Location Choices under Quality Uncertainty’. CEPR Discussion paper no 4323.
- [6] Dasgupta, Partha and Eric Maskin (1986): “The Existence of Equilibrium in Discontinuous Economic Games,” *Review of Economic Studies*, 53, 1-41.
- [7] d’Aspremont, Claude, J. Jaskold Gabszewicz and Jacques-Francois Thisse (1979): “On Hotelling’s ‘Stability in Competition’, ” *Econometrica* Vol 47 (5) 1145-1150.

- [8] de Palma, A., V. Ginsburgh, Y. Y. Papageorgiou and J.-F. Thisse (1985): "The Principle of Minimum Differentiation Holds under Sufficient Heterogeneity," *Econometrica*, 53, 767-81.
- [9] Devletoglou, N. (1965) "A Dissenting View of Duopoly and Spatial Competition," *Economica*: 140-160.
- [10] Dos Santos Ferreira, R. (1998); "Launhardt and Hotelling" *Recherches Economic de Louvain* 64(1):97-109)
- [11] Dos Santos Ferreira, R. Thisse, J.-F. (1996), "Horizontal and Vertical Differentiation: The Launhardt Model" *International Journal of Industrial Organization* 14(4):485-506.
- [12] Eaton, B. (1972) "Spatial Competition Revisited," *Canadian Journal of Economics* 5: 268-277.
- [13] Eaton, B. C. and R. G. Lipsey (1975): "The Principle of Minimum Differentiation Reconsidered: Some New Developments in the Theory of Spatial Competition," *Review of Economic Studies*, 42, 27-49.
- [14] Friedman, J. and Thisse, J.-F. (1993), "Partial Collusion Fosters Minimum Product Differentiation", *The RAND Journal of Economics* vol 24 (4) (Winter) 631-645.
- [15] Fudenberg, D. Tirole, J. (1984), "The Fat-Cat Effect, the Puppy-Dog Ploy, and the Lean and Hungry Look", *American Economic Review*, 1984, 74, 361-8.
- [16] Gal-Or, E. (1982): "Hotelling's Spatial Competition as a Model of Sales," *Economics Letters*, 9: 1-6.
- [17] Graitson, D. (1982): "Spatial Competition a la Hotelling: A Selective Survey," *Journal of Industrial Economics*, Vol XXXI: 13-25
- [18] Hébert, R. F. (1972): 'A Note on the Historical Development of the Economic Law of Market Areas', *Quarterly Journal of Economics* Vol 86, No 4. 563-571.

- [19] Hébert, R. F. (1981), "Richard Cantillon's Early Contributions to Spatial Economics", *Economica* Vol 48(189): 71-77.
- [20] Hinloopen, J. (2002), "Price regulation in a spatial duopoly with possible non-buyers", *Annals of Regional Science* 36:19-39.
- [21] Hinloopen, J. and C. van Marrewijk (1999), "On the Limits and possibilities of the principle of minimum differentiation" *International Journal of Industrial Organization* 17 :735-750.
- [22] Hotelling, Harold (1929): "Stability in Competition," *Economic Journal*, 39, 41-57.
- [23] Launhardt, W. (1885), *Mathematische Begründung der Volkswirtschaftslehre* Verlag Engelmann Leipzig.
- [24] Lederer P. and A. Hurter (1986): "Competition of Firms: Discriminatory Pricing and Location", *Econometrica* vol 54 (3) (May): 623-640.
- [25] Lerner, A. P. and H. W. Singer (1937): "Some Notes on Duopoly and Spatial Competition," *Journal of Political Economy* Vol 25: 145-190.
- [26] Rothschild, R. "The Effect of Sequential Entry on Choice of Location," *European Economic Review* 12 (1979), 227-41.
- [27] Shaked, A. (1982): "Existence and Computation of Mixed Strategy Nash Equilibria for 3-Firms Location Problem," *Journal of Industrial Economics*, 31, 93-96.
- [28] Shaked, A. (1975): "Non-Existence of Equilibrium for the Two-Dimensional Three Firms Location Problem, " *Review of Economic Studies*, 42: 51-56.
- [29] Spulber, D. (1981), "Spatial Nonlinear Pricing", *American Economic Review* Vol 71(5) 923-933.

- [30] Stahl, K. (1982): “Differentiated Products, Consumer Search, and Locational Oligopoly,” *Journal of Industrial Economics* Vol 31 (1/2) Symposium on Spatial Competition and the Theory of Differentiated Markets: 97-113.