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Gravitino Dark Matter and the Flavour Structure of R-violating Operators

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ABSTRACT

We study gravitino dark matter and slow gravitino decays within the framework of R-violating supersymmetry, with particular emphasis on the flavour dependence of the branching ratios and the allowed R-violating couplings. The dominant decay modes and final state products turn out to be very sensitive to the R-violating hierarchies. Mixing effects can be crucial in correctly deriving the relative magnitude of the various contributions, particularly for heavy flavours with phase space suppression. The study of the strength of different decay rates for the gravitino is also correlated to collider signatures expected from decays of the Next-to-Lightest Supersymmetric Particle (NLSP) and to single superparticle production.

1 Introduction

Recently, there has been renewed interest in the possibility of having gravitino dark matter within the framework of R-violating supersymmetry [1, 2], which occurs if the gravitino decays are slow enough for its lifetime to be larger than the age of the universe [3, 4]. This is an exciting possibility that allows supersymmetric dark matter, even if the symmetries of the fundamental theory result in an unstable Lightest Supersymmetric Particle (LSP) [5–7].

This is what happens if, in addition to the couplings that generate the fermion and Higgs masses

$$\mu H_1 H_2 + m^e L_i \bar{E}_j H_1 + m^d Q_i \bar{D}_j H_1 + m^u Q_i \bar{E}_j H_2, \tag{1.1}$$

we also have R-violating couplings of the form

$$h L_i H_2 + \lambda L_i L_j \bar{E}_k + \lambda' L_i Q_j \bar{D}_k + \lambda'' \bar{U}_i \bar{D}_j \bar{D}_k. \tag{1.2}$$

In the above, $H_{1,2}$ are the Higgs superfields, $L(Q)$ are the left-handed lepton (quark) doublet superfields, and $\bar{E}$ $(\bar{D}, \bar{U})$ are the corresponding left-handed singlet fields. The first three couplings in (1.2) violate lepton number, while the fourth violates baryon number.

The stricter bounds on R-violating operators come from proton stability, and R-parity [8], which forbids all lepton and baryon number violating operators, is one of the possible solutions. However, this is not the only symmetry that can guarantee proton stability; baryon or lepton parities [9, 10] can also exclude the simultaneous presence of dangerous $LQ\bar{D}$ and $\bar{U}\bar{D}\bar{D}$ couplings [11]. Experimental constraints from the non-observation of modifications to Standard Model rates, or of possible exotic processes [12] also impose additional bounds[1].

R-violating supersymmetry results in a very rich phenomenology. In the presence of the additional operators, the NLSP can decay into conventional Standard Model particles. The missing energy signature of the Minimal Supersymmetric Standard Model (MSSM) [14] is substituted by multi-lepton and/or multi-jet events. In addition to the consequences for collider searches, R-violation implies that any gravitinos that have been thermally produced after a period of inflation, are also unstable.

Gravitinos have three main decay modes: via tree-level three-body decays to fermions [4], via two-body decays to neutrino and photon due to neutrino–neutralino mixing [1, 3], and via one-loop decays to neutrino and photon, generated by the trilinear couplings [2] [2]. In all three cases, the very large suppression $1/M_p$ of the gravitino vertex, where M_p is the reduced Planck scale, plus additional suppression from phase space, mixing and loop factors, respectively, result in large gravitino lifetimes. For a wide set of R-parity violating couplings and gravitino masses these exceed the age of the universe. Moreover, the photon flux from these decays could be able to explain the apparent excess in the extragalactic diffuse gamma-ray flux in the re-analysis of the EGRET data [15, 16].

[1] Additional strong constraints can be obtained from the observation of NLSP decays to a gravitino LSP, with a photon or lepton plus missing energy signature [13].

[2] For heavy gravitinos, there is also the possibility of producing massive gauge bosons. However, for trilinear couplings and the range of parameters considered here, these contributions are subdominant.

The branching ratios for gravitino decays are sensitive to the flavour structure of the R-violating operators. In the case of O (GeV) gravitinos, the presence of tau or bottom quarks in the final state significantly enhances the branching ratio of radiative decays with respect to the tree-level ones, while for "super-light" gravitinos, as in [17], gravitinos are essentially stable with respect to the three-body decays. Moreover, in the case of non-zero $\lambda'' \bar{U}_3 \bar{D}_j \bar{D}_k$ only — with a top quark final state — gravitinos lighter than m_t have a maximal stability, modulo mixing effects, which we will discuss in a subsequent section[3].

In [2], gravitino decays were studied for $LL\bar{E}$ operators that give rise to both loop and tree-level decays, with a tau or a muon in the loop. Here, we extend the results in the following way:

(i) We look at flavour effects in more detail, making the link with fermion mass hierarchies. Within this framework we comment on the relative magnitudes for bilinear and trilinear R-violation and what are the implications for gravitino decays.

(ii) We extend the discussion to all 45 $LL\bar{E}$, $LQ\bar{D}$ and $\bar{U}\bar{D}\bar{D}$ operators, paying particular attention to the different features of the various decay modes and possible bounds from gamma-ray measurements.

(iii) We consider possible implications of mixing effects, which in certain cases can be quite significant. For instance, for the $\bar{U}_3 \bar{D}_j \bar{D}_k$ operator, the expected decay depends very sensitively on the right quark mixing (for which little information is available).

(iv) We link the above with probes of R-parity violation at the LHC, in particularly NLSP decays, which may yield interesting signatures.

We begin in Section 2 by describing the various modes of gravitino decays with trilinear couplings and the calculation of the resulting extragalactic diffuse photon flux. In Section 3 we discuss possible flavour structures for R-parity violating operators, before we look at the consequences for gravitino decays in Section 4, with particular attention to bounds from gamma-ray measurements. We continue with the corresponding prospects for hadron colliders in Section 5, before concluding in Section 6.

2 Gravitino Decays

As already discussed, trilinear R-violating operators may cause gravitinos to decay via two different channels:

- Via two-body radiative loop decays to neutrino and photon (Fig. 1) [2].
- Via tree-level decays to fermions (Fig. 2) [4].

[3] For an operator of the form $\lambda' L_i Q_3 \bar{D}_k$ this argument does not hold, since, when we pass from superfields to component fields the $L_i Q_3$ part can become $\ell_i t$ or $\nu_i b$.

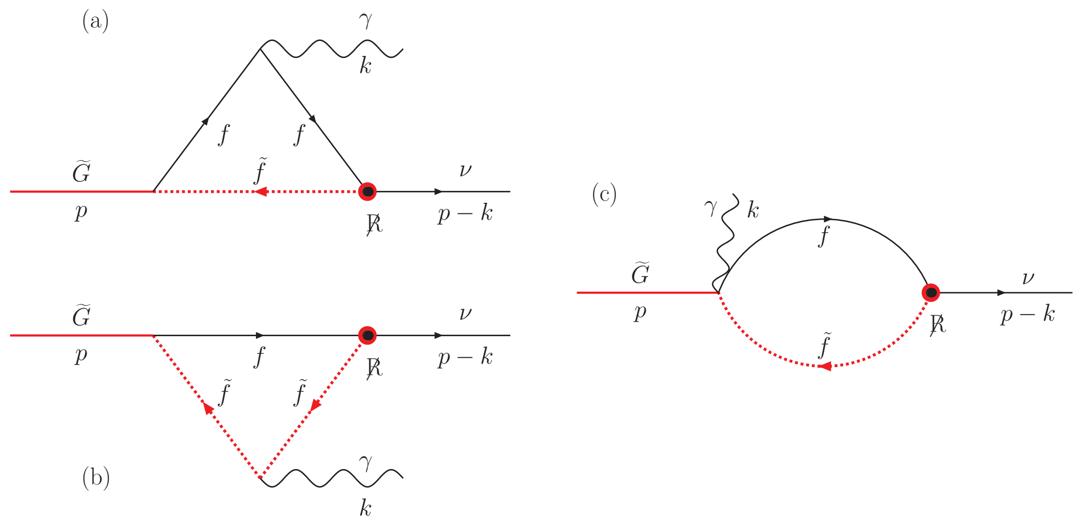

Figure 1: *Basic set of Feynman diagrams for radiative gravitino decay, shown for (s)fermion loops. In the case of (s)quarks, the neutrino is coupled to down-type quark to preserve SU(2) invariance. Arrows denote flow of fermion number for left-chiral fields.*

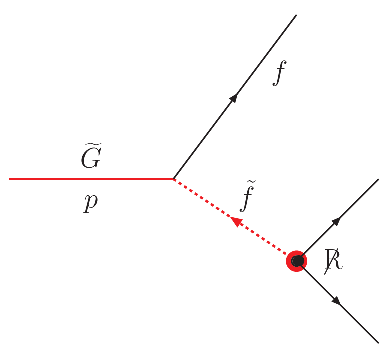

Figure 2: *Three-body decay of a gravitino via an R-parity violating coupling. There are three contributing diagrams where the sfermion carries any one of the three indices i, j and k of the corresponding operators.*

The decay rates have been presented in detail in the original references, and for completeness are briefly summarised in Appendices A and B, respectively. For light gravitino masses and appropriate fermions in the loop the radiative decays may dominate. Indeed, as we shall see, even when the three-body decay involving an intermediate sfermion $\tilde{f}$ is well above the kinematical threshold at $2m_f$, the radiative dominance is still present. The behaviour of the decay rates is controlled by the mass dependence of the decay width: for the three-body decay $\Gamma_{\tilde{G}} \propto m_{\tilde{G}}^7$, while for the radiative decay $\Gamma_{\tilde{G}} \propto m_{\tilde{G}}$ at low gravitino masses. The latter occurs since the gravitational coupling compensates for the relatively high loop mass by its increasing strength for higher loop momenta. Because of the helicity structure of the couplings, the two-body decay width is also strongly dependent on the mass of the fermion in the loop, $\propto m_l^2$ at low gravitino masses, implying significantly shorter lifetimes for dominant third generation couplings.

To constitute a realistic dark matter candidate, the gravitino lifetime should exceed the age of the universe. Moreover, the photon flux from gravitino decays has to be consistent with observations. The diffuse extra-galactic gamma ray flux of energy E from the gravitino decays is described by a integral over red-shift z given by [18]

$$F(E) = E^2 \frac{dJ}{dE} = \frac{2E^2}{m_{\tilde{G}}} C_\gamma \int_1^\infty dy \frac{dN_\gamma}{d(Ey)} \frac{y^{-3/2}}{\sqrt{1+\kappa y^{-3}}}, \tag{2.1}$$

where $y = 1 + z$ and dN_γ/dE is the gamma ray spectrum from the gravitino decay. Here

$$C_\gamma = \frac{\Omega_{\tilde{G}} \rho_c}{8\pi \tau_{\tilde{G}} H_0 \Omega_M^{1/2}} \quad \text{and} \quad \kappa = \frac{\Omega_\Lambda}{\Omega_M}. \tag{2.2}$$

For the radiative gravitino decay $dN_\gamma/dE = \delta(E - m_{\tilde{G}}/2)$ and Eq. (2.1) simplifies to [1]

$$F(E) = E^2 \frac{dJ}{dE} = \mathrm{BR}(\tilde{G} \to \gamma\nu) C_\gamma (1 + \kappa x^3)^{-1/2} x^{5/2} \theta(1 - x), \tag{2.3}$$

where $x = 2E/m_{\tilde{G}}$. In the case of three-body decays the hadronization of the produced particles and the resulting photon spectrum have been calculated using PYTHIA 6.4 [19]. The photons from the three-body decays come mostly from internal bremsstrahlung off leptons and from π^0 decays.

Using the original EGRET analysis [15], with a power law description of the extragalactic flux as

$$E^2 \frac{dJ}{dE} = 1.37 \cdot 10^{-6} \left(\frac{1 \text{ GeV}}{E} \right)^{0.1} \text{GeV cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1}, \tag{2.4}$$

in the energy range 30 MeV to 100 GeV, severe bounds on gravitino decays and thus on the allowed combinations of gravitino masses and R-violating couplings can be derived. For comparison, predictions for photonic spectra from gravitino decays through neutrino–neutralino mixing, and also possible antimatter signatures of gravitino dark matter, have recently been studied in [18] and [20].

3 Flavour Structure and Hierarchies of R-violating Operators

The implication of radiative gravitino decays as compared to the tree-level ones, clearly depends on the flavour structure of the R-violating operators involved. For higher generations the radiative decay widths become larger and the tree-level diagrams suppressed due to limited phase space. Most phenomenological studies assume a single operator-dominance. This can be motivated by the fact that the Yukawa couplings that generate fermion masses also have large hierarchies. However, in principle, one may try to relate R-violating hierarchies to those of fermion masses [21, 22], using models with family symmetries. When exact, the latter allow only the third generation fermions to become massive, while the remaining masses are generated by the spontaneous breaking of this symmetry (see below). If R parity is violated, couplings with different family charges will also appear with different powers of the family symmetry-breaking parameter, and thus with different magnitudes.

Moreover, one would have to appropriately take into account mixing effects. Indeed, even with the common assumption of single R-violating operator dominance, this would be true only for the basis of current eigenstates for quarks and leptons, while, in the mass-eigenstate basis, there would be several operators corresponding to the original dominant one in the current basis. In addition, the fact that there are strict bounds on some operators, implies that mixing effects may in given models generate additional bounds on couplings that at a first glance look less constrained. This has been analysed in detail in [22], where it was shown that in theories with strong correlations between operators (such as left-right symmetric models), the effects can be particularly significant.

The starting point in such considerations, is to assume a $U(1)$ flavour symmetry, with the charges of the Standard Model fields denoted as in Table 1.

	Q_i	$\bar{U}_i$	$\bar{D}_i$	L_i	$\bar{E}_i$	N	H_2	H_1
U(1)	α_i	β_i	γ_i	c_i	d_i	e_i	$-\alpha_3-\beta_3$	w

Table 1: *Notation for possible U(1) charges of the various Standard Model fields, where i is a generation index.*

The flavour charge of H_2 is chosen so that the operator that generates the top quark mass ($Q_3\bar{U}_3H_2$) has a zero U(1) flavour charge and thus is allowed at zeroth order, as it should be, since the top quark is significantly heavier than the rest. The remaining matrix elements may be generated when the U(1) symmetry is spontaneously broken [23, 24] by fields θ, $\bar{\theta}$ that are singlets of the Standard Model gauge group, with U(1) charges that are in most cases taken to be ±1, respectively. For instance, for $\alpha_i=\beta_i$ and $|\alpha_3-\alpha_2|=\pm1$ as in [9], the charm mass comes about by a term $Q_2\bar{U}_2H_2\left(\langle\theta\rangle/M\right)$ or $Q_2\bar{U}_2H_2\left(\langle\bar{\theta}\rangle/M\right)$ where M is the heavy scale of the theory.

One may generalise the above to non-abelian flavour symmetries, and, as an example, the following mass matrices have been proposed [25] :

$$M^{\rm up}\propto\begin{pmatrix}0&\epsilon^3&\epsilon^3\\ \epsilon^3&\epsilon^2&\epsilon^2\\ \epsilon^3&\epsilon^2&1\end{pmatrix},\quad M^{\rm down}\propto\begin{pmatrix}0&\bar{\epsilon}^3&\bar{\epsilon}^4\\ \bar{\epsilon}^3&\bar{\epsilon}^2&\bar{\epsilon}^2\\ \bar{\epsilon}^3&\bar{\epsilon}^2&1\end{pmatrix},\quad M^{\ell}\propto\begin{pmatrix}0&\bar{\epsilon}^3&\bar{\epsilon}^4\\ \bar{\epsilon}^3&\bar{\epsilon}^2&\bar{\epsilon}^2\\ \bar{\epsilon}^3&\bar{\epsilon}^2&1\end{pmatrix}.$$

When diagonalising these matrices, the fermion mass hierarchies and mixing are well reproduced for appropriate values of ϵ, $\epsilon\sim\bar{\epsilon}^2\sim0.04$. In general, in the models appearing in the literature, the relative flavour charges in Table 1 and thus the exact structure of the mass matrices are determined by the GUT multiplet structure (and the requirement that particles in the same GUT multiplet have the same charge). Nevertheless, in all cases, the observed fermion hierarchies require smaller charges for the operators of the higher generations (zero for the top Yukawa mass terms, but also for the bottom and tau in a supersymmetric model with large $\tan\beta$). This implies that, independently of the specific flavour and GUT structure of the theory, *and unless extra fields with a non-zero flavour charge are involved in the generation of R-violating couplings* [22], operators that contain fields of the third generation should be naturally larger.

One has also to worry about the overall suppression of the R-violating couplings with respect to the dominant Yukawa ones. However, this may arise either from a small $\tan\beta$ in supersymmetric models, from the form of the Kähler potential, or from additional, model dependent, features of the theory that may involve extra fields and symmetries.

Along these lines, one may also understand how it could be possible to only have dominant $\bar{U}_3\bar{D}_j\bar{D}_k$ operators. The obvious step, to also ensure the absence of any unacceptable proton decay, is to first eliminate lepton-number violating operators by imposing a lepton triality, under which the fields transform as

$$Z_3 : (Q, \bar{U}, \bar{D}, L, \bar{E}, H_1, H_2) \to (1, 1, 1, a, a^2, 1, 1). \tag{3.1}$$

This allows only the baryon-number-violating operators and the mass terms, while forbidding lepton-number-violating ones[4]. In this construction bilinear R-violation would also be disallowed.

To allow only lepton-number violating operators, we could work instead with a baryon triality, such as in [9]

$$Z_3 : (Q, \bar{U}, \bar{D}, L, \bar{E}, H_1, H_2) \to (1, a^2, a, a^2, a^2, a^2, a). \tag{3.2}$$

Such a baryon triality would allow for bilinear R-violation. However, one may also envisage different structures where the symmetries forbid an LH_2 term while allowing trilinear lepton-number violating operators. An example of this is given by

$$Z_3 : (Q, \bar{U}, \bar{D}, L, \bar{E}, H_1, H_2) \to (1, a, 1, 1, 1, 1, a^2). \tag{3.3}$$

It is interesting to observe that in this case the term $\mu H_1 H_2$ would also be forbidden. This is due to charge correlations that arise from the above requirements, plus the need to allow Yukawa couplings that generate fermion masses. In this case, the μ-term would have to arise either radiatively [26], or through the Kähler potential [27]. The μ term could also be generated within the framework of the NMSSM [28], via a singlet field with appropriate charge; in which case a term SLH_2 would also be allowed. Baryon number violating operators would be allowed at subdominant orders, due to a term $SS\bar{U}\bar{D}\bar{D}$ which is significantly suppressed; moreover, this is not the complete picture, since to explain fermion mass hierarchies one would have to introduce flavour dependent charges, which could further suppress R-violating operators, particularly for the lighter generations that are dangerous for proton decay (see discussion below).

[4] A flavour-dependent generalisation of this symmetry has been discussed in [10]. In that scenario, consistent solutions were found containing only a subclass of operators violating lepton number ($LL\bar{E}$) and baryon number ($\bar{U}\bar{D}\bar{D}$). Thus it is possible to have both lepton and baryon number violation without disturbing proton stability.

From the above, it is clear that whether bilinear or trilinear R-violation dominates is directly linked to the symmetries of the underlying theory, and phenomenological information would be a valuable probe of this symmetry structure.

Would these considerations be sufficient to understand the structure of the R-violating operators on the basis of positive experimental results? As already discussed, even in the case of one dominant operator, for fermions in the basis of current eigenstates, mixing effects will induce non-zero coefficients for related operators in the basis of mass eigenstates. These will be suppressed by the mixing parameters with respect to the dominant operator, but will not be zero, and this may affect phenomenological and cosmological predictions. We should also keep in mind that experiments only provide information on the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix $V^{\rm CKM} = V_u^{L\dagger} V_d^L$ [29], and that one can construct theoretical models where the left quark mixing is in either the up or the down sector, or both. Similarly, lepton mixing comes from the product of matrices of charged leptons and neutrinos, with the additional complication that, for the latter, we have the possibility of both Dirac and Majorana mass terms (the recent neutrino data indicate the existence of neutrino masses and contain the possibility that right-handed neutrinos do exist). For instance, in the above mass matrices from [25], the quark mixing is given by

$$V_u^{L,R} \approx \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ -\epsilon & 1 & \epsilon^2 \\ \epsilon^3 & -\epsilon^2 & 1 \end{pmatrix}, \quad V_d^{L,R} \approx \begin{pmatrix} 1 & \bar\epsilon & \bar\epsilon^4 \\ -\bar\epsilon & 1 & \bar\epsilon^2 \\ \bar\epsilon^4 & -\bar\epsilon^2 & 1 \end{pmatrix}.$$

Due to this mixing, an R-violating operator is in fact a sum of terms. For instance, the mixing matrices above would lead to the following interesting mixings:

$$\begin{array}{l} (\bar U_3 \bar D_i \bar D_j)' = \bar U_3 \bar D_i \bar D_j - \epsilon^2 \bar U_2 \bar D_i \bar D_j + \epsilon^3 \bar U_1 \bar D_i \bar D_j + \dots \\ (L_1 Q_3 \bar D_3)' = L_1 Q_3 \bar D_3 - \bar\epsilon^2 L_1 Q_3 \bar D_2 + \bar\epsilon^4 L_1 Q_3 \bar D_1 + \dots \end{array} \tag{3.4}$$

These mixings are particularly important since the dominant couplings here have massive final states. As we shall see, mixing also opens up for final states forbidden by the gauge symmetry of the couplings. However, more generically, we observe the following:

(i) The right-handed quark mixing (relevant for $\bar U$ and $\bar D$) is essentially not constrained by the data. Therefore, in a model with left-right asymmetric mass matrices, one could also imagine a theory with a minimal mixing in the right-handed sector, in which case a dominant $\bar U_3 \bar D_i \bar D_j$ flavour would be the only relevant one, and a gravitino with $m_{\tilde G} < M_t$ would be essentially stable.

(ii) For the left quark mixing (relevant for Q), we know the numerical values from V_{CKM} (where for instance the 2-3 mixing is a factor of ≈ 0.04). Thus, a coupling $\lambda' L_3 Q_3 \bar D_3$, would in principle also imply the coupling $0.04\lambda' L_3 Q_2 \bar D_3$.

(iii) The left lepton mixing (relevant for L) is constrained by the lepton data (large 1-2 and 2-3 mixing, and small 1-3 mixing).

We see that there are several flavour choices that can lead to significant effects on the decays under discussion, particularly in the cases where the available phase space is limited. This will be explored in the next Section.

4 Flavour Effects in Gravitino Decays

4.1 Flavour Effects for $LL\bar{E}$ Operators

From the nine R-violating $LL\bar{E}$ operators, six can potentially give rise to both loop and tree-level decays (a common flavour in $\bar{E}$ and one of the L fields is needed to form the loop):

$$L_{2,3}L_1\bar{E}_1, \quad L_{1,3}L_2\bar{E}_2, \quad L_{1,2}L_3\bar{E}_3, \tag{4.1}$$

while three have only three-body decays

$$L_2L_3\bar{E}_1, \quad L_1L_3\bar{E}_2, \quad L_1L_2\bar{E}_3. \tag{4.2}$$

The cases with a muon or a tau in the loop were discussed in [2]. For an electron in the loop, the photonic gravitino decays are very suppressed due to the electron mass, and the tree-level decays dominate unless the gravitino becomes extremely light. This is demonstrated in Figure 3, where we plot λ_{max}, the maximum allowed coupling, versus the gravitino mass, assuming a common slepton mass of 200 GeV. In doing so, we demand that:

(i) there is one dominant coupling,

(ii) the gravitinos can be dark matter, with a lifetime of at least 10 times the current age of the universe and that

(iii) photon production from the gravitino decays, as calculated by Eq. (2.1), is consistent with the bounds on the photon spectrum given in Eq. (2.4).

We see that while the photon flux from two-body loop decays puts strong bounds on couplings that lead to loops with muons (blue) or taus (red), the couplings with electron loops (green) are only affected by the three-body decay photons down to very small gravitino masses. For the couplings (4.2) with no loop diagrams the bounds are thus correspondingly weak, and follow the bound for $L_2L_1\bar{E}_1$. As expected, the neutrino flavour has no effect on the bounds from the radiative decay, so results for e.g. $L_1L_2\bar{E}_2$ and $L_3L_2\bar{E}_2$ are virtually identical, save for minute differences near the slepton threshold.

It is also interesting to note that in the terms with only three-body decays in (4.2), there is always the possibility for tau production in the final state. Indeed, for $L_1L_2\bar{E}_3$ an SU(2) singlet τ is always produced if kinematically allowed, while for $L_2L_3\bar{E}_1$ and $L_1L_3\bar{E}_2$ an SU(2) doublet τ is produced, unless the gravitino mass becomes lower or comparable to the tau. In this case the factor $L_{1,2}L_3$ would only contribute to the tree-level decay via the $\nu_\tau e$ or $\nu_\tau\mu$ term. This is observed in Figure 3, where in the three-body dominated region, bounds on e.g. $L_1L_3\bar{E}_3$ are stricter than the bounds on $L_1L_2\bar{E}_2$ and $L_2L_1\bar{E}_1$, due to the extra photons from the tau decay. One can also notice that the bound on $L_2L_1\bar{E}_1$ is slightly better than on $L_1L_2\bar{E}_2$; this is due to more bremsstrahlung from electrons than from muons in the final state.

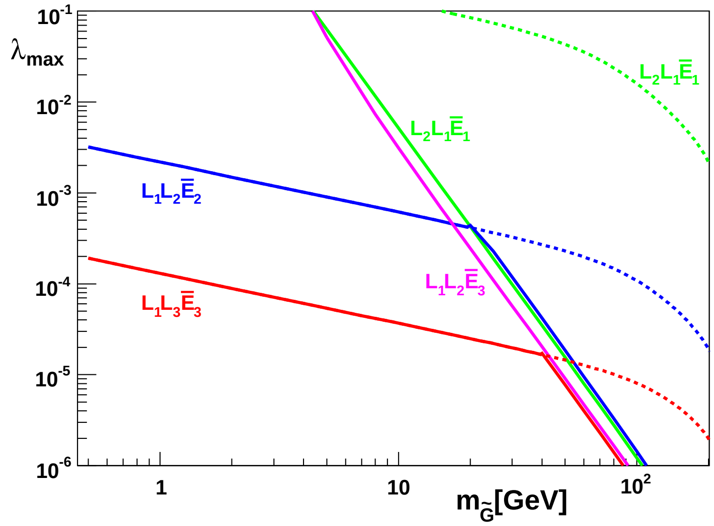

Figure 3: *Maximum value* $\lambda_{\max}$ *of R-violating couplings versus gravitino mass, for* $LL\bar{E}$ *operators. Bounds shown as dashed lines are when considering radiative loop decays only, solid lines include photons from three-body decays. The sparticle masses are 200 GeV.*

4.2 Flavour effects in $LQ\bar{D}$ operators

Out of the 27 R-violating $LQ\bar{D}$ operators, only the following nine can potentially give rise to both loop and tree-level decays (a common flavour in Q and $\bar{D}$ is needed to form the loop):

$$L_{1,2,3}Q_1\bar{D}_1, \quad L_{1,2,3}Q_2\bar{D}_2, \quad L_{1,2,3}Q_3\bar{D}_3, \tag{4.3}$$

while the remaining 18 have only three-body decays.

In Figure 4 we show a comparison of the partial lifetime for the loop and tree-level decays for the second and third generation. We choose L_3, but this has little significance. Comparing to the results for the $LL\bar{E}$ operators in [2], we observe that with a b quark instead of a τ in the loop, radiative decays dominate over the three-body ones for a significantly wider range of gravitino masses, up to 40 GeV, for the same sparticle masses (200 GeV). This arises both due to the higher fermion mass in the loop, but also due to the two bottom masses in the final state. The coupling $L_iQ_3\bar{D}_3$ gives rise to either $\ell_i t\bar{b}$ or $\nu_i b\bar{b}$, and the first term is forbidden by phase space up to high gravitino masses, which can be seen as a bump in the $L_3Q_3\bar{D}_3$ three-body lifetime near threshold.[5]

As in the previous subsection we can put constraints on the couplings of the $LQ\bar{D}$ operators from gamma rays and gravitino lifetime, as a function of the gravitino mass. The resulting bounds for operators with both loop and tree-level decays are shown in Figure 5. Due to the increased dominance of two-body decays compared to the pure lepton operators,

[5]Close to the $b\bar{b}$ threshold at ~ 10 GeV hadronization effects will become important for the three-body decay. This is not considered here as the two-body decay clearly dominates in this mass range.

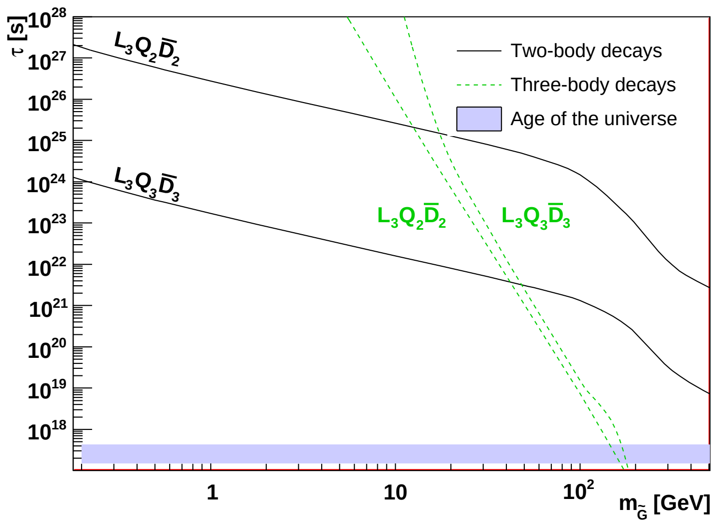

Figure 4: *Comparison of partial lifetime versus gravitino mass for two-body loop decays and three-body tree-level decays for the* $L_2Q_3\bar{D}_3$ *and* $L_3Q_3\bar{D}_3$ *couplings. Couplings have all been set to* $\lambda' = 0.001$.

we have even stronger coupling bounds, in particular for the $L_iQ_3\bar{D}_3$ couplings, and there is now also a significant constraint on the first generation loops, i.e. $L_iQ_1\bar{D}_1$, for low gravitino masses.

If the mass of the gravitino is close to the lightest possible meson for one particular operator, we may no longer neglect hadronisation effects from the formation of single mesons, as opposed to the QCD jet interpretation of the quarks in the three-body decay. In the simplest case we would have a two-body final state with a lepton and a meson, such as a pion or a kaon, or even heavier mesons if allowed by the structure of the R-violating operator and the mass of the gravitino. For instance, the operator $L_3Q_1\bar{D}_1$ will lead to $\tau\pi^+(\tau u\bar{d})$ or $\nu_\tau\pi^0(\nu_\tau d\bar{d})$, and similar considerations hold for other flavour combinations.

However, since the decay into single mesons is only relevant for low gravitino masses, this issue can be neglected for operators allowing loop decays. This is because the constraint from the loop decay to photon and neutrino is in all cases a lot more stringent than the constraint arising from the decay into mesons at these gravitino masses.

For operators not permitting loop decays the situation is different, but in the cases with light mesons the resulting gamma ray constraints are so weak that other constraints on the couplings are more important [12]. Thus the only cases where decays into single mesons can have some effect are for operators which give heavy mesons, *i.e.* B or D mesons. In these cases there can be small modifications on the constraints in the range of gravitino masses close to the heavy quark masses, but the nature of heavy quarks as kinematically equivalent to their corresponding mesons should limit this effect.

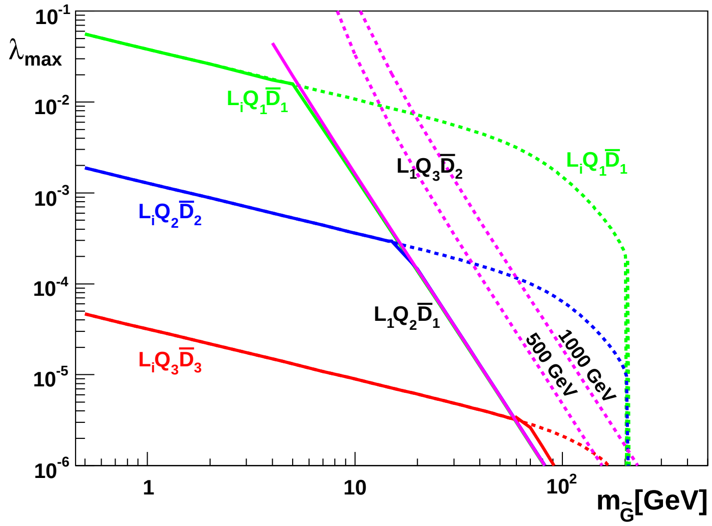

Figure 5: *Maximum value $\lambda_{\max}$ of R-violating couplings versus gravitino mass, for $LQ\bar{D}$ operators. The sparticle masses are 200 GeV, except where indicated.*

4.3 Flavour effects in $\bar{U}\bar{D}\bar{D}$ operators

In this case, we only have tree-level gravitino decays, and of particular interest is the possibility of gravitino decays via a dominant $\bar{U}_3\bar{D}_j\bar{D}_k$ operator. For light gravitinos, since top production in the final state is kinematically forbidden, decays will arise either

(i) due to t-c mixing and other possible mixings,

(ii) or from four-body final states with an off-shell top quark and possibly an off-shell W, and with at least one massive final state particle (b-quark).

The first case is expected to dominate since the second is very suppressed, and the dominant decay width should be a function of the right-handed $\bar{U}_3 - \bar{U}_2$ mixing. In this case $\lambda_{\max}$ can be large, with interesting phenomenological implications that we discuss in the next Section. Another interesting feature of mixing is that it opens up gravitino decay channels that were disallowed by the flavour structure of the superpotential, e.g. the possibility of two b (or $\bar{b}$) in the final state.

Both of these effects are shown in Figure 6, where we plot the partial lifetime for a selection of gravitino decay modes as a function of gravitino mass. We assume a dominant coupling $\lambda''_{312} = 1.0$ that for low gravitino masses relies on mixing effects in the decays. The coupling is chosen large to minimize the lifetime. We illustrate a possible realization of mixing with the mixing matrices in Eq. (3), taking $\epsilon = 0.04$ and $\bar{\epsilon} = 0.20$. As expected, it is the t-c mixing that dominates gravitino decays at low masses, and the gravitino is long lived enough to be dark matter for a large range of masses. Only for gravitino masses above 200 GeV, when the top production threshold has been passed with good margin, do the top channels dominate and the gravitino becomes disallowed as a dark matter candidate due to its short lifetime.

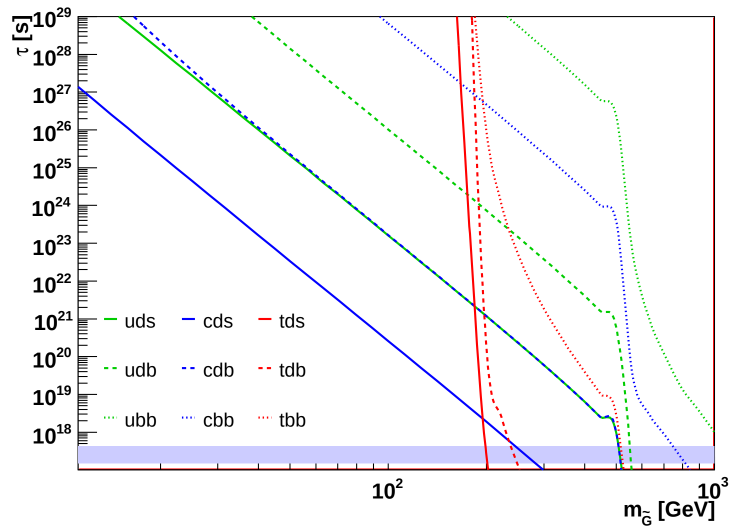

Figure 6: *Partial lifetime versus gravitino mass for gravitino decays into various quark final states with* $\lambda''_{312} = 1.0$*,* $\epsilon = 0.04$ *and* $\bar{\epsilon} = 0.20$*. All squark masses have been set to* 500 *GeV.*

We find that changing between the three possible λ''_{3jk} couplings only changes the relative importance of the down type quarks in the gravitino decay, e.g. for $\lambda''_{313} = 1.0$, $\tilde{G} \to cdb$ is the dominant decay channel for low masses. Among the channels that are closed in the absence of mixing, we only show decays to b quark pairs. The lighter pairs have very similar behaviour to other light quark pairs. We see that the probability of two b quarks in the final state is negligible because of the large suppression due to mixing and kinematics when compared to other decay channels. Other choices for the mixing matrices only change the relative importance of the different decay channels, not the behaviour as a function of gravitino mass.

In Figure 7 we also show the resulting bounds on the λ'' couplings when considering the photon spectrum as in the previous subsections. We notice that the first two generations have a log-linear behaviour in terms of the gravitino mass, with equal slopes. The difference in scaling is due to different squark masses. With the same squark mass, the two curves would be indistinguishable. The importance of mixing effects are again shown for the λ''_{312} coupling: the opening up of decays through mixing strengthens the bounds on that coupling.

In general, due to the structure of these operators we produce either three quarks or three anti-quarks. If there is sufficient phase space, one could imagine that we can end up with two-body final states with a baryon and a meson for very light gravitino masses. However, we need to keep in mind that the lightest flavours for λ'', in particular λ''_{112} and λ''_{113}, are extremely constrained from double nucleon decay and neutron-antineutron oscillations respectively [30].

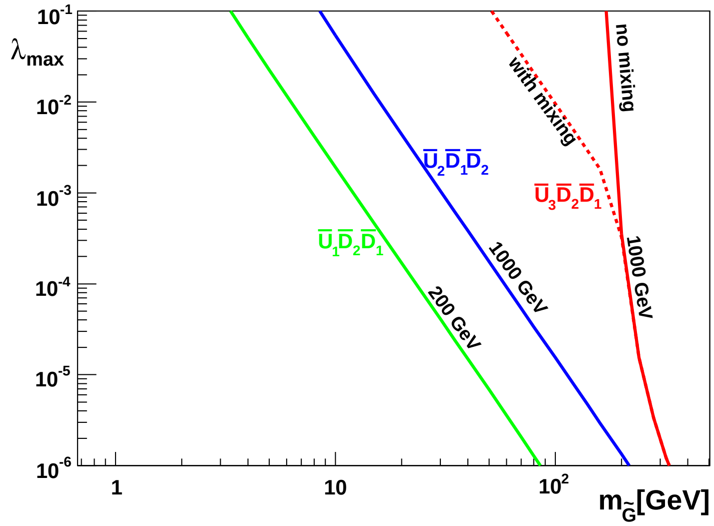

Figure 7: *Maximum value* $\lambda_{\max}$ *of R-violating couplings versus gravitino mass, for* $\bar{U}\bar{D}\bar{D}$ *operators. The squark masses are as indicated.*

5 Prospects for R-violation in colliders

For R-violating couplings above 10^{-6} for 100 GeV sparticle masses, and with a scaling that for most operators is a simple proportionality relation, one would expect interesting signatures like multi-lepton and/or multi-jet events in the final state of sparticle production in a collider. Depending on the flavour of the R-violating operator, the nature of the NLSP, and the respective $\lambda_{\max}$ that we found in the previous section, one would generically expect either:

(i) possible observable single superparticle productions, if λ can be sufficiently large [5, 6, 31],

(ii) MSSM production of sparticle pairs followed by R-violating decays of the NLSP, for the flavours where $\lambda_{\max}$ is smaller than $\sim 10^{-2}$, or

(iii) no R-violatig decays of the NLSP inside detectors for very small $\lambda_{\max}$ (smaller than $\sim 10^{-6}$), with some cross-over region where displaced vertices could be observed.

From the results shown in the previous Section, the observation of single sparticle production at the LHC is almost entirely excluded in the gravitino dark matter scenario for operators that give loop decays with second or third generation loop-particles, due to the strict bounds from gamma rays. For dominant three-body decays the same conclusion holds unless the gravitino mass is small ($m_{\tilde{G}} \lesssim 10$ GeV). Thus the possible astrophysical observation of gravitino decays will have important consequences for LHC expectations, and vice versa. It is worth noting that this conclusion, for the case of dominant two-body decay, is only weakly dependent on the assumed masses of the other sparticles, as can be seen from the insensitivity of $\lambda_{\max}$ to large changes in intermediate sparticle mass, see Figure 5 of [2].

For no operator do the constraints considered here eliminate the possibility of seeing R-violating decays in colliders, but the $L_i L_3 \bar{E}_3$ and $L_i Q_3 \bar{D}_3$ operators allow only a very restricted coupling range for intermediate to high gravitino masses. Indeed, even for couplings of the order of 10^{-6} it should be possible to detect the R-violating NLSP decays [31].

The discovery of supersymmetry at the LHC and the reconstruction of a neutralino NLSP has been shown to be possible [32] at the same level or better than for R-parity conserving scenarios when one considers the lepton number violating operators. This is due to the numerous leptons expected in the final state. However, for the case of a $\bar{U}\bar{D}\bar{D}$ operator, assumptions have to be made, either for the production of additional leptons in the event from cascade decays, or for heavy flavours that can be tagged. The heavier the flavours, the better the detection prospects due to flavour tagging or top reconstruction.

Decays of the NLSP are highly dependent on the combination of NLSP flavour and dominant R-parity violating operator flavour. If these flavours are the same, all NLSP decays should be rapid two-body decays if kinematically allowed. In other scenarios, three or even four-body decays are the leading decays, with resulting suppression due to phase space and heavy virtual particles. For an $\bar{U}_3 \bar{D}_j \bar{D}_k$ operator, we have the following particular implications:

(i) we have the possibility of large R-violating coupling with resulting resonant single stop production [34] or single gluino top production [35]. Moreover, for a large $\lambda''_{3jk}\lambda''_{i3k}$ product, one may observe interesting signatures in single top–bottom production [36].

(ii) if the NLSP is a neutralino with a mass larger than the top, it should have a rapid three-body decay with a top in the final state, on the other hand, if the neutralino is lighter than the top, then it should decay via either subdominant operators or mixing effects, which may well enhance its decay rate enough for it to decay within the detector, giving a displaced vertex.

Taken together this would imply the interesting possibility of sparticle production via one operator, and decay via a different one.

6 Conclusions

We have studied slow gravitino decays originating from lepton or baryon number violating operators in R-violating supersymmetry, focusing on the flavour structure of the theory. We found that the dominant decay modes, and thus the final state products are particularly sensitive to the hierarchies of R-violating operators and exhibit distinct correlations, which we have analysed. Already the dominance of trilinear R-violating couplings over bilinear modes implies the presence of symmetries that, among others, have interesting implications for the μ-term.

A more detailed study of the flavour dependence of the operators has determined the ratio between (i) the tree-level gravitino decays to three fermions and (ii) the two-body loop decays into a photon and a neutrino, which in turn puts strong bounds on the maximal value of the allowed R-violating couplings. Bounds from photon spectra are much stricter than the ones from the requirement on the gravitino lifetime, and thus strongly constrain the respective operators, particularly $L_i L_3 \bar{E}_3$ and $L_i Q_3 \bar{D}_3$ that involve a τ and a bottom-quark in the loop. On the other hand, for operators without photonic decays larger coupling constants are possible, particularly in the case of phase space suppressions due to the presence of heavy fermions in the final state. Moreover, mixing effects turn out to be crucial in correctly deriving the relative magnitude of the various contributions, and play a significant role for decay modes with phase space suppression and particularly for the ones generated by $\bar{U}_3 \bar{D}_j \bar{D}_k$.

In all cases, the bounds on the R-violating couplings from the cosmological requirements are compatible with visible signatures at colliders, which can vary from single superparticle production (for flavours where a larger coupling constant is allowed) to MSSM production and R-violating decays (for the smaller couplings). Particularly for the operator flavours that would lead to predominantly photonic gravitino decays, giving strong constraints on the couplings, interesting event properties such as vertex displacement might be expected.

A Photonic Gravitino Decays

The photonic decays of the gravitinos have been calculated in [2]. The rate for the radiative decay $\tilde{G} \to \gamma\nu$ with the loop fermion f is given by

$$\Gamma = \frac{\alpha\lambda^2\, m_{\tilde{G}}}{2048\pi^4}\,\frac{m_f^2}{M_p^2}\overline{|\mathcal{F}|^2}, \tag{A.1}$$

where $M_p = (8\pi G_N)^{-1/2} = 2.4 \times 10^{18}$ GeV is the reduced Planck mass, and[6]

$$\overline{|\mathcal{F}|^2} = \frac{1}{12}|c_1|^2 + \frac{2}{6}|c_2|^2 + \frac{1}{6}\mathrm{Re}\,(c_1^* c_2), \tag{A.2}$$

with

$$\begin{aligned} c_1 &= 2[(m_{\tilde{G}}^2 - m_{\tilde{f}}^2 + m_f^2)C_0^{(a)} + \Delta B_0^{(1)}], \\ c_2 &= 2[m_f^2 C_0^{(a)} + m_{\tilde{f}}^2\, C_0^{(b)} + \Delta B_0^{(2)}], \end{aligned} \tag{A.3}$$

where, in the notation of `LoopTools` [37, 38], we have

$$\begin{aligned} C_0^{(a)} &= C_0(m_{\tilde{G}}^2, 0, 0, m_{\tilde{f}}^2, m_f^2, m_f^2), \\ C_0^{(b)} &= C_0(m_{\tilde{G}}^2, 0, 0, m_{\tilde{f}}^2, m_f^2, m_{\tilde{f}}^2), \end{aligned}$$

[6] Here, we correct a minor error in that paper due to a misprint in the gravitino spin-sum taken from [33], where the sign in Eq. (4.31) should be $(\not{p} + m_{3/2})$.

$$
\begin{aligned}
\Delta B_0^{(1)} &= 2B_0(m_{\tilde{G}}^2, m_{\tilde{f}}^2, m_f^2) - B_0(0, m_{\tilde{f}}^2, m_f^2) - B_0(0, m_f^2, m_f^2), \\
\Delta B_0^{(2)} &= B_0(m_{\tilde{G}}^2, m_{\tilde{f}}^2, m_f^2) - B_0(0, m_{\tilde{f}}^2, m_f^2).
\end{aligned}
\tag{A.4}
$$

The C_0 are three-point functions corresponding to Fig. 1 (a) and (b), whereas the ΔB_0 are finite differences of two-point functions.

B Three-body Gravitino Decays

The three-body decays of gravitinos have been calculated in [4], where extensive analytic formulas were derived. Here, we only comment on the spin summed squared amplitudes, and refer to the original paper for the full computation.

The full squared amplitude (summed over spins) for the gravitino decay $\tilde{G} \to l_{ijk} \to \nu_i \ell_j \bar{\ell}_k$ is the sum of three individual squared amplitudes plus three interference terms. These arise since the gravitino can couple to all the particles involved in the R-violating operator. Then, for the case where the gravitino couples to a neutrino and a sneutrino, one has

$$
\begin{aligned}
|M_a|^2 = \frac{1}{3} \frac{l_{ijk}^2}{M_p^2 (m_{jk}^2 - m_{\tilde{\nu}_i}^2)^2} &(m_{\tilde{G}}^2 - m_{jk}^2 + m_{\nu_i}^2)(m_{jk}^2 - m_{\ell_j}^2 - m_{\ell_k}^2) \\
&\times \left(\frac{(m_{\tilde{G}}^2 + m_{jk}^2 - m_{\nu_i}^2)^2}{4 m_{\tilde{G}}^2} - m_{jk}^2 \right),
\end{aligned}
\tag{B.1}
$$

where $m_{jk}^2 = (p_j + p_k)^2$, with $p_{j,k}$ the four-momenta of the respective particles. The remaining squared amplitudes $M_{b,c}$, where the gravitino couples to the charged lepton of the doublet and the singlet charged lepton respectively, are given by the same formula, when substituting the appropriate flavours in the vertices and the propagator. The interference terms are of the form

$$
\begin{aligned}
2\mathrm{Re}\,(M_a M_b^\dagger) = \frac{1}{3} \frac{l_{ijk}^2}{M_p^2 (m_{jk}^2 - m_{\tilde{\nu}_i}^2)(m_{ik}^2 - m_{\tilde{\ell}_j}^2)} &\Big[(m_{ik}^2 m_{jk}^2 - m_{\tilde{G}}^2 m_{\ell_k}^2 - m_{\nu_i}^2 m_{\ell_j}^2) \\
\times \left((m_{\tilde{G}}^2 + m_{\ell_k}^2 - m_{\nu_i}^2 - m_{\ell_j}^2) - \frac{1}{2 m_{\tilde{G}}^2} (m_{\tilde{G}}^2 + m_{jk}^2 - m_{\nu_i}^2)(m_{\tilde{G}}^2 + m_{ik}^2 - m_{\ell_j}^2) \right) \\
+ \frac{1}{2} (m_{ij}^2 - m_{\nu_i}^2 - m_{\ell_j}^2)(m_{jk}^2 - m_{\ell_j}^2 - m_{\ell_k}^2)(m_{ik}^2 - m_{\nu_i}^2 - m_{\ell_k}^2) \\
- \frac{m_{\nu_i}^2}{2} (m_{jk}^2 - m_{\ell_j}^2 - m_{\ell_k}^2)^2 - \frac{m_{\ell_j}^2}{2} (m_{ik}^2 - m_{\nu_i}^2 - m_{\ell_k}^2)^2 \\
- \frac{m_{\ell_k}^2}{2} (m_{ij}^2 - m_{\nu_i}^2 - m_{\ell_j}^2)^2 + 2 m_{\nu_i}^2 m_{\ell_j}^2 m_{\ell_k}^2 \Big].
\end{aligned}
\tag{B.2}
$$

For $LQ\bar{D}$ operators the results are similar, and found by replacing the SU(2) doublet field L by Q, and the SU(2) singlet $\bar{E}$ by $\bar{D}$, and summing over colours. For $\bar{U}\bar{D}\bar{D}$ operators we also have similar amplitudes and interference terms. Again the contributions can be read off from Eq. (B.1) and (B.2), modulo colour and symmetry factors that arise from the possibility of two identical particles in the final state.

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