

# Monitoring Sessile Droplet Evaporation on a Micromechanical Device

A. Prasad,<sup>a</sup> A. T.-H. Lin<sup>b</sup>, V. R. Rao<sup>c</sup> and A. A. Seshia<sup>a</sup>

A bulk acoustic mode micro-electro-mechanical dual resonator platform is utilised to study the evaporation of sub-microliter water droplets from the surface of the resonator. An analytical formulation for the observed frequency shift and the measured dependence of resonant frequency on the modes of evaporation is presented and seen to be consistent with optically derived data. The resonators access only a thin layer of the liquid through shear contact and, hence, the response is not affected by the bulk mass of the droplet to first order. A relation between droplet contact area and elapsed time was established for the evaporation process and is used to derive a value of the diffusion coefficient of water in air that is found to be in reasonable agreement with literature values. This work introduces a new tool for the electro-mechanical monitoring of droplet evaporation with relevance to applications such as biosensing in liquid samples of sub-microliter volumes.

## INTRODUCTION

Micro-Electro-Mechanical Systems (MEMS) technology enables batch fabrication of miniaturized mechanical devices that may be co-integrated with chip-scale electronics and microfluidic handling systems to realize miniaturized and portable platforms for bio-chemical sensing and environmental monitoring [1]. Such platforms may utilize significantly smaller quantities of sample and reagents and potentially offer low cost routes towards volume manufacturing. Micro-/nano-mechanical sensors based on the modulation of mechanical properties on interaction with the measurand, have evoked much academic and translational interest in recent years [2]. Particular implementations of such mechanical transducers are micro/nanomechanical resonators where the interaction between analyte/surrounding media and the resonator surface produces a shift in the resonant frequency response. These ‘dynamic’ mode sensors have shown exquisite degree of sensitivity in vacuum enabling single molecule detection [3], zeptogram-scale mass sensing [4], and sub-attoneutron force detection [5]. However, when these sensors are operated in air and liquid environments, they suffer from degradation in performance due to high levels of fluidic damping. Viscous drag, squeeze film damping and acoustic radiation loss limit achievable resonator quality factors and, hence, the sensor resolution. Further, the presence of liquid can potentially interfere with electrical transduction of the mechanical response. There has been much recent research in attempting to address the problems associated with operation in liquids and several innovative designs, for example embedding fluidic channels inside the resonator as in the case of suspended microchannel resonators [6-8], isolating the transduction element from the liquid as in the

case of some piezoelectric devices [9], and electro-thermal actuation and piezoresistive sensing methods [10] have been employed with promising results. However, these techniques require complex fabrication processes, and may be limited by transport issues in confined channel geometries and the lack of precise chemical definition of the sensor surface [11].

An alternative approach to overcome the aforementioned issues has been previously reported by employing a dual-resonator platform (DRP) [12]. In this approach, two identical micromachined square plate resonators are coupled by a thin mechanical beam; one resonator is referred to as the *Transducer* resonator (TR) and the other as the *Sensor* resonator (SR). While the SR is interfaced with the liquid, the TR is electrically addressed to enable readout of the modulated frequency response. This scheme also allows relatively easy handling of liquid samples without significant constraints on electrical interfacing and device performance.

Apart from applications involving sensing of attached mass in air or vacuum environments (e.g. thin-film monitoring, particulate monitoring etc.), the DRP can also be employed in situations where monitoring of physical processes in liquid media in contact with the SR is necessary. One such application is monitoring the evaporation of sessile droplets of liquids. Studies on the evaporation of droplets on flat surfaces have gathered tremendous interest over past decades with various studies being carried out to model the evaporation process [13-17]. Evaporation of droplets is studied for understanding the general wetting behaviour of liquids [18], in the industrial context of spray cooling and inkjet printing [19-23], in the context of nanodispensing [24], in biology for mapping DNA on flat substrates [25], in microfluidics for optimizing device biochip designs [26],

and dynamics of drying drops on nanostructured surfaces [27], and for understanding of various day-to-day phenomena involving the physical process of evaporation e.g. the coffee stain effect [28]. These studies are also of relevance in a biochemical sensing context in fluid, where the very small volumes of fluid involved may evaporate rapidly from the sensor surface in a stop-flow mode.

The Quartz Crystal Microbalance (QCM) has been previously used to investigate sessile droplet evaporation [29-30]. For instance, Joyce *et al.* used the QCM to monitor the extreme modes of droplet evaporation of a series of alcohols by modelling the evaporation process and its interaction with the radial sensitivity of the crystal. Previous work has also addressed M/NEMS based evaporation studies for investigating evaporation of microdroplets of diameters ranging from a few microns [31] to tens of microns [32-33]. These devices, based on the excitation of out-of-plane flexural resonances, are suitable for cases where droplet volumes are in picoliter and femtoliter range. In contrast due to the in-plane nature of the excited mode shapes in the resonators used in our work, regardless of the bulk volume of the droplet, only a small thin layer of the droplet in shear contact with the resonator is effectively probed. Any changes in this thin layer (for e.g. due to changes in contact area with the resonator and in some cases thickness of the layer itself) can be effectively sensed. Thus, these resonators find application where modes of evaporation (associated with shrinking droplet contact area) must be studied. The device platform reported in this paper can also be applied to study of the evaporation process of much larger droplet volumes (on the order of a few microliters or below), relevant to several practical applications outlined earlier, that are difficult to quantitatively analyze using existing optical techniques. Thus, the paper reports the use of the DRP to investigate evaporation of sessile droplets of DI water by direct electrical open-loop resonance frequency monitoring and the process of correlating the measured response due to the evaporating droplet with an analytical model of the physical process.

## THEORY

### Liquid mass loading on the Dual-Resonator Platform

The DRP consists of two identical thin silicon square plates, mechanically coupled by a thin coupling beam. The coupling beam is connected to the center points of the edges of the plate directly facing each other. The coupled system is driven in its coupled square-extensional (SE) modes and for analysis and experiments, the in-phase mode is monitored (Fig. 1).

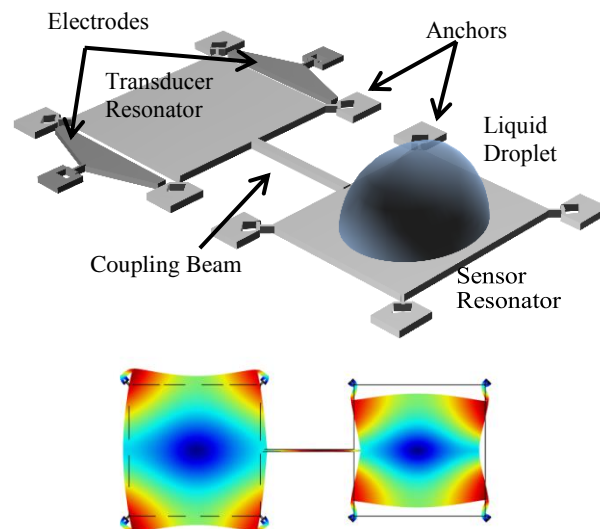


FIG. 1 (top) Schematic of the Dual Resonator Platform with a liquid droplet spotted on the Sensor Resonator, and (bottom) COMSOL simulation of out-of-phase square-extensional (SE) mode.

Using the Rayleigh-Ritz energy principle, the modal frequency  $\omega$  of such a vibrating system can be approximately determined by equating the maximum potential energy ( $PE_{Max}$ ) to the maximum kinetic energy ( $KE_{Max}$ ) of the system. For a single square plate of side length  $L$  and thickness  $h$ , SE mode shape can be approximated by the superposition of the following displacement functions [34]:

$$u_x(x, t) = U_0 \sin\left(\frac{\pi x}{L}\right) \sin \omega t, -\frac{L}{2} < x < \frac{L}{2} \quad (1a)$$

$$u_y(y, t) = U_0 \sin\left(\frac{\pi y}{L}\right) \sin \omega t, -\frac{L}{2} < y < \frac{L}{2} \quad (1b)$$

where,  $u_x(x, t), u_y(y, t)$  are the time dependent in-plane displacements and  $U_0$  is the maximum displacement along the axes. As the mode shape is symmetric about the center  $(0, 0)$  of the plate,  $PE_{Max}$  and  $KE_{Max}$  can be evaluated by calculating twice of the maximum energy components along either of the axes. Strain energy or potential energy stored per unit volume due to displacements along the X- axis is given by

$$\overline{PE}_x = \frac{1}{2} E_{Si} \epsilon_{xx}^2 \quad (2)$$

where,  $E_{Si}$  is the Young's Modulus of silicon for the SE mode and  $\epsilon_{xx}$  is the strain in the X- direction expressed in terms of displacement as:

$$\epsilon_{xx} = \frac{\partial u_x}{\partial x} \quad (3)$$

Hence, from eq. (2) and (3), we get:

$$\overline{PE}_x = \frac{E_{Si}\pi^2 U_0^2 \cos^2\left(\frac{\pi x}{L}\right) \sin^2 \omega t}{2L^2} \quad (4)$$

Strain energy stored in a differential element of volume  $dV(= dx dy dz)$  at a distance  $x$  from the center is  $d\overline{PE}_x(= \overline{PE}_x dV)$ . Integrating for the whole plate, we get the component of stored strain energy in the vibrating plate which is then multiplied by a factor of 2 to get the time-dependent relationship of the stored  $PE$  for the DRP:

$$PE = 2 \int_V \int \overline{PE}_x dV = \frac{E_{Si}\pi^2 h U_0^2 \sin^2 \omega t}{2} \quad (5)$$

Hence, 
$$PE_{Max} = \frac{1}{2} E_{Si}\pi^2 h U_0^2 \quad (6)$$

Similarly, to calculate the total kinetic energy of the vibrating square plate, we again consider the differential element of mass  $dm(= \rho_{Si} dV)$  and proceed as following:

$$\begin{aligned} d\overline{KE}_x &= \frac{1}{2} dm \left( \frac{\partial u_x}{\partial t} \right)^2 \\ &= \frac{1}{2} \rho_{Si} \omega^2 U_0^2 \sin^2\left(\frac{\pi x}{L}\right) \sin^2 \omega t dV \end{aligned} \quad (7a)$$

$$KE = 2 \int_V \int d\overline{KE}_x = \frac{\rho_{Si} h L^2 U_0^2 \omega^2 \sin^2 \omega t}{2} \quad (7b)$$

Hence, 
$$KE_{Max} = \frac{1}{2} \rho_{Si} h L^2 U_0^2 \omega^2 \quad (8)$$

By equating the maximum values of  $PE$  and  $KE$  from eq. (6) and eq. (8), we get

$$\omega = \frac{\pi}{L} \sqrt{\frac{E_{Si}}{\rho_{Si}}} \quad (9a)$$

or,

$$f_{SE} = \frac{1}{2L} \sqrt{\frac{E_{Si}}{\rho_{Si}}} \quad (9b)$$

For the case of dual-resonator platform, the in-phase SE modal frequency can be approximated as  $\omega_{DRP}^2 = 2PE_{Max}/2KE_{Max}$  or, simply

$$f_{DRP} = \frac{1}{2L} \sqrt{\frac{E_{Si}}{\rho_{Si}}} \quad (10)$$

Here, we have assumed the contribution of the coupling beam to be negligible as compared to the two vibrating plates for the mode in consideration. When a solid mass  $\Delta m(\ll m_{DRP})$  is added to the coupled system, the effective mass and stiffness of the coupled system changes thereby changing the system resonance frequency. There is little or no shear wave formation in the attached solid mass where the whole mass vibrates in unison with the resonator surface. For such a case, Sauerbrey's equation holds true, and frequency shift is given by

$$\delta f = \frac{1}{2} \left( \frac{\Delta k}{k} - \frac{\Delta m}{m_{DRP}} \right) f_{DRP} \quad (11)$$

When resonator surface comes in contact with a liquid deposit, an evanescent shear wave is formed at the liquid near the solid-liquid boundary. This shear wave dies off exponentially normal to the surface. Hence, to obtain the mass loading by the liquid and the resulting frequency shift, the modified  $PE$  and  $KE$  of the system must be re-calculated by applying Rayleigh's principle again as before. An effective thickness  $\delta_L$ , also known as the penetration depth, of the liquid layer is defined [35] as:

$$\delta_L = \sqrt{\frac{\eta_L}{\pi f_L \rho_L}} \quad (12)$$

where,  $f_L$  is the resonance frequency when in contact with liquid,  $\eta_L$  is the dynamic viscosity and  $\rho_L$  is the density of the liquid. Penetration depth represents the distance travelled by the evanescent wave normal to the solid surface and in the liquid medium before the amplitude attenuates by a factor of  $e$ . For the resonators we have used in this paper, the estimated penetration depth is approximately 320 nm. There is negligible change in the penetration depth for the frequency shifts observed in the experiments, hence, for calculation purposes the value stated above is used (please refer to the supplementary material). It is assumed that irrespective of the total volume/mass of the droplet, the resonator only "sees" a thin disc of liquid of thickness  $\delta_L$  and contact radius  $r_c$ . Hence, in the case where the droplet diameter exactly matches the resonator edge length  $L$ , the theoretical mass of liquid in shear contact is around 492 ng corresponding to a volume of 492 pL while the total mass and volume of the droplet is 448  $\mu g$  and 448 nL respectively (please refer to the supplementary material). Assuming that there is insignificant change in the  $PE$  of the DRP due to the liquid deposit, only the  $KE$  of the resonator is perturbed. Velocity distribution in the fluid layer due to shear contact with the resonator is found by solving the Navier-Stokes equation which results in:

$$\frac{\partial v_x(z, t)}{\partial t} = \frac{\eta_L}{\rho_L} \frac{\partial^2 v_x(z, t)}{\partial z^2} \quad (13)$$

This is a one dimensional heat conduction equation whose solution is given by:

$$v_x(z, t) = V_0 \exp\left(-\frac{z(1+j)}{\delta_L}\right) \cos \omega_L t \quad (14a)$$

or,

$$v_x(z, t) = V_0 \exp\left(-\frac{z}{\delta_L}\right) \exp\left(-\frac{jz}{\delta_L}\right) \cos \omega_L t \quad (14b)$$

At the point of contact ( $z=0$ ), assuming no-slip, relative velocity between the liquid and the vibrating resonator surface should be zero. Hence, from eq. (1) and eq. (14b) we get:

$$v_x(0, t) = \frac{\partial u_x}{\partial t} \\ V_0 = \omega U_0 \sin\left(\frac{\pi x}{L}\right) \quad (15)$$

For a circular deposit of radius  $r_c$  spotted at the center of one of the square plates of the DRP, the KE of a differential element within the liquid layer of thickness  $\delta_L$  along X- axis is given by:

$$d\overline{KE}_{xL} = \frac{1}{2} dm_L (Re\{v_x\})^2 \\ = \frac{1}{2} \rho_L dV \omega_L^2 U_0^2 \sin^2\left(\frac{\pi x}{L}\right) \exp\left(-\frac{2z}{\delta_L}\right) \cos^2\left(\frac{z}{\delta_L}\right) \cos^2 \omega_L t \quad (16)$$

Hence,

$$\overline{KE}_{xL} = \iiint d\overline{KE}_{xL} = \rho_L \omega_L^2 U_0^2 I(r_c) J_z \cos^2 \omega_L t \quad (17)$$

where,

$$I(r_c) = \int_{-r_c}^{r_c} \sqrt{r_c^2 - x^2} \sin^2\left(\frac{\pi x}{L}\right) dx \\ J_z = \int_0^z \exp\left(-\frac{2z}{\delta_L}\right) \cos^2\left(\frac{z}{\delta_L}\right) dz$$

$I(r_c)$  is evaluated numerically and is plotted against droplet radii in Fig. 2. For droplets with height much larger than  $\delta_L$ ,  $J_z = J_{\delta_L}$ . Maximum KE of the liquid deposit is

$$\overline{KE}_{MaxL} = 2\rho_L \omega_L^2 U_0^2 I(r_c) J_{\delta_L} \quad (18)$$

Hence, from eq. (6), (8) and eq. (18), we have

$$2\overline{PE}_{Max} = 2\overline{KE}_{Max} + \overline{KE}_{MaxL}$$

$$f_L (= \frac{\omega_L}{2\pi}) = \frac{1}{2L} \sqrt{\frac{E_{Si}}{\rho_{Si} + \frac{2\rho_L I(r_c) J_{\delta_L}}{hL^2}}} \quad (19)$$

Therefore, for a measured “unloaded” frequency  $f_U$ , frequency shift due to liquid loading is approximately given by

$$\Delta f = -S(r_c, z) f_U \quad (20)$$

where, differential sensitivity factor  $S(r_c, z)$  is defined as  $\rho_L I(r_c) J_{\delta_L} / \rho_{Si} hL^2$ . This analysis clearly indicates that the frequency is independent of the total mass of the droplet spotted on the surface, but depends on the thin interfacial layer of liquid where the shear waves propagate. However, for larger droplets, gravity may impact the pure “in-plane” nature of the mode by inducing additional stresses in the resonator resulting in out-of-plane bending or variations in stiffness. Due to the high stiffness of the silicon and the small volumes of the droplets involved, these effects are considered to be negligible in the present analysis.

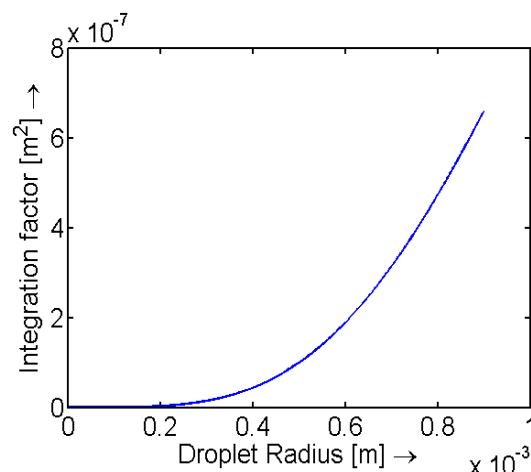


FIG. 2 Integration factor,  $I(r_c)$ , vs. droplet radius. Note with larger drops the value of  $I(r_c)$  increases non-linearly which is due to the fact that mass loading becomes more prominent towards the edges of the resonator (anti-nodal region).

### Modes of evaporation of the liquid droplet

Evaporation of a liquid droplet is a diffusion driven process, where the concentration of the molecules ( $c_s$ ) reach saturation just above the droplet surface while far away the concentration ( $c_\infty$ ) depends on ambient conditions. This concentration gradient drives molecules away from the surface of the liquid droplet. Established evaporation rate is governed by Fick's law of diffusion where diffusion flux  $\phi$  is related to the concentration gradient and diffusion coefficient  $D$  by  $\phi = -D\nabla c$ . Depending on the substrate surface and the liquid properties, there are two pure modes of evaporation for a slowly evaporating sessile droplet: pinned contact line mode and constant contact angle mode. The droplet can follow either of the two modes, may follow a mixed mode in which both the modes occur in succession

or a hybrid mode in which both occur at the same time. When the droplet contact line is fixed, the height and the contact angle of the droplet reduce with time whereas in the second constant contact angle mode, the contact area and height reduce with time while preserving the contact angle of the droplet. When the mixed mode of evaporation is observed, the droplet reaches a critical contact angle during the first mode, at which point, the second mode is triggered. For a sessile droplet of small volume, the shape at any instance can approximated using a spherical cap geometry (Bond number,  $Bo = \rho_L g r_c^2 / \gamma \cong 0.067$ , is less than 1 indicating the dominance of surface tension forces over gravitational). In our case, as indicated in the experimental section later, it is observed that the contact line somewhat remains unchanged (pinned contact line mode) in the first phase of the experiment and then the evaporation seemingly transitions to a receding contact line mode for the remaining phase. As initial height of the droplet is much larger than the penetration depth, we hypothesize small corresponding change in the resonant frequency of the coupled system. Assuming that the second phase is the constant contact angle mode of evaporation, for a droplet of contact angle  $\theta$ , spherical cap radius  $R_s$ , and droplet volume  $V_c$ , following relations can be established:

$$r_c = R_s \sin \theta, \quad V_c = \frac{1}{3} \pi \beta R_s^3 \quad (21)$$

where  $\beta = (1 - \cos \theta)^2 (2 + \cos \theta)$

Now, the volume evolution of the drop is given by

$$V_c^{\frac{2}{3}} = V_0^{\frac{2}{3}} - \frac{8\pi^{\frac{2}{3}} 3^{\frac{1}{3}} D (c_s - c_\infty) f(\theta)}{3\rho_L \beta^{\frac{1}{3}}} t \quad (22)$$

where,  $f(\theta) = -\cos \theta / 2 \ln(1 - \cos \theta)$  is function of contact angle derived by Shanahan *et al.* [36] to compensate for the horizontal contact surface. Simplifying the above equation in terms of the contact area  $A_c (= \pi r_c^2)$ , we get

$$A_c = A_{c0} - \frac{8\pi D (c_s - c_\infty) \sin^2 \theta f(\theta)}{\rho_L \beta} t \quad (23)$$

$= A_{c0} - kt$

where,  $k = 8\pi D (c_s - c_\infty) \sin^2 \theta f(\theta) / \rho_L \beta$ . Hence, contact area is proportional to the evaporation and wetting parameters (diffusivity, temperature, humidity and contact angle). Unlike the first phase, where there is no change in the interfacial layer, the coupled system should “see” a change in mass with shrinking droplet radius during this second phase of evaporation (fig. 3).

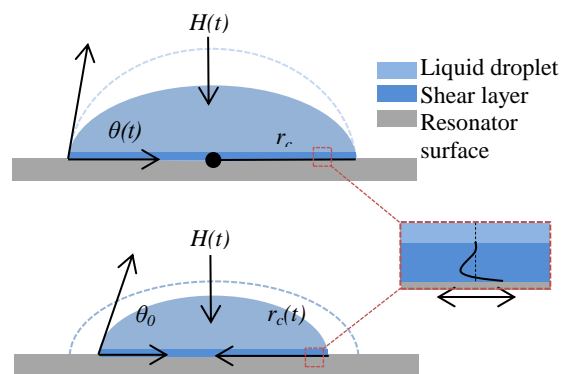


FIG. 3 Schematic showing two modes of evaporation of the droplet from the resonator surface (top) Pinned contact line mode, (bottom) Constant angle mode, and (right) shear evanescent wave formation in the liquid

## EXPERIMENTAL METHOD

### Device description

As explained earlier, the DRP consists of two identical micromachined square plate resonators that are coupled by a thin mechanical beam; one resonator is termed the Transducer resonator (TR) and the other is termed the Sensor resonator (SR). The device is fabricated by MEMSCAP Inc. USA using a standard Silicon-on-Insulator (SOIMUMPS) process. The resonators are suspended through T-shaped anchors at the corners while gold pads are patterned on the anchors and as well as on the electrodes for electrical interfacing. The geometrical parameters of the device are specified in Table 1. The device surface is not otherwise specifically treated and is used as it is received from the foundry.

Table 1. Device design parameters for the Dual Resonator Platform

Device design parameters	Values
Resonator edge length (L)	<b>1400 <math>\mu\text{m}</math></b>
Resonator thickness (h)	<b>25 <math>\mu\text{m}</math></b>
Coupling beam length	<b>1400 <math>\mu\text{m}</math></b>
Coupling beam width	<b>22 <math>\mu\text{m}</math></b>
SE in-phase resonance frequency (air, unloaded)	<b>3142969 Hz</b>
Quality factor (air, unloaded)	<b>10200</b>

### Actuation and Sensing

Coupled resonators are excited in their in-plane square extensional modes which inherently provide better quality factors as compared to their out-of-plane flexural counterparts. The TR is electrically interfaced through the actuation electrodes, surrounding the resonator body, and the

anchor pads. While the coupled device is capacitively excited, in open loop, the motional current from the resonator body, through one of the anchor pads, is fed to the network analyzer (Agilent 4396B, RF Network Analyzer) following pre-amplification by a trans-impedance amplifier circuit (Fig 4). The network analyzer is configured for obtaining S21 (electrical transmission) parameters of the device and is connected to a PC where the data is recorded for off-line analysis. As the signal is highly attenuated and damped, it is difficult to extract the resonance peak and other information from the output motional signal using conventional capacitive sensing techniques. Hence, to boost the signal, a piezoresistive sensing scheme is employed. When the resonators are excited in the square-extensional mode, anti-nodes result at the corners which are also the points of suspension of the resonator body. These suspension tethers connecting to the anchors are so designed so that the strain in these regions is maximized when the displacement of the square plate is maximized. By scaling the contribution of the piezoresistive signal from the T-shaped anchor relative to the resonator body, the piezoresistive response can be decoupled from the geometry of the resonator.

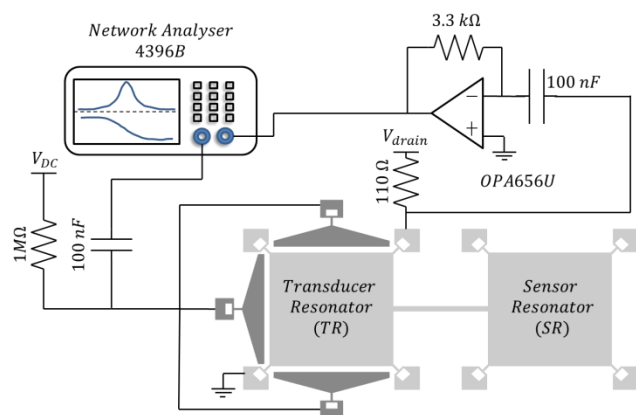


FIG. 4. Circuit schematic of Dual-resonator Platform

### Droplet dispensing and optical measurements

Distilled water droplets of volume 0.5-1  $\mu\text{L}$  are dispensed on the SR by a micropipette. As the resonator surface area ( $1400 \times 1400 \mu\text{m}^2$ ) and the droplet volumes are relatively large, the manual placement of the sessile drop on the resonator surface is relatively straightforward through this process may be automated for smaller dimensions and droplet volumes using microfluidics or robotic spotting techniques. A digital microscope (Keyence VHX-500F, Digital Microscope) is used to observe the evaporation of droplets from the top without interfering with the electronics setup. Images are taken periodically at an interval of 30 seconds and are later processed using ImageJ to obtain the radius of the droplet at various time intervals.

### RESULTS AND DISCUSSION

The DRP is first tested in air to obtain its unloaded frequency response. The resonance frequency  $f_U$  and quality factor of the device are then extracted from the S21 (transmission) response in the unloaded state. The device is then extracted and spotted with a sessile droplet of DI water. Following this, the electrical transmission response and the optical images are recorded as a function of time as the droplet evaporates. The procedure is repeated several times to ascertain repeatability.



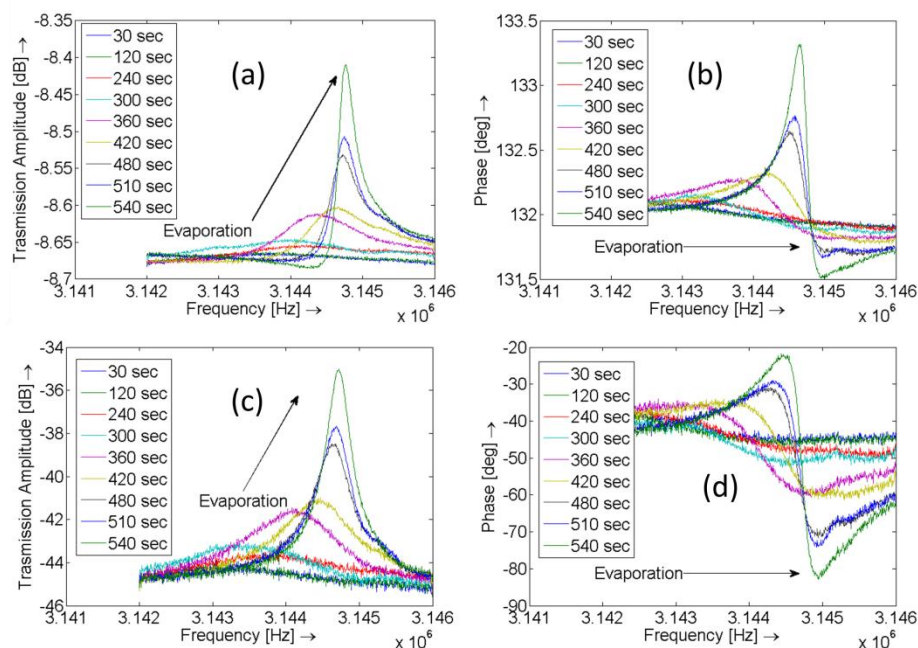


FIG. 5. Transmission(S21) amplitude and phase plots at various time instances during evaporation of the droplet. (a)-(b) Raw data plots, and (c)-(d) Feedthrough cancelled data plots

Frequency response plots at various time instants for one of the evaporation experiments is shown in Fig. 5 (a)-(b). As the motional current is swamped by the feedthrough current (parasitic and direct overlap capacitance), the original frequency and phase data is processed by subtracting the feedthrough component, obtained by supplying only the AC voltage to the device, from it. The corresponding feedthrough cancelled amplitude and phase frequency response curves are shown in Fig. 5 (c)-(d).

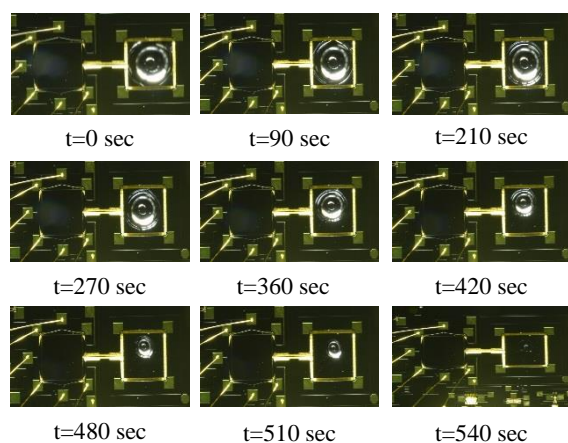


FIG. 6 Time-lapse images of evaporating sessile water droplet on the DRP. In the 1st row, the contact line remains pinned; in the 2nd row, the droplet evaporates with retreating contact line; in the 3rd row, the droplet completely evaporates.

Optical micrographs showing droplet evaporation from the DRP at different time instants is shown in Fig 6. For the first 200 seconds (approximately), the size of the drop appears unchanged. Following this the drop rapidly starts to shrink and it eventually evaporates from the surface.

A scatter plot for the obtained frequency shifts,  $\Delta f$ , is plotted against time as shown in Fig 7 (a). The plot can be divided into three distinct zones, in the first zone, the resonant frequency is seen to fluctuate around a nominally value, in the second zone the frequency shift varies with time, and finally the frequency shift response flattens reaching a value close to zero indicating near-complete evaporation. The nature of frequency shifts throughout the experiments clearly indicates that the change in mass loading on the surface of the SR is different during different phases of the experiment. Further elaboration on this observation is discussed below when the optical images of droplet evaporation are compared with the observed frequency shifts. Anomalous frequency values and outliers were removed from these scatter plots.

The contact area is calculated using the optically measured droplet radii values (from ImageJ) and is plotted as a function of time as shown in Fig. 7 (b) (inset). From the frequency shift data, using eq. (20),  $S(r_c, z)$  and, hence,  $I(r_c)$  is calculated. Values of integration factor are then used to obtain the expected contact area ( $\pi r_c^2$ ). Linear least-square fits for the optical and the modelled or frequency shift data for different regions is performed. In both cases, the corner points are excluded from the fitting schemes (please refer to the supplementary material). Sets of slopes

for each individual experiment are extracted and average values designated  $k_{1,Opt}$ ,  $k_{2,Opt}$  and  $k_{1,Mod}$ ,  $k_{2,Mod}$  respectively are used for comparison. For the first phase  $k_{1,Mod} = -8.942 \times 10^{-10}$  as compared to  $k_{1,Opt} = -7.561 \times 10^{-11}$  demonstrating a good match between the two techniques. The mode of evaporation during this stage can be said to be pinned contact line mode though contact line deformations can be seen occurring throughout this process. A reason for this could be because of slight non-uniformity of the surface energy distribution for the sensor as a result of contamination and exposure to the ambient. Hence, as the height of the droplet reduces in the pinned contact line mode, the drop readjusts to maintain the smallest possible contact area. For the second stage,  $k_{2,Mod} = -5.4983 \times 10^{-9}$  as compared to  $k_{2,Opt} = -5.156 \times 10^{-9}$  which is within 6.64% of the obtained value from optical observations of droplet evaporation. For the given ambient conditions,  $T=295.5$  K and Humidity=31%, and the observed receding contact angle value of  $(48.85 \pm 1.45)^\circ$  (please refer to the supplementary material), we can estimate the value of diffusion coefficient from the obtained slope  $k_{2,Mod}$ . The experimentally obtained values for the diffusion coefficient of water are  $(2.8341 \pm 0.105) \times 10^{-5} [m^2 s^{-1}]$  as compared to the reported value of  $2.492 \times 10^{-5} [m^2 s^{-1}]$  in the literature [37]. While the match between the measured and theoretical values is relatively good (9%-17% of the theoretical value), it should be emphasized here that the evaporation model itself is non-linear for smaller contact angles (please see the supplementary material). Also, diffusion constant is very responsive to local conditions and no one experiment is precisely equivalent to the other. However, it should be noted that by controlling the environmental conditions, the accuracy of the estimate could be further improved.

This analysis illustrates the usefulness of the technique in obtaining evaporation parameters. Alternatively, if the sample liquid is known (and, hence, its diffusion coefficient), one can easily find receding contact angle by analysing the slope obtained from the plot of contact area against time. For example, in our case, if we assume theoretical value of diffusion coefficient of water vapour in air for the measurements, then receding contact angle can be inferred to be approximately  $44.17^\circ$  compared to the observed value of  $48.85^\circ$ . This could be especially helpful in cases where sub-microliter/nanoliter droplet volumes need to be analysed where general imaging instruments and other techniques cannot be used. Also, as the surface properties of silicon can be readily modified, one possible application of the proposed device could be to studying wetting properties of different liquids for different surfaces.

An interesting observation, revealed by comparing the time histories of the optical images and frequency response data, is that a change in the resonance frequency is observed even after the optical images appear to indicate that the droplet

has completely evaporated. This can be attributed to the fact that since the thickness of the water layer seen by the sensor is only a few hundred nanometres, hence, a thin layer of water which is not captured by the microscope is still present and contributes to the measured DRP frequency response until it evaporates completely. After evaporation of the droplet a visible residue is seen on the surface which contributes to “dry” mass loading.

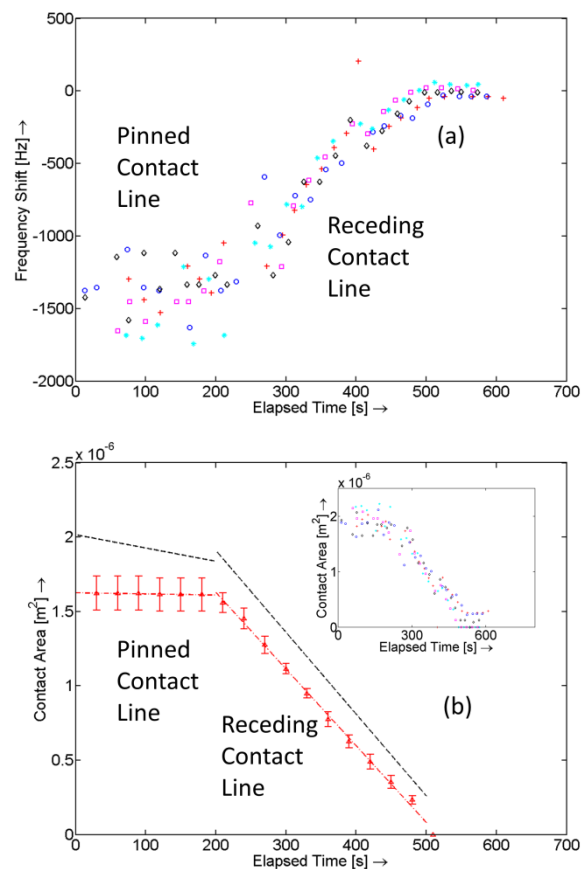


FIG 7. (a) Frequency scatter plot vs. time for various experiments, (b) Contact area obtained from the frequency data (black, dashed line) and the optical images (along with the stand. Dev. error bars) plotted vs. time, and (b-inset) scatter plot of contact area from the frequency data ( $\square, \diamond, \circ, +, *$  - from different experiments;  $\Delta$  - from optical images).

Another observation from the data is that the expected contact area is always higher than what is measured optically. This implies that the coupled sensor is actually seeing more “mass” which could potentially be explained by the assumptions underlying the model described here. First, it is assumed that the liquid droplet is modelled by a symmetrically centred spherical cap geometry which, in reality, is not the case. The resulting difference in the spatial distribution of the droplet volume might result in a difference in the frequency shift observed experimentally. Also, surface roughness could result in trapping of water molecules, and, hence, higher mass loading. It has also been



observed that inorganic residue can be deposited on the surface of the resonator resulting in additional mass loading due to miniscule amounts of contaminant residue [38]. Also, fluctuations in temperature as the droplet evaporates have been neglected though such effects are expected to be small. A thorough analysis of such effects is left for future work.

The minimum optically observable radius of the droplet for the experiments where a change in frequency was recorded was around  $250\ \mu\text{m}$ . Below this radius, the droplet evaporated quickly thereby making it difficult to obtain a consistent value for droplet radius. Assuming this to be the minimum contact radius that can be detected by the DRP, the corresponding mass and volume of the water in the shear layer is approximately  $63\ \text{ng}$  and  $63\ \text{pL}$  respectively and the total mass and volume of the droplet is approximately  $20.4\ \mu\text{g}$  and  $20.4\ \text{nL}$  respectively (please refer to the supplementary material). In practice, the lowest detectable mass is related to the minimum detectable value of  $\Delta f$  as limited by the noise floor of the device and is smaller than the values reported above [12].

One of the challenges associated with this technique is to control the positioning of droplet on the resonator. Theoretically, if the droplet evaporates with contact line pinned, the positioning of the droplet does not affect the response as the contact area, and, hence, the mass sensed does not change throughout the evaporation process. In the case where the contact line recedes, if the recession is symmetric and the centre of the droplet remains fixed, then it is possible to account for the asymmetric positioning in the model if the initial position of the droplet is known (e.g. through a calibration process aided by surface structuring or a microfluidic interface to specify the initial position of the droplet) as the gradient of frequency shift with respect to mass decrement is always positive. However, if the droplet moves over the surface of the resonator during evaporation or contracts asymmetrically during evaporation, then the interpretation of the response must account for this motion. In the current setup, these effects could be averaged out to a certain extent by using data across repeated experiments. Another drawback of this technique is the limited bandwidth of the open-loop frequency monitoring scheme. This can be addressed by embedding the DRP in the feedback loop of an electronic amplifier to construct an oscillator [39] whose output tracks the resonance frequency enabling larger bandwidth applications. This approach would eliminate errors resulting from numerical extraction of parameters from the measured open-loop response. Finally, the device dimensions can be scaled to investigate evaporation of even smaller volumes of liquid and related physical processes, not accessible by standard optical techniques.

## Conclusions

The dual-resonator platform, with its inherent advantages of relative ease of liquid handling without significant

deterioration in response, offers a unique opportunity to investigate liquid-mediated surface processes as a miniaturised microfabrication-compatible analogue of the QCM. The DRP is shown to be capable of detecting two modes of evaporation – viz. the pinned contact line mode and the receding contact line mode successfully and an analytical formulation of the frequency shift in the DRP is derived to understand the underlying correlation between the evaporating sessile droplet and the observed frequency shift. It was also shown that the device is very sensitive to small changes in shape of the droplet as well as very small volumes of liquid. A very good agreement was found in the experimentally derived value of diffusion coefficient of water derived from the measured data with literature values establishing the initial basis for this technique.

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## Notes and references

<sup>a</sup> Nanoscience Centre, Department of Engineering, University of Cambridge, 11 JJ Thomson Avenue, Cambridge, CB3 0FF, United Kingdom.

<sup>b</sup> Institute of Microelectronics, A\*STAR, 11 Science Park Road, Singapore Science Park II, Singapore 117685.

<sup>c</sup> Centre for Excellence in Nanoelectronics, Department of Electrical Engineering, IIT Bombay, Powai, Mumbai 400076, India

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