NNLO SOFT AND VIRTUAL CORRECTIONS FOR ELECTROWEAK, HIGGS, AND SUSY PROCESSES *

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I present applications of a master formula for next-to-next-to-leading order soft and virtual QCD corrections to various electroweak, Higgs, and supersymmetric processes. They include Drell-Yan and charged Higgs production, single-top production in flavor-changing neutral-current processes, and squark and gluino production.

1 Introduction

Soft and virtual QCD corrections to processes of electroweak or supersymmetric (SUSY) origin can be substantial. The calculations of the cross sections, total or differential, in hadron-hadron and lepton-hadron colliders can be represented in factorized form by

$$\sigma = \sum_f \int \left[\prod_i dx_i \, \phi_{f/h_i}(x_i, \mu_F^2) \right] \, \hat{\sigma}(s, t_i, \mu_F, \mu_R) \tag{1}$$

with σ the physical cross section, $\hat{\sigma}$ the partonic cross section, ϕ_{f/h_i} the parton distribution for parton f in hadron h_i, and μ_F, μ_R the factorization and renormalization scales, respectively.

The perturbatively calculable $\hat{\sigma}$ includes soft and virtual corrections from soft-gluon emission and virtual diagrams. These corrections appear as plus distributions and delta functions in $\hat{\sigma}$. In single-particle-inclusive (1PI) kinematics the plus distributions are $\mathcal{D}_l(s_4) \equiv [\ln^l(s_4/M^2)/s_4]_+$ with $s_4 = s+t+u-\sum m^2$, where s, t, u are kinematical invariants and m the masses of the particles in the

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scattering, and M any relevant hard scale. In pair-invariant-mass (PIM) kinematics they are $\mathcal{D}_l(z) \equiv [\ln^l(1-z)/(1-z)]_+$ with $z = Q^2/s$, where Q^2 is the pair mass squared. Note that s_4 (sometimes called s_2) $\to 0$ and z (sometimes called x) $\to 1$ at threshold.

A unified approach and a master formula for calculating these corrections at next-to-next-to-leading order (NNLO) for any process in hadron-hadron and lepton-hadron colliders have been recently presented in Ref. [1]; they follow from threshold resummation studies [2, 3, 4, 5]. Here I describe various applications to processes which are of electroweak or supersymmetric origin at lowest order.

2 NLO and NNLO corrections

I begin by presenting the generalized next-to-leading-order (NLO) master formula for processes with simple color flows, which is appropriate for many electroweak and SUSY processes. The NLO soft and virtual corrections in the $\overline{\rm MS}$ scheme in either 1PI or PIM kinematics take the form

$$\hat{\sigma}^{(1)} = \sigma^B \frac{\alpha_s(\mu_R^2)}{\pi} \{c_3\, \mathcal{D}_1(x_{th}) + c_2\, \mathcal{D}_0(x_{th}) + c_1\, \delta(x_{th})\} \, , \tag{2}$$

with x_{th} the threshold variable s_4 (in 1PI kinematics) or $1-z$ (in PIM kinematics), where σ^B is the Born term, $c_3 = \sum_i 2C_{f_i}$, $c_2 = T_2 - \sum_i C_{f_i} \ln(\mu_F^2/s)$ with

$$T_2 = 2\,{\rm Re}\Gamma'^{(1)}_S - \sum_i \left[C_{f_i} + 2C_{f_i}\,\delta_K\, \ln\left(\frac{-t_i}{M^2}\right)\right] \, , \tag{3}$$

and $c_1 = c_1^\mu + T_1$, with

$$c_1^\mu = \sum_i \left[C_{f_i}\,\delta_K\, \ln\left(\frac{-t_i}{M^2}\right) - \gamma_i^{(1)}\right] \ln\left(\frac{\mu_F^2}{s}\right) + d_{\alpha_s} \frac{\beta_0}{4} \ln\left(\frac{\mu_R^2}{s}\right) \, . \tag{4}$$

We note that we sum over incoming partons i and the C_{f_i}'s are color factors, $C_F = 4/3$ for quarks and $C_A = 3$ for gluons. Also δ_K is 0 (1) for PIM (1PI) kinematics. Γ_S are soft anomalous dimensions which describe the color exchange in the hard scattering, γ_i are parton anomalous dimensions, $\beta_0 = (11C_A - 2n_f)/3$ is the lowest-order beta function, and d_{α_s} equals 0,1,2 if the Born cross section is of order $\alpha_s^0, \alpha_s^1, \alpha_s^2$, respectively. More detais are given in Ref. [1].

At NNLO the $\overline{\rm MS}$ scheme master formula for the soft and virtual corrections is $\hat{\sigma}^{(2)} = \sigma^B (\alpha_s^2(\mu_R^2)/\pi^2) {\hat{\sigma'}}^{(2)}$ with

$$
\begin{aligned}
{\hat{\sigma'}}^{(2)} &= \frac{1}{2} c_3^2 \, \mathcal{D}_3(x_{th}) + \left[\frac{3}{2} c_3 \, c_2 - \frac{\beta_0}{4} c_3\right] \mathcal{D}_2(x_{th}) \\
&+ \left\{ c_3 \, c_1 + c_2^2 - \zeta_2 \, c_3^2 - \frac{\beta_0}{2} \, T_2 + \frac{\beta_0}{4} c_3 \ln\left(\frac{\mu_R^2}{s}\right) + \sum_i C_{f_i} \, K \right\} \mathcal{D}_1(x_{th}) \\
&+ \left\{ c_2 \, c_1 - \zeta_2 \, c_2 \, c_3 + \zeta_3 \, c_3^2 - \frac{\beta_0}{2} T_1 + \frac{\beta_0}{4} \, c_2 \ln\left(\frac{\mu_R^2}{s}\right) + 2 \, {\rm Re} {\Gamma'}_S^{(2)} - \sum_i \nu_{f_i}^{(2)} \right. \\
&\left. + \sum_i C_{f_i} \left[\frac{\beta_0}{8} \ln^2\left(\frac{\mu_F^2}{s}\right) - \frac{K}{2} \ln\left(\frac{\mu_F^2}{s}\right) - K \, \delta_K \, \ln\left(\frac{-t_i}{M^2}\right)\right] \right\} \mathcal{D}_0(x_{th}) \\
&+ R_{\delta(x_{th})} \, \delta(x_{th}) \, . \qquad (5)
\end{aligned}
$$

More details and extensions of the master formulas to the more general case of complex color flows are given in Ref. [1].

3 Applications to electroweak and SUSY processes

Using the NNLO master formula I have rederived known NNLO results for Drell-Yan and Higgs production and for $W^+\gamma$ production, and I have produced new results for many other processes [1]. Here I give a few examples.

3.1 The Drell-Yan process, $q\bar{q} \to V$

The NLO corrections are given by Eq. (2) with $x_{th} = 1 - x = 1 - Q^2/s$, and $c_3 = 4C_F$, $c_2 = -2C_F \ln(\mu_F^2/Q^2)$, $c_1 = -(3/2)C_F \ln(\mu_F^2/Q^2) + 2C_F\zeta_2 - 4C_F$. Using Eq. (5) we rederive the NNLO soft and virtual corrections in Ref. [6] and thus also derive previously unknown two-loop anomalous dimensions in Eq. (5). Similar results are given in Ref. [1] for $W^+\gamma$ production, $q\bar{q} \to W^+\gamma$ [7], and related results are derived in [1] for Standard Model Higgs production, $gg \to H$.

3.2 Charged Higgs production, $\bar{b}g \to H^+\bar{t}$

The NLO corrections are given by Eq. (2) and the NNLO corrections by Eq. (5), with $x_{th} = s_2 = s + t + u - m_{H^+}^2 - m_{\bar{t}}^2 - m_{\bar{b}}^2$, and $c_3 = 2(C_F + C_A)$, $c_2 =$

$C_F[\ln(m_H^4/(sm_t^2)) - 1 - \ln(\mu_F^2/s)] + C_A[\ln(m_H^4/(t_1u_1)) - \ln(\mu_F^2/s)]$, and $c_1^\mu = \ln(\mu_F^2/s)[C_F \ln(-u_1/m_H^2) + C_A \ln(-t_1/m_H^2) - 3C_F/4 - \beta_0/4] + (\beta_0/4)\ln(\mu_R^2/s)$.

3.3 FCNC single-top production, $eu \to et$

Here we consider single-top production mediated via flavor-changing neutral currents (FCNC) through a term in the effective Langrangian of the form $\kappa_{tq\gamma} e\bar{t}\sigma_{\mu\nu} q F^{\mu\nu}/\Lambda$ [8].

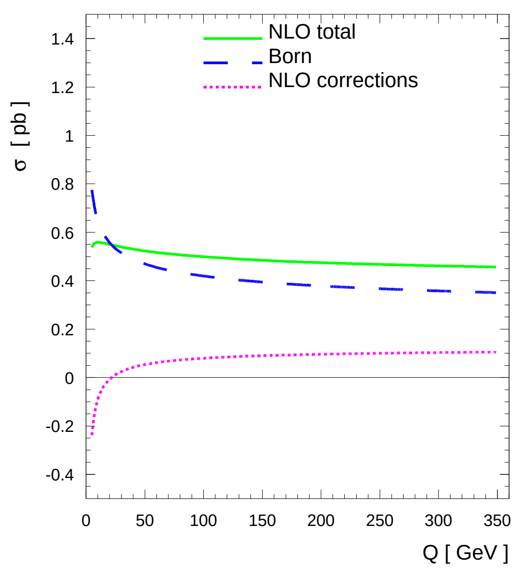

Figure 1: Born cross section, NLO corrections, and total NLO (Born+NLO corrections) cross section for FCNC single-top production at HERA with m_t=175 GeV/c^2, $\kappa_{tu\gamma} = 0.1$, and $\sqrt{S} = 300$ GeV. Here $Q = \mu_F = \mu_R$.

The NLO corrections are given by Eq. (2) with $x_{th} = s_2 = s + t + u - m_t^2 - 2m_e^2$, $c_3 = 2C_F$, $c_2 = C_F[-1 - 2\ln((-u + m_e^2)/m_t^2) + 2\ln((m_t^2 - t)/m_t^2) - \ln(\mu_F^2/m_t^2)]$, and $c_1^\mu = [-3/4 + \ln((-u + m_e^2)/m_t^2)]C_F \ln(\mu_F^2/s)$. They stabilize the FCNC single top cross section at HERA as a function of scale (see fig. 1)

[8]. The NNLO corrections are given by Eq. (5).

3.4 Squark and gluino production

We now consider squark and gluino production. We start with squark pair production. For the process $q\bar{q} \to \tilde{q}\bar{\tilde{q}}$ and $qq \to \tilde{q}\tilde{q}$ the c_i coefficients are the same as for the $q\bar{q} \to Q\bar{Q}$ channel in heavy quark pair hadroproduction [1]; for $gg \to \tilde{q}\bar{\tilde{q}}$ they are the same as for $gg \to Q\bar{Q}$.

We continue with gluino pair production. For the process $q\bar{q} \to \tilde{g}\tilde{g}$ the c_i's are the same as for $q\bar{q} \to \tilde{q}\bar{\tilde{q}}$; for $gg \to \tilde{g}\tilde{g}$ they are the same as for $gg \to \tilde{q}\bar{\tilde{q}}$.

Finally we study squark-gluino production, $qg \to \tilde{q}\tilde{g}$. Here $x_{th} = s_4 = s + t + u - m_{\tilde{q}} - m_{\tilde{g}}$, $c_3 = 2(C_F + C_A)$, $c_2 = -C_F - C_A - 2C_F \ln(-u_1/m^2) - 2C_A \ln(-t_1/m^2) - (C_F + C_A)\ln(\mu_F^2/s)$, and $c_1^\mu = \ln(\mu_F^2/s)[C_F \ln(-u_1/m^2) + C_A \ln(-t_1/m^2) - 3C_F/4 - \beta_0/4] + (\beta_0/2)\ln(\mu_R^2/s)$, with $m = m_{\tilde{q}}$ or $m_{\tilde{g}}$.

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