Market-induced Asset Specificity: Redefining the Hold-up Problem

Shira Lewin-Solomons

January 2003

DAE Working Paper No. 0304

Market-induced asset specificity: Redefining the hold-up problem

Shira B. Lewin – Solomons

Department of Applied Economics
University of Cambridge

January 2003

ABSTRACT

In a standard hold-up problem, individuals are vulnerable to hold-up because it is impossible to write complete contracts to cover the lifespan of relationship-specific investments. Hold-up occurs only when investments are to some degree nongeneric, and the extent of the problem increases with the time-span over which an investment must pay off, since long-term contracts are more difficult to write than short-term contracts. This result appears inconsistent with the real life experience of contract suppliers in two respects. First, suppliers often consider themselves "vulnerable" to hold-up even when investments are generic. Second, such a sense of vulnerability is often greatest precisely when assets are short-lived rather than long-lived. This paper provides a model that solves this apparent paradox by looking beyond the isolated problem of bilateral monopoly to the market context in which contracting takes place. When we do so, we find that the very meaning of asset specificity comes into question.

JEL codes: L22, M2, D4

Key words: asset specificity, hold-up, market structure, contracts

Market-induced asset specificity: Redefining the hold-up problem

Shira Lewin - Solomons

Department of Applied Economics
University of Cambridge

January 2003

I. Introduction

The hold-up problem is a standard element of the toolbox of today's economist. As developed variously by Klein, Crawford and Alchian [1978], Williamson [1985], Hart and Moore [1990] and others, this approach teaches us that economic actors may at times fail to pursue opportunities for productive investment because they expect the gains from these investments to be appropriated by others through ex post bargaining. The problem occurs only when it is impossible to write complete contracts that cover the entire productive life of an investment that is to some extent specialised to a relationship. If complete contracts are possible, then contracts can ensure that investment occurs. If investments are completely generic, then contracts are unnecessary.

Although the relevance of this approach to hold-up is well established, in today's outsourcing economy, the real life experience of many contract suppliers appears to contradict this simple story. True, most such suppliers would agree that the existence of sunk investments provides their customers with bargaining power. However, the reason for such bargaining power often has little to do with product or investment specialization. Rather, if a supplier feels locked into her relationship with a buyer, the source of this lock-in is commonly a concern that finding a new buyer will be difficult. A good example is the broiler chicken industry in the United States, in which growers become deeply locked into relationships with the integrators who purchase their output, despite the very standard nature of both the product and the investments. In such relationships, investments become relationship-specific not because of their inherent characteristics (as in conventional asset specificity), but because of the market context in which contracting takes place. We will call this phenomenon *market-induced asset specificity,* as contrasted with *value-induced asset specificity.*

This paper argues that the type of asset specificity at work in economic relationships may have a great impact on the nature of optimal contracting, as well as on which types of investments are more prone to hold-up. In particular, we will find below that when asset specificity is of the market-induced type, the risk of hold-up may actually fall (rather than rising) when investments pay off over a longer term

The ideas in this paper evolved over several years, and benefited from the input of numerous colleagues. Most particularly, I would like to thank Andrew Fearne for pointing out that conventional asset specificity is frequently absent in real-world hold-up. I thank Cecile Aubert and Daniel Sgroi for encouraging me to write this paper, Oliver Hart for his feedback and advice, and the Department of Applied Economics for their financial support. All errors remain my own. Correspondence should be addressed to shirabatya@post.harvard.edu.

rather than over a shorter term.[1] For example, a chicken grower will be all the more anxious to maintain her relationship with an integrator after the installation of "state-of-the-art" ventilation equipment that will remain "state-of-the-art" for only a couple of years. By contrast, an investment in a chicken house that can easily be adapted for use over many years does not induce the same urgency to maintain production levels. As a result, the latter investment is less relationship-specific in the market-induced sense of the term. It is hoped that, by providing a rigorous theoretical framework for the analysis of both value-induced and market-induced asset specificity, this paper will facilitate a deeper understanding of the determinants of contracts, thus spurring others to re-examine the relevance of the market dimension for the theory of contracts and firm structure.

A. Relation to previous work

The notion that market structure influences contracting is not new to economics. In a 1951 article, George Stigler notes that vertical integration rates are high both in infant industries with undeveloped markets, and in declining industries characterized by high market concentration. By contrast, in established industries with smoothly functioning, competitive markets, vertical integration is less common. Economics research on the theory of contracts and firms has to date tended to neglect this link between firm structure and market structure. Rather, research has focused on the important implications of value-induced asset specificity.

A few exceptions to this focus bear mention. A literature does exist on turnover costs, most particularly in labour markets[2] . Turnover costs are related to the phenomenon studied here, as their size depends on market structure, and their effects are similar to those of specific investments. For example, MacLeod and Malcomson [1993] construct a model of bilateral trade in which even general investments may be subject to hold-up due to turnover costs. Their model explores the problem of optimal investment when switching partners is costly so that commitment is needed but renegotiation must be allowed because trade is not always efficient. When only general investments are made, the hold-up problem is solved with a contract specifying a fixed trading price and a breach payment if one party ceases trade. Such a solution is not possible in this paper, since we will assume that output is only partially contractible, so that parties are limited in their ability to commit to a price in advance.

The paper whose approach comes closest to this one is Ramey and Watson [2001], which looks at the effect of market conditions on bilateral investments. To the author's knowledge, Ramey and Watson provide the only other model (besides that presented here) that has to date analyzed how market conditions effect levels of noncontractible investments for the production of goods that are also not fully contractible. Their model considers the problem of providing incentives for investment when both investments and output are entirely noncontractible. Incentives for investment and effort arise solely from the threat to end trade, in which case both parties will be forced to search for new trading partners.

Although very similar in approach to that presented here, Ramey and Watson's model differs from this one in two main respects. First, its emphasis is on agency issues, rather than on designing contracts to encourage investment. In fact, contracts exist in their model only after investment has occurred. By contrast, here

[1] As Williamson [1985] explains, classic hold-up occurs because, among other problems, the world is uncertain. Such uncertainty makes it extremely difficult to write effective long-term contracts. Short-term contracts are easier to write, but if investments are long-term, such contracts may not protect the investor sufficiently from the opportunistic behavior of others. It is a natural corollary of this argument that if investments are only short-term, the risk of hold-up is reduced, since long-term contracts are not needed.

[2] Examples include Lindbeck and Snower 1988, 1991, MacLeod and Malcomson 1993, Ramey and Watson 2001, and Vetter and Andersen 1994. For a good survey of this literature, see Malcomson 1997.

output (and therefore investment) is partially contractible, and the time over which an investment pays off therefore plays an important role in the trade-off between contracting and the market as mechanisms for encouraging investment. Second, Ramey and Watson place a greater emphasis on determining a steady state equilibrium in the market as a whole. Thus, the degree of friction is endogenous, just like unemployment in an efficiency wage model. This paper presents an alternative interpretation of the probability of securing a trading partner, so that a supplier may not secure a new buyer immediately even though suppliers are in short supply.

This model is therefore the first to look at the problem of constructing contracts that maximize incentives to invest when output is neither fully contractible nor fully noncontractible and obtaining a new trading partner may take time. This model is also the first to explore how the relationship between asset life-span and hold-up may be affected by the type of asset specificity at work.

B. Outline

Our approach in this paper is to create a general model such that conventional, value-induced specificity is a special case. Thus, we begin in section II by reformulating the standard hold-up problem, so that it lends itself to generalization. This section also provides a rigorous definition of noncontractibility, allowing us to assume that output is neither fully contractible nor fully noncontractible. In this framework, noncontractibility of investments and noncontractibility of output are inherently linked. In order to maximize incentives to invest, the optimal contract term depends on the relative effectiveness of the market and contracting as incentive devices. This contract term increases initially as the investment life-span rises, and is then a constant. As expected, in this standard model, longer-term investments are more prone to hold-up than shorter-term investments.

The framework in section II serves as a foundation for the general model of hold-up presented in section III. Here, we introduce market-induced asset specificity, so that a supplier must search for an alternative buyer if bargaining breaks down. We find here that the available market price depends on the remaining investment life-span, so that the trade-off between the market and contracting is far more complicated. Section IV then analyses the consequences of this general model for optimal contracting. We find here that a longer investment life-span need not always increase the likelihood of hold-up, and, paradoxically, that unless contracts last until investments become obsolete, longer-term investments lead to shorter contracts. Section VI concludes.

II. The standard hold-up problem

We begin by reformulating the standard model so that it lends itself to generalization. The standard hold-up problem is one of providing sufficient incentives for a relationship-specific investment when contracts are incomplete. By relationship-specific, we mean here that the product of an investment is worth more to its intended buyer than to others.

A. Setup

Consider a buyer B who contracts to purchase a product from a supplier S. This product, if produced, provides B with a flow value v for $t \in [0, T]$. If the relationship breaks down, then output can be sold for only $P = \gamma v$ where $\gamma \in [0, 1]$ is an inverse measure of the relationship-specificity of output. In order to provide this output, S must make an investment I in production equipment. For simplicity, let I be the only production cost. I is more efficient than any alternative investment. Let the discount rate be $r > 0$ for both B and S. Let S's bargaining power be $\rho \leq 1$, so that if bargaining occurs, any surplus is split between S and B in a ratio $\rho : 1 - \rho$.

We want to reach conclusions about the likelihood of investment when this investment is efficient. Thus, we require a measure of the costliness of investment. We need this measure to be robust to changes in the investment term, the rate of discounting, and the units with which we measure time, in order that fair comparisons can be made between the likelihood of hold-up with shorter- and longer-term investments.[3] Therefore, recalling that the potential output of investment is $v \int_0^T e^{-rt}dt$, let the costliness of investment λ be defined by

$$\lambda = I \Big/ v \int_0^T e^{-rt}dt$$

Thus, as long as $\lambda < 1$, $I < v \int_0^T e^{-rt}dt$ and investment is efficient. The hold-up problem will sometimes mean that no investment occurs even though $\lambda < 1$. The extent of the hold-up problem is therefore measured by how low λ needs to be for investment to occur. For example, with no contracts at all, complete asset specificity and even bargaining power $\left(\rho = \frac{1}{2}\right)$, investment occurs as long as $\lambda \leq \frac{1}{2}$, since the benefits of investment are split equally between B and S. Thus, in many cases, efficient investment fails to occur.

In order to secure investment by S, three incentives exist. First, B can contract with S to provide the product. Such contracting is limited by uncertainty, which prevents the parties from specifying in advance the exact attributes of the product to be supplied. Thus, assume that at each time t, the contractibility of output is $\delta_t \leq 1$ so that any contract can guarantee only a product valued at $\delta_t v$. For some choice $T^{\mathrm{c}} \leq T$, a contract is then an agreement to pay p_t for all $t \leq T^{\mathrm{c}}$, contingent on output valued at $\delta_t v$ at each moment. Second, S can obtain additional compensation for providing more than the contract requires. Thus, if S supplies the full value v rather than only $\delta_t v$ then she can expect an additional payment $\rho\,(1-\delta_t)\,v$ from bargaining. Third, after the contract ends, S can still sell her output at a price determined by relative bargaining power and the open market price P. Thus, after T^{c}, even though no contract holds sway, S obtains a price $\tilde{P} = P + \rho\,(1-\gamma)\,v = (\gamma + \rho\,(1-\gamma))\,v$ through bargaining. Putting all these incentives together, individual rationality is satisfied (investment occurs) if and only if

$$I \leq \int_0^{T^{\mathrm{c}}} (p_t + \rho\,(1-\delta_t))\,ve^{-rt}dt + \int_{T^{\mathrm{c}}}^{T} (\gamma + \rho\,(1-\gamma))\,ve^{-rt}dt \tag{1}$$

1. A perfect world

As in other models of hold-up, we find here that when S has complete bargaining power, when output is not at all relationship-specific, or when it is possible to contract perfectly on either output or the investment I, then we obtain the first-best and hold-up is not a problem.

Proposition 1 *Investment occurs whenever it is efficient (whenever $\lambda \leq 1$) if any of the following hold:*

1. $\delta_t \equiv 1$. *(Output is perfectly contractible.)*
2. $\rho = 1$. *(Suppliers have total bargaining power.)*
3. *The investment I is perfectly contractible.*
4. $\gamma = 1$. *(Output is not relationship-specific.)*

Proof. See appendix. ■

[3] If we look only at the investment cost, then by construction, long-term investments will be superior since they provide output for longer. Thus, some normalisation of this cost is needed.

2. Defining noncontractibility

Given proposition 1, to make hold-up possible, we need to assume that I is not perfectly contractible. By this we must mean that if B attempts to pay S to choose I, S may choose an alternative that is contractually equivalent but undesirable. In other words, an alternative investment I' exists that allows S to satisfy the letter of her contract with B until some date, so that output with value $\delta_t v$ is produced at each time t. If not provided incentives to do otherwise, S may then choose I' rather than choosing I. Note that if output is perfectly contractible so that $\delta_t \equiv 1$, then I becomes effectively contractible, since any contractually equivalent alternative to I will cost at least as much and therefore S will have no incentive to choose I' over I. Thus, *contractibility of output and contractibility of investments are intrinsically linked.*

To be precise, assume that for each $\tau \leq T$, S may choose an alternative investment with cost I'_τ that allows production valued at $\delta_t v$ on $[0, \tau]$. This alternative is inefficient[4] so that for $\lambda' > 1$ we have $I'_\tau = \lambda' v \int_0^\tau e^{-rt} dt$. (Thus, the cost of investment exceeds the value of the output.) Depending on the contract, S may nevertheless choose I'_τ over I as it may be cheaper and it involves a smaller commitment to future sales. In other words, fear of hold-up may lead to underinvestment. Notice that if the contract ends at T^c then I'_{T^c} dominates I'_τ for $\tau > T^c$. Thus, incentive compatibility requires only that choosing I dominates choosing I'_τ $\forall \tau \leq T^c$.

3. Increasing uncertainty

The time scale over which investments pay off is important in hold-up because the distant future is less certain than the immediate future, so that at points further in the future, specifying product characteristics accurately becomes more and more difficult. For simplicity of exposition, we therefore make the following assumption:

Assumption 1 *The contractibility of output decreases continuously over time from* 1 *to* 0*. Thus,* δ_t *is a strictly decreasing and continuous function of* t *such that* $\delta_0 = 1$ *and* $\lim_{t \to \infty} \delta_t = 0$.

B. The earliest payment scheme

For most parameter values, no unique optimal incentive scheme exists to motivate investment, since some latitude always exists in the scheduling of payments. To narrow the search and to maximize the usefulness of any option to sell outside, we will look for the "earliest" contract. In other words, among contracts that provide S with the lowest possible payoff while satisfying incentive compatibility, we will select the contract that ends as early as possible while paying as much as possible as early as possible within the contract term.

We begin therefore by looking for a contract such that the individual rationality constraint is binding, so that S is indifferent between I and I'_0 (the null investment) and obtains a zero payoff for both. In this case, when the contract has term T^c, S prefers I to all I'_τ if and only if the payoff of I'_τ is nonpositive for all $\tau \leq T^c$. Thus we need

$$\int_0^\tau p_t e^{-rt} dt \leq \lambda' \int_0^\tau \delta_t v e^{-rt} dt \ \forall \tau \leq T^c \tag{2}$$

Contractual payments can at no time exceed the cost of an alternative investment that allows S to satisfy the letter of her contract in a given period. The earliest payment scheme therefore involves payments $p_t = \lambda' \delta_t v$.

To summarize, for $t \leq T^c$, payments $p_t = \lambda' \delta_t v$ are made and S obtains an additional payment $\rho (1 - \delta_t) v$ through bargaining. For $t > T^c$, no contract exists,

[4] The results below will hold for $\lambda' \leq 1$ as long as $\lambda' > \rho$. However, exposition is simplified by assuming that $\lambda' > 1$.

and S obtains $(\gamma + \rho(1-\gamma))\, v > \rho(1-\delta_t)\, v$ through bargaining. Define

$$\psi(t) = \lambda' \delta_t + \rho(1-\delta_t)$$

to be the fraction of value obtained by S at time t in a contract and define

$$\bar{\mu} = \gamma + \rho(1-\gamma)$$

to be the fraction of value obtained by S at all times without a contract. Thus, if the contract ends at τ, S obtains

$$\int_0^\tau \psi(t)\, e^{-rt} dt + \int_\tau^T \bar{\mu} e^{-rt} dt = \lambda(\tau, T) \int_0^T v e^{-rt} dt$$

where $\lambda(\tau, T)$ is the S's portion of total value produced when $T^{\mathrm{c}} = \tau$, given by

$$\lambda(\tau, T) = \frac{\int_0^\tau \psi(t)\, e^{-rt} dt + \int_\tau^T \bar{\mu} e^{-rt} dt}{\int_0^T e^{-rt} dt}$$

Individual rationality is satisfied as long as the total received by S is at least I, or equivalently as long as the fraction of value received by S is sufficiently large relative to the efficiency of the investment. Thus investment occurs if and only if

$$\lambda(T^{\mathrm{c}}, T) \geq \lambda \tag{3}$$

To make the contract as short as possible, we set

$$T^{\mathrm{c}} = \inf\{\tau | \lambda(\tau, T) \geq \lambda\}$$

No contract shorter than T^{c} can satisfy individual rationality and incentive compatibility, since to do so, it would have to violate (2).

Let $\lambda^{\max} = \max_\tau \lambda(\tau, T)$ denote the maximum level of λ such that investment is possible and let $T^* = \arg\max_\tau \lambda(\tau, T)$ denote the contract length that maximizes incentives to invest so that $\lambda^{\max} = \lambda(T^*, T)$. To determine T^* and $\lambda^{\max}$, note that $\lambda(\tau, T)$ is increasing in τ as long as contracting dominates the market outcome at τ, or equivalently whenever

$$\psi(\tau) \geq \bar{\mu} \iff \delta_\tau \geq \frac{1-\rho}{\lambda' - \rho}\gamma \tag{4}$$

Therefore [5]

$$T^* = \sup\left\{\tau \leq T | \delta_\tau \geq \frac{1-\rho}{\lambda' - \rho}\gamma \iff \psi(\tau) \geq \bar{\mu}\right\} \tag{5}$$

When I is sufficiently small, $\lambda \leq \lambda(0, T)$ and therefore no contract is necessary and S can obtain a rent of $(\lambda(0,T) - \lambda) \int_0^T v e^{-rt} dt$ above her reservation payoff. Once λ rises above $\lambda(0, T)$, some contracting becomes necessary and the individual rationality constraint (3) binds. Thus we get $T^{\mathrm{c}} > 0$ with $\lambda = \lambda(T^{\mathrm{c}}, T)$. Finally, at $\lambda = \lambda^{\max}$, we reach the highest possible payoff that S can receive since $\lambda(\tau, T)$ cannot exceed $\lambda^{\max}$. Thus $\lambda = \lambda^{\max} = \lambda(T^*, T)$ and $T^{\mathrm{c}} = T^*$. One can also create earliest payment schemes that provide a payoff above the reservation level. In this case, B provides S with a signing bonus. This bonus has no incentive effect, since it increases the payoff of all investment options. Therefore, one cannot use such bonuses to achieve $\lambda > \lambda_{\max}$.

The analysis here is simplified by the fact that as T increases, beyond some point, T^* is constant.

[5] Note that the set $\left\{\tau | \delta_\tau \geq \frac{1-\rho}{\lambda' - \rho}\gamma\right\}$ is never empty since $\delta_0 = 1 > \frac{1-\rho}{\lambda' - \rho}\gamma$.

Proposition 2 *If $\gamma = 0$ or $\rho = 1$ then define $T^{\bar{\mu}} = \infty$. Otherwise define $T^{\bar{\mu}}$ by*

$$\psi\left(T^{\bar{\mu}}\right) = \bar{\mu} \iff \delta_{T^{\bar{\mu}}} = \frac{1-\rho}{\lambda' - \rho}\gamma \tag{6}$$

For all $T > 0$, $T^* = \min\left(T, T^{\bar{\mu}}\right) > 0$.

Proof. See appendix. ■

The following proposition summarizes how the various parameters affect the likelihood of hold-up.

Proposition 3 *Suppose that the supplier has an outside option to sell at γv.*

1. *If γ rises, then $T^{\bar{\mu}}$ falls and $\lambda^{\max}$ rises weakly. The increase in $\lambda^{\max}$ is strict when, initially, $T^* \left(= T^{\bar{\mu}}\right) < T$ and $\rho < 1$. In other words, when asset specificity falls, the contract that maximizes incentives becomes shorter and investment is more likely.*

2. *If δ_t shifts upwards to $\left\{\hat{\delta}_t\right\}$ such that $\hat{\delta}_t > \delta_t \forall t > 0$, then both $T^{\bar{\mu}}$ and $\lambda^{\max}$ increase. In other words, when contractibility improves, incentives are maximized when contracts become longer, and investment becomes more likely.*

3. *If ρ rises, then both $T^{\bar{\mu}}$ and $\lambda^{\max}$ rise as well. An improvement in supplier bargaining power makes investment more likely and makes contracts more useful.*

Proof. See appendix. ■

Note also that in this framework, increased contractibility of output (higher δ_t values) improves the possibility of investment indirectly by increasing the cost of the alternative investments I'_τ (if we keep λ' constant) and thus making possible higher payments p_t. The more clearly one can specify contractual requirements, the more difficult it is to satisfy the letter but not the spirit of an agreement, and therefore the more likely it is that the efficient option is preferred. In the extreme case of perfect contractibility, S will never choose an investment other than I, since I is the most efficient method of providing v.

C. The dimension of time

The results above are all as we would expect them, and most will continue to hold when we add market-induced asset specificity. The exceptions are the results with respect to time. These are far more complex and will not hold in the same way in a general model. As illustrated in figure 1, the supplier's share from contracting $\psi(t)$

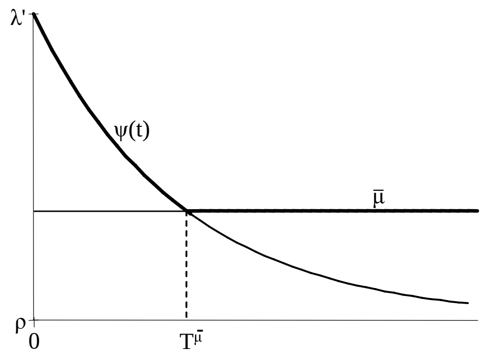


Figure 1: Contracting versus the market with value-induced asset specificity

begins at $\lambda' > 1$ and falls over time, with a limit at ρ. The supplier's share in the market is constant at $\bar{\mu} \in (\rho, 1)$. The two curves intersect at $T^{\bar{\mu}}$, the contract length at which $\lambda(\cdot, T)$ is maximized for high levels of T. T^* is constant at $T^{\bar{\mu}}$ for $T \geq T^{\bar{\mu}}$ and $\lambda^{\max}$ is the discounted average of the function given by the heavy line. If the investment becomes longer-term, more contracting takes place at the (lower) market rate. As a result, $\lambda^{\max}$ falls as T increases.

Proposition 4 *Consider the model above, in which a supplier is assured of finding an alternative buyer who will pay γv. $\lambda^{\max}$ is strictly decreasing in T. Thus hold-up becomes more likely as the term of an investment lengthens.*

Proof. See appendix. ∎

Since uncertainty increases over time, longer-term investments are more vulnerable to hold-up, since longer-term contracts are more difficult to write. The simplest way to assume decreasing contractibility over time is to set $\delta_t = e^{-ut}$ where $u \geq 0$ is the level of uncertainty. As long as $u > 0$, perfect contractibility is elusive, and this problem becomes more severe as the investment life-span increases.

III. Market-induced asset specificity

The model in section II shows how the degree of value-induced asset specificity affects optimal contracting and the risk of hold-up. Although insightful, this model is based on a number of unwritten assumptions which we will seek here to loosen, yielding a more general model of asset specificity, with very different conclusions about the relationship between investment life-span and hold-up.

First, note that when a supplier leaves a buyer in section II, the potential value v is completely lost to the buyer, with no chance of replacement. If we define β to be the probability that a buyer will replace a supplier after bargaining breaks down, this model assumes that $\beta = 0$. More problematically, section II also assumes that when a supplier and buyer part company, the supplier finds a new buyer immediately, albeit one whose valuation is reduced by a factor γ. Thus, if we define σ to be the probability that a supplier can find a new buyer after bargaining breaks down, section II assumes that $\sigma = 1$.

We are therefore led to ask: *Why should we expect that in all markets, at all times, $\beta = 0$ and $\sigma = 1$?* Assuming that $\beta = 0$ makes sense when buyers are large parties that can purchase from multiple suppliers. When one source of supply is lost, this loss does not open up other sources of supply, since all other profitable opportunities are already being exploited. This assumption also makes sense when buyers must entice suppliers to make investments by offering contracts. In such a setting, suppliers are in short supply, and once hired by buyers, do not in equilibrium become available to others. Since we are concerned here with the problem of providing incentives for investment, and since allowing $\beta > 0$ opens up a host of complex theoretical issues that will distract us from our main point, we will retain $\beta = 0$ here.

However, the assumption that $\sigma = 1$ is far stronger and less easy to justify. This assumption makes sense only when product markets are frictionless, so that a supplier can find a buyer without delay. Whenever suppliers are concerned with the problem of finding replacement buyers (as with most small suppliers), $\sigma < 1$. In such a case, market-induced asset specificity exists.

In order to include section II as a special case, let us therefore consider a market in which $\beta = 0$ and $0 \leq \sigma \leq 1$. Again, we will assume that a supplier S can invest I in order to provide a flow value v to a target buyer B on $[0, T]$. Again, the product is worth only γv to alternative buyers on $[0, T]$, where $0 \leq \gamma \leq 1$. Thus γ and σ are inverse measures of the degree of value-induced and market-induced asset specificity, respectively. To simplify the algebra, define $\phi = -\ln(1-\sigma)$. Thus, if a supplier lacks a buyer at time 0, the supplier has found a buyer by time t with probability $1 - e^{-\phi t}$ where we adopt the convention $\ln(0) = -\infty$ and $e^{-\infty} = 0$. Note that for

$\sigma = 0$, $\phi = 0$ so that $1 - e^{-\phi t} = 0$ as expected. Also, for $\sigma = 1$, $\phi = \infty$, so that $1 - e^{-\phi t} = 1$ as expected.

A. Market pricing in time

The analysis of market bargaining is far more complex here than in section II above, because the negotiated price now depends on how much production time remains. Therefore, we cannot derive single prices P and $\tilde{P}$ as above and the fraction of value μ obtained without a contract is now dependent on both t and T.

Proposition 5 *Suppose that a supplier can find an alternative buyer with valuation γv with probability σ per unit time and let $\phi = -\ln(1-\sigma)$.*

1. *The function ψ (which gives the highest payoff possible with a contract) is the same as with $\sigma = 1$.*

2. *However, when $\sigma < 1$, the fraction of value obtained by the supplier from her original buyer at time τ without a contract is now given by*

$$\mu(\tau, T) = \rho + \gamma(1-\rho)\frac{\phi\rho}{r+\rho\phi}\left(1 - e^{-(r+\rho\phi)(T-\tau)}\right)$$

3. *With an unrelated buyer, at time $T - \tau$, the supplier now obtains $P_\tau = \Pi_\tau \gamma v$ where $\Pi_\tau = \rho + (1-\rho)\frac{\phi\rho}{r+\rho\phi}\left(1 - e^{-(r+\rho\phi)\tau}\right)$ is the fraction of value received in the open market with time τ remaining.*

Proof. See appendix. ■

Corollary 1 1. *When $\sigma > 0$ (so that resale is possible), P_τ and $\mu(\tau, T)$ are increasing in γ and ϕ, and decreasing in r. Thus, when either value-induced or market-induced asset specificity rises or when individuals become more short-sighted, suppliers obtain a lower market price.*

2. *As $\phi \to \infty$, $\Pi_\tau \to 1$, $P_\tau \to \gamma v$ and $\mu(\tau, T) \to \gamma + \rho(1-\gamma) = \bar{\mu}$. Thus when resale is assured, then market prices are the same as in section II, which can be viewed as a special case.*

3. *Π_τ and P_τ are increasing in τ and $\mu(\tau, T)$ is decreasing in τ and increasing in T such that $\mu(\tau + t, T) = \mu(\tau, T - t)$.*

4. *$\mu(T, T) = \rho$ so that when no time remains, the supplier's bargaining power is eroded. If $\tau < T$, then $\mu(\tau, T) > \rho$. Finally, if $\sigma < 1$ then as $T \to \infty$, $\mu(0, T) \to \rho + \gamma(1-\rho)\frac{\phi\rho}{r+\rho\phi} \in (\rho, \bar{\mu})$.*

B. Asset longevity and the hold-up problem

As we would expect, the price negotiated by suppliers is higher when the degree of either value-induced or market-induced asset specificity is lower. What is more surprising is that this negotiated price also depends on the time horizon. In corollary 1, point 3 tells us that the longer the life of an investment, the greater is the bargaining power of a supplier. Such bargaining power exists because a failure to make a sale in the current period is not a major loss when one will continue producing long into the future, since the chances of finding an alternative buyer are good.

This result is very different from that with only value-induced asset specificity, in which case the product is worth $P = \gamma v$ in the outside market each period and thus each $P_t = P_{t+1} = \gamma v$ and $\tilde{P}_t = \tilde{P}_{t+1} = \gamma v + \rho v(1-\gamma) = \bar{\mu} v$. In this case, the life-span of the investment makes no difference to the outcome without contracting, but simply makes contracting more difficult, leading to an increased hold-up risk.

By contrast, if asset specificity is market-induced rather than value-induced, then a longer asset life may actually alleviate the hold-up problem rather than worsening it. The effect of asset longevity depends on the relative magnitude of two effects. First, the problem of contractibility increases the hold-up potential. Second, the increased chance of finding a future buyer decreases the need to have a contract at all since the outcome of bargaining improves.

Although the mathematics are slightly different, one may think of a short-term nonspecific investment as being analogous to a generic, highly perishable good. If a distributor obtains such a good, and its buyer reneges on the contract, then this distributor will incur large losses, as obtaining a new buyer on short notice may not be possible, even if the good in question is widely used. By contrast, if the good is long-lasting, then the distributor is less concerned with losing any one buyer.

IV. Deriving the optimal contract

One can derive the optimal contract and levels of T^{μ}, T^{*} and $\lambda^{\max}$ in the general model as in section II.B. However, the results are complicated by the fact that now the open market price is decreasing over time rather than constant. As a result, all outcomes depend on the asset life-span T.

A. Choosing the contract length

Recall that the fractions obtained by a supplier before and after the contract ends are given by

$$\psi(\tau) = \rho + \left(\lambda' - \rho\right)\delta_{\tau}$$

$$\mu(\tau, T) = \begin{cases} \rho + (1-\rho)\,\gamma\frac{\phi\rho}{r+\rho\phi}\left(1 - e^{-(r+\rho\phi)(T-\tau)}\right) & \text{if} \quad \phi < \infty \;\; (\sigma < 1) \\ \rho + (1-\rho)\,\gamma & \text{if} \quad \phi = \infty \;\; (\sigma = 1) \end{cases}$$

Again, the optimal contract will maximize $\lambda(\tau, T)$ which now takes the form

$$\lambda(\tau, T) = \frac{\int_0^{\tau} \psi(t)\, e^{-rt} dt + \int_{\tau}^{T} \mu(\tau, T)\, e^{-rt} dt}{\int_0^{T} e^{-rt} dt}$$

However, to maximize $\lambda(\tau, T)$, we cannot here, analogous to what we did before, simply define T^{μ} to be the place where these two curves ψ and μ cross. This case is more complicated because $\psi(0) = \lambda' > 1 > \mu(0, T)$ and $\psi(T) \geq \rho = \mu(T, T)$. In other words, contracting dominates the market at both the beginning and the end of the asset life.

Consider the following example. Let $\delta_t = e^{-ut}$ so that contractibility decreases at a rate u over time. Figure 2 illustrates the effect of a reduction in σ in the case $\gamma = .5$, $\lambda' = 1.2$, $u = .5$, $\rho = \frac{1}{2}$, $r = .05$, and $T = 6$. ψ is unchanged from its value

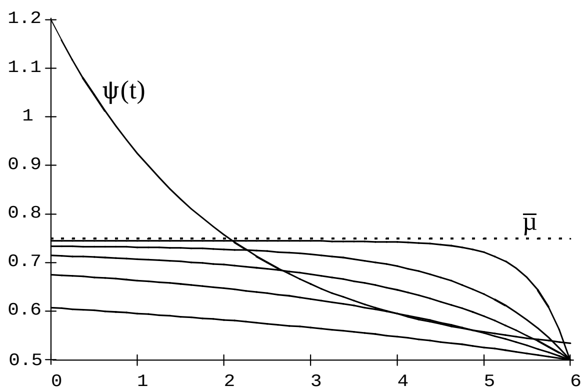


Figure 2: Adding market-induced specificity

with $\sigma = 1$. The dashed line represents μ when $\sigma = 1$ ($\phi = \infty$) as in the previous section, so that $\mu(t, 6) = \bar{\mu} \equiv \frac{1}{2} + \frac{1}{2}\gamma = .75$. Underneath (in descending order) are graphs of $\mu(\cdot, 6)$ at $\sigma = .99$, .8, .6, .4, and .2. A reduction in σ has two effects on the market price $\mu(\cdot, 6)$. First, μ shifts downward, since suppliers have more difficulty finding alternative buyers, and therefore have lower effective bargaining power. Second, the degree of this shift depends on time, so that now, like ψ, the curve μ slopes downward, eventually reaching $\rho < \psi(t)$ at $t = T$.

Therefore, unless they are tangent, if $\psi(\cdot)$ and $\mu(\cdot, T)$ intersect, then they intersect (at least) twice.[6] Let T^{μ} be the point where $\psi(\cdot)$ and $\mu(\cdot, T)$ first cross.

$$T^{\mu} = \sup\{\tau \leq T | \psi(t) \geq \mu(t, T)\, \forall t \leq \tau\}$$

Returning to the example, when $\sigma = .2$, $\mu < \psi$ at all values, and therefore $T^* = T^{\mu} = T = 6$. For the other values of σ, $\mu(\cdot, 6)$ and $\psi(\cdot)$ intersect twice, making the optimal contract length more complicated to determine. In particular, at $\sigma = .4$, the curves of $\mu(\cdot, 6)$ and $\psi(\cdot)$ are almost tangent, so that $\lambda(T, T) = .736 > .734 = \lambda(T^{\mu}, T)$ and therefore $T^* = T$. In this case, the advantage of the market over contracting (the difference between μ and ψ) is small and temporary, so that incentives are highest with a contract lasting the entire investment term. By contrast, when $\sigma = .8$, $\lambda(T, T) = .736 < .776 = \lambda(T^{\mu}, T)$ and therefore $T^* = T^{\mu} < T$. In this case, the market option provides larger benefits for a longer term, and therefore a shorter contract becomes optimal. Thus, as proposition 6 states below, T^{μ} is optimal in some cases, but in others,we have $T^* = T$.

Proposition 6 *Suppose that $\gamma > 0$ and that δ_t is strictly decreasing and convex, as when $\delta_t = e^{-ut}$ for $u > 0$. Then*

1. *$T^* \in \{T^{\mu}, T\}$*
2. *Suppose that $T^{\bar{\mu}} < T$.*
   - *(a) $\lim_{\sigma \to 1} T^{\mu} = T^{\bar{\mu}}$ and for σ sufficiently close to 1, $T^* = T^{\mu}$.*
   - *(b) T^{μ} is weakly decreasing in both σ and γ, and strictly so at σ and γ such that $T^{\mu} < T$. In other words, a decrease in either type of asset specificity leads to shorter contracts.*
3. *If $T^{\bar{\mu}} \geq T$ then $T^{\mu} = T\ \forall \sigma$.*
4. *If σ is sufficiently close to 0 then $T^* = T$.*

In other words, when market-induced asset specificity rises, the optimal contract length rises as well, and if market-induced asset specificity is sufficiently high, then contracting is used for the entire investment life-span.

Proof. See appendix. ■

B. Asset life-span and contract length: an upside-down result

Now that the optimal contract length is defined properly, we are ready to explore the most interesting implication of this model: the effect of investment life-span on contract length. Consider the same example as above, so that $\gamma = .5$, $\lambda' = 1.2$, $u = .5$, $\rho = \frac{1}{2}$, and $r = .05$, but now fix $\sigma = .3$ and consider what happens as we vary the investment life-span T. As illustrated in figure 3, $\mu(t, 3)$ and $\mu(t, 6)$ never exceed $\psi(\tau)$ so that $T^* = T^{\mu} = T$. Thus, for low levels of T, T^{μ} and T^* are increasing in T. However, moving from $T = 6$ to $T = 9$, T^{μ} jumps downward from 6 to 3.17. Moving on to $T = 12$, T^{μ} decreases further to $T^{\mu} = 2.82$. Finally, at $T = \infty$, we get $T^{\mu} = 2.55$. Thus, T^{μ} (and with it T^*) initially increases with T, but eventually begins to fall. As a result, the optimal contract length may become shorter when the investment life-span becomes longer.

[6] If δ_t is strictly convex as with $\delta_t = e^{-\delta t}$ then $\psi(\cdot)$ and $\mu(\cdot, T)$ intersect no more than twice.

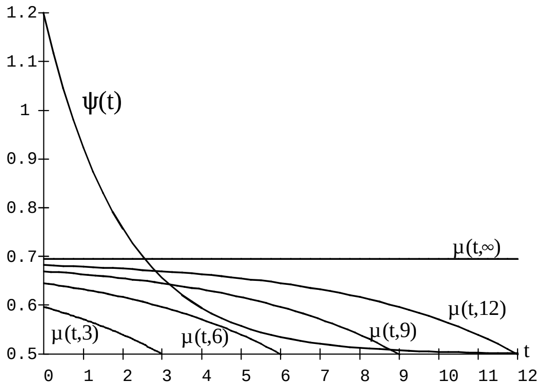


Figure 3: varying the investment lifespan

Proposition 7 *Suppose that $0 < \sigma < 1$, $\gamma > 0$ and δ_t is strictly decreasing and convex.*

1. *For T sufficiently small, $T^* = T^\mu = T$. Thus for short-lived assets, the optimal contract term is the same as the investment life-span, and thus increases as the investment life-span increases.*

2. *For T sufficiently large, $T^\mu < T$. Moreover, at such levels of T, T^μ is decreasing in T.*

3. *For T sufficiently large, $T^* = T^\mu < T$.*

In other words, if the investment life-span increases enough, the optimal contract becomes shorter.

Proof. See appendix. ■

Corollary 2 *Suppose that the optimal contract between a buyer and a supplier does not extend to the end of the life-span of the investment involved and that δ_t is strictly decreasing and convex. Then, ceteris paribus, if the investment life-span increases, the optimal contract becomes shorter.*

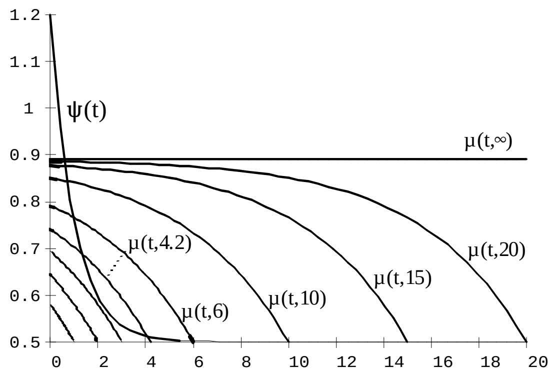
These results, in particular the corollary, are counter-intuitive. However, they make sense if we consider the importance of market-induced asset specificity. When an investment has a long working life, a supplier is less vulnerable, so that the outcome of market bargaining is improved. In a sense, a longer asset life-span is analogous to a reduction in asset specificity. (See proposition 6 above.) In both cases, the advantages of contracting are reduced, so that optimal contracts are shorter.

C. When longer investments prevent hold-up

In contrast with standard models of asset specificity, here, incentives for investment may actually be greater when suppliers invest for the long run. Such long-term investments provide a commitment value to suppliers, making threats to leave an existing buyer more credible, since the benefits of moving to a better customer will be enjoyed for longer. If we take into account market-induced asset specificity, the investments most prone to hold-up are not long term, but those that pay off in the medium term. For such investments, contracting is difficult, but the prospects of recontracting are not sufficiently good to serve as a substitute.

To see why this reversal occurs, consider what happens as we vary T. For T very small, $\lambda^{\max}$ is close to $\lambda' > 1$, since contracts are almost complete. For such levels of T, contracting is used for the entire investment life-span. As T rises, $\lambda^{\max}$ falls, since $\psi(t)$ is decreasing. For T beyond some $\hat{T}$, contracting for the entire life-span is no longer optimal. Thus, to maximize incentives to invest, market bargaining is used for $t \geq T^{\mu}$. A further increase in T has two effects: First, as T^{μ} falls while T rises, the contract term becomes a smaller and smaller fraction of the total investment life-span. Since the initial contracting interval provides S with the highest payoff, the result is a fall in $\lambda^{\max}$. Second, as T rises, $\mu(\cdot, T)$ shifts upward (which is why T^{μ} falls). Therefore, S's payoff from market bargaining rises. The result is a rise in $\lambda^{\max}$. Therefore, although $\lambda^{\max}$ is initially decreasing in T, beyond some point, this trend may reverse itself, so that $\lambda^{\max}$ begins to rise.

A small reversal of this type occurs in the illustration above, so that $\lambda^{\max} = .720$ when $T = \infty$, whereas for $T = 12$ we have $\lambda^{\max} = .695$. This reversal is much more striking when value-induced asset specificity is small (high γ), when contractibility falls rapidly (high u) and when suppliers have low bargaining power (low ρ). Thus retain $\lambda' = 1.2$, $r = .05$, $\sigma = .3$, and $\rho = \frac{1}{2}$, but let $\gamma = 1$ and $u = 1$ so that no value-induced asset specificity exists and so that contractibility falls rapidly, as illustrated in figure 4. In this example, $\lambda_{\max}$ is smallest, so that investment is least likely,



T	ε	1	2	3	4.2	6	10	15	20	∞
T^*	ε	1	2	3	1.31	.96	.71	.62	.60	.58
$\lambda^{\max}$	1.2	.94	.81	.73	.70	.72	.76	.81	.84	.90

Figure 4: Reversal of decline in $\lambda^{\max}$ with rapidly falling contractibility

when $T = 4.2$, for which $\lambda^{\max} = .70$. By contrast, for very long-term investments, $\lambda^{\max}$ approaches .90, which is 29% larger.

We obtain even stronger results when suppliers have lower bargaining power, which may be a reasonable assumption in many applications. For example, for $\rho = .25$, we obtain the following outcome:

T	ε	1	2	3	4	5.6	8	10	15	20	30	∞
T^*	ε	1	2	3	4	1.52	1.17	1.02	.83	.75	.70	.68
$\lambda^{\max}$	1.2	.85	.67	.56	.50	.48	.49	.52	.57	.61	.67	.74

In this case, $\lambda^{\max}$ has a minimum of .48 at $T = 5.6$, but for very long investments, $\lambda^{\max}$ approaches .74, which is 54% larger.

The reversal of the decline in $\lambda^{\max}$ occurs at an earlier time when either σ or u is higher, since either change increases the advantages of the market over contracting. For example, if we set $\lambda' = 1.2$, $r = .05$, $\sigma = .7$, $\gamma = 1$ and $u = 2$, we get the following results for $\rho = .5$ and $\rho = .25$:

T	ε	.5	1	1.6	2.1	3	5	10	15	∞
T^* (.5)	ε	.5	1	.563		.303	.231	.209	.208	.208
$\lambda^{\max}$ (.5)	1.2	.94	.81	.73		.78	.84	.91	.93	.96
T^* (.25)	ε	.5	1		.656	.459	.302	.211	.198	.195
$\lambda^{\max}$ (.25)	1.2	.85	.66		.52	.54	.61	.74	.80	.89

For $\rho = .5$, $\lambda^{\max}$ has its minimum of .73 at $T = 1.6$, rising by 32% to .96 as $T \to \infty$. For $\rho = .25$, $\lambda^{\max}$ has its minimum of .52 at $T = 2.1$, rising by 71% to .89 as $T \to \infty$. In general, we can prove the following:

Proposition 8 *Suppose that* $0 < \sigma < 1$, $\gamma > 0$ *and* $\delta_t = e^{-ut}$ *where* $u > 0$.[7]

1. *If* $T > 0$ *then for* u *sufficiently large,* $T^* = T^\mu < T$. *Moreover, as* $u \to \infty$, $T^\mu \to 0$.
2. *Let* $\lambda^{\max}(T, u)$ *denote the maximum share of the supplier for investment life-span* T *and uncertainty level* u. *If* $\hat{T} > 0$ *then for* u *and* T *sufficiently large,* $\lambda^{\max}(T, u) > \lambda^{\max}\left(\hat{T}, u\right)$.

Thus, when contracting is sufficiently ineffective, incentives for investment are higher for long-lived assets than for those with an intermediate life-span.[8]

Proof. See appendix. ■

V. The power of commitment

The conclusions of section C above are counter-intuitive. Surely, one might reason, rather than incurring a long-run investment, if an equivalent short-term alternative exists, this will be superior. After all, such an investment provides one with more flexibility and allows shorter, more effective contracts. Although the repeated short-term investment strategy may appear attractive, a closer analysis reveals that such an approach can never reduce the risk of hold-up. We obtain this result because in such a situation, flexibility actually creates a strategic disadvantage.

For example, suppose that one has a choice between an investment that lasts 20 periods, and an alternative investment that lasts 4 periods and has equivalent efficiency (so that λ is identical for the two options). Let $\lambda_4^{\max}$ represent the supplier's maximal share with an investment of term 20, and let $\lambda_4^{\max}$ represent the same with an investment of term 4. If the 4-period investment is repeated five times, then the result is a 20-period investment, but with shorter contracts. Figure 5 illustrates the supplier's share with an investment with term 20. Here, $\lambda_{20}^{\max}$ is the discounted average of the function represented by the thick black line. The supplier may attempt

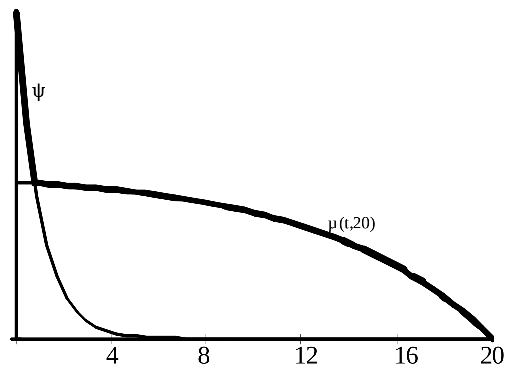


Figure 5: supplier's share with a long-term investment

to do better than $\lambda_{20}^{\max}$ by announcing his intention in advance to repeatedly invest

[7] We obtain the same outcome if $\delta_\tau = f(\tau, \delta)$ such $f(\cdot, \delta)$ is strictly decreasing in δ on $(0, \infty)$ and always strictly decreasing and convex in τ, and such that for all $\tau > 0$, $\lim_{\delta \to \infty} f(\tau, \delta) = 0$.

[8] The author conjectures that $\lambda_{\max}$ is always either decreasing in T or (for sufficiently high δ) U-shaped. However, a rigorous proof of this conjecture has proved elusive and the proposition stated here is the strongest that appears to be provable. Any ideas on how to prove a stronger result are most welcome.

for four periods at a time. At the beginning of each investment, a new contract can be signed, allowing a higher share to be received as illustrated in figure 6. Again, the thick line represents the share received.

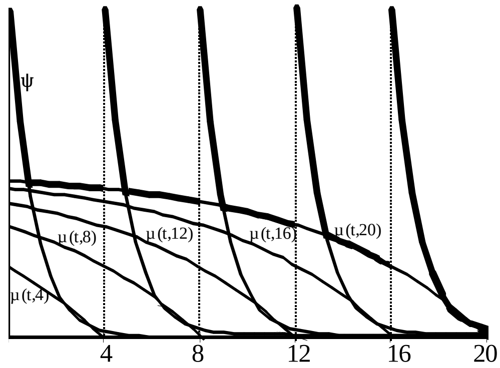


Figure 6: Repeated short-term investments

Unfortunately, unless a four-period investment is sustainable on its own (unless $\lambda < \lambda_4^{\max}$), such a strategy falters as it depends on the credibility of the promise to re-invest. Graphically, $\lambda_4^{\max}$ is represented here by the discounted average of the thick line in the last four periods, between $t = 16$ and $t = 20$. Suppose that $\lambda_4^{\max} < \lambda$. Then when we reach $t = 16$, re-investment will not occur, as the supplier's share does not cover investment costs. As a result, the market option falls from $\mu(t, 20)$ to $\mu(t, 16)$ since investment is expected for only 16 periods. Using backwards induction, it is now clear that re-investment will not occur at $t = 12$ either. Continuing this reasoning, the entire strategy unravels — unless investment for four periods is sustainable on its own.

We see here that the very inflexibility of a long-term investment provides a strategic advantage. It is true that the last few periods of such an investment yield a very low payoff, or even a loss. However, the fact that those last periods exist increases a supplier's payoff earlier on. Therefore, the supplier benefits from committing to a long investment term. The ability to change one's mind is in this case undesirable. We conclude that, although a supplier may sometimes obtain a strategic advantage by engaging in repeated short-term investments, such a strategy will never solve the hold-up problem, allowing investment to occur when it otherwise would not occur.

VI. Conclusion

This paper has presented a theoretical framework that allows us to analyze situations in which output is neither fully contractible nor fully noncontractible and asset specificity is an outcome of both product characteristics and market conditions. We have found that within this framework, it is perfectly reasonable to expect that investment for longer-term projects may be less prone to hold-up than that for medium-term contracts, and that the term of an investment may play an important role in determining the balance of bargaining power between contracting parties.

Much room exists for both practical and theoretical work building upon this framework. In the practical realm, a re-examination of common applications of the hold-up model is likely to reveal that market-induced asset specificity plays a role in almost all of these. It is hoped that this framework will enhance our understanding of such applications, so that the gap between theory and observation can be narrowed. Among these applications, the most obvious is the analysis of outsourcing arrangements such as grower contracts, whose characteristics served as the motivation for this paper. Another application would examine the strategic implications of firms' investment strategies. From a conventional hold-up perspective, committing to produce over a long term will increase one's vulnerability to hold-up since contracts become less effective at providing protection. However, if one's vulnerability is the result of market conditions, then investing for the long-run may actually enhance bargaining power and thereby prevent hold-up by making contracts less necessary.

Much theoretical work also remains to be done extending this framework as well as adapting it to different contracting scenarios since, to highlight clearly the most important effects of market-induced specificity, this paper presents quite a simplified theoretical framework. First, although allowing for both value-induced and market-induced asset specificity, this paper considers their effects independently. More realistically, the two effects are linked. By this we mean that alternative buyers vary in their valuations, so that a supplier can choose between accepting a low price quickly or searching carefully for a high-valuation customer. As a result, asset specificity may actually be multi-dimensional, rather than two-dimensional as described here.

Second, in contrast with Ramey and Watson (2001), this paper has treated as exogenous the probability σ of finding a new buyer. Endogenizing σ in this framework (as opposed to theirs) is complicated by the fact that here, suppliers are in short supply and a buyer secures an alternative supplier primarily not by searching, but by enticing someone else to invest by offering a new contract. Thus, to derive a market equilibrium, one would need to consider all potential suppliers as well as actual suppliers. Nevertheless, this sort of framework may help to shed light on efficiency wage scenarios in which investments serve as incentive devices. Third, only unilateral investment is considered here. If both parties invest, then in some circumstances a rise in market-induced asset specificity might actually increase incentives for the *buyer* to invest, as in Ramey and Watson (2001).

It is the author's hope that this paper will be only a beginning, so that the relevance of market structure becomes a standard consideration in the economic theory of contracts.

References

Hart, O., and Moore, J., 1990. Property rights and the nature of the firm. *Journal of Political Economy* 98 (6): 1119-58.

Klein, B., Crawford, R., and Alchian, A.. 1978. Vertical Integration, appropriable rents, and the competitive contracting process. *Journal of Law and Economics* 21: 297-326.

Lindbeck, A., and Snower, D. J. 1988. Cooperation, harassment, and involuntary unemployment: An insider-outsider approach. *American Economic Review.* 78(1): 167-88.

Lindbeck, A., and Snower, D. J. 1991. Interactions between the efficiency wage and insider-outsider theories. *Economics Letters* 37(2): 193-96.

MacLeod, W. B., and Malcomson, J. M. 1993. Investments, hold-up, and the form of market contracts. *American Economic Review* 83(4): 811-837.

Malcomson, J. M. 1997. Contracts, hold-up, and labor markets. *Journal of Economic Literature* 35(4): 1916-57.

Ramey, G., and Watson, J. 2001. Bilateral trade and opportunism in a matching market. *Contributions to Theoretical Economics* 1(1): Article 3.

Stigler, G. J. 1951. The division of labor is limited by the extent of the market. *Journal of Political Economy* LIX(3):185-193.

Vetter, H., and Andersen, T. M. 1994. Do turnover costs protect insiders? *Economic Journal* 104(422): 124-30.

Williamson, O. E., 1985. *The Economic Institutions of Capitalism.* New York: The Free Press.

Appendix. Proofs

Proof of proposition 1. (1) Suppose that output is perfectly contractible so that $\delta_t \equiv 1$. Suppose that the contract requires provision of v on $[0, T]$ in return for a lump sum $\int_0^T ve^{-rt}dt$ paid at T. This satisfies individual rationality (1) if and only if $\lambda \leq 1$. Moreover, no alternative investment can dominate I since we have assumed that I is the cheapest way to produce v on $[0, T]$.

(2) Suppose that $\rho = 1$ and that investment is efficient so that $\lambda \leq 1$. Then set $T^c = 0$. Without any lump-sum, in return for investing, B receives a flow value of $(\gamma + (1 - \gamma))\, v = v$. Therefore, S invests as long as $\int_0^T ve^{-rt}dt \geq I \iff \lambda \leq 1$.

(3) Now suppose that the investment I is perfectly contractible, though we might have $\delta_t < 1$. Then again set $T^c = 0$ but let B pay S a lump sum $(1 - \gamma - \rho(1 - \gamma)) \int_0^T ve^{-rt}dt$ at time 0 in return for choosing I. The total received by S is then

$$\begin{aligned} & (1 - \gamma - \rho(1 - \gamma)) \int_0^T ve^{-rt}dt + (\gamma + (1 - \gamma)) \int_0^T ve^{-rt}dt \\ = & \int_0^T ve^{-rt}dt = \frac{I}{\lambda} \end{aligned}$$

Therefore S invests as long as $\lambda \leq 1$.

(4) Finally suppose that $\gamma = 1$. Then again set $T^c = 0$. Without any lump-sum, in return for investing, B receives a flow value $(1 + \rho(1 - 1))\, v = v$. Therefore, S invests as long as $\int_0^T ve^{-rt}dt \geq I \iff \lambda \leq 1$. ■

Proof of proposition 2. Recall that δ_t is strictly decreasing and continuous with $\delta_0 = 1$ and $\lim_{t\to\infty} \delta_t = 0$. Therefore $\psi(0) = \lambda' > 1$ and $\lim_{t\to\infty} \psi(t) = \rho$. Also recall that if $\gamma > 0$, then $\bar{\mu} > \rho$. Therefore, when $\gamma > 0$ and $\rho < 1$, there exists a unique $T^{\bar{\mu}} > 0$ such that $\psi(T^{\bar{\mu}}) = \bar{\mu}$. If $T \leq T^{\bar{\mu}}$ then $\psi(t) > \bar{\mu}\ \forall t \leq T$ and therefore $T^* = T$. If $T > T^{\bar{\mu}}$ then $\psi(t) > \bar{\mu} \iff t < T^{\bar{\mu}}$ and therefore $T^* = T^{\bar{\mu}}$.

Finally, note that if $\gamma = 0$ then $\bar{\mu} = \rho > \psi(t)\,\forall t$ and if $\rho = 1$ then $\psi(t) = 1 + (\lambda' - 1)\,\delta_t > 1 = \bar{\mu}$. In either case, contracting dominates the market so $T^* = T$. Since $T^{\bar{\mu}} = \infty$, $\min(T, T^{\bar{\mu}}) = T = T^*$ as needed. ■

Proof of proposition 3. (1) Consider the effect of a rise in γ to $\gamma + \varepsilon$ so that $\bar{\mu}_\varepsilon = \gamma + \varepsilon + \rho(1 - \gamma - \varepsilon) = \bar{\mu} + \varepsilon(1 - \rho) > \bar{\mu}$. Consider how $T^{\bar{\mu}}$ and $\lambda^{\max}$ compare to the corresponding values $T^{\bar{\mu}_\varepsilon}$ and $\lambda_\varepsilon^{\max}$. Since δ_t is decreasing in t, since $\psi(T^{\bar{\mu}}) = \bar{\mu} < \bar{\mu}_\varepsilon = \psi(T^{\bar{\mu}_\varepsilon})$, we must have $T^{\bar{\mu}_\varepsilon} < T^{\bar{\mu}}$.

Now comparing $\lambda_\varepsilon^{\max}$ and $\lambda_\varepsilon^{\max}$, note that because $\bar{\mu}_\varepsilon > \bar{\mu}$, $\lambda(\tau, T) \geq \lambda_\varepsilon(\tau, T)$ $\forall \tau$, and this inequality is strict if $\rho < 1$ and $\tau < T$. Therefore $\lambda^{\max} \leq \lambda_\varepsilon^{\max}$ with strict inequality if $\rho < 1$ and $T^* < T$.

(2) Now consider the effects of an upward shift in $\{\delta_t\}$ to $\left\{\hat{\delta}_t\right\}$ such that $\hat{\delta}_t > \delta_t \forall t > 0$. This shift does not affect $\bar{\mu}$, but it causes ψ to shift upward to

$$\hat{\psi}(t) = \lambda'\hat{\delta}_t + \rho\left(1 - \hat{\delta}_t\right) > \psi(t)\,\forall t > 0$$

Define $\hat{T}^{\bar{\mu}}$ such that $\hat{\psi}\left(\hat{T}^{\bar{\mu}}\right) = \bar{\mu}$. Since ψ and $\hat{\psi}$ are decreasing in t and $\psi(T^{\bar{\mu}}) = \hat{\psi}\left(\hat{T}^{\bar{\mu}}\right)$, it must be that $\hat{T}^{\bar{\mu}} > T^{\bar{\mu}}$. When contractibility improves, one can raise incentives by lengthening the contract. Finally, define $\hat{\lambda}(\tau, T)$ to be S's share when the contract ends at τ and contractibility is $\left\{\hat{\delta}_t\right\}$. Since $\hat{\psi}(t) > \psi(t)\,\forall t > 0$, $\forall t > 0$, $\hat{\lambda}(\tau, T) > \lambda(t, T)$. Therefore, using $T^* > 0$, $\hat{\lambda}^{\max} = \max_\tau \hat{\lambda}(\tau, T) \geq \hat{\lambda}(T^*, T) > \lambda(T^*, T) = \lambda^{\max}$.

(3) Consider the effects of an increase in ρ. First, $\frac{1-\rho}{\lambda'-\rho}\gamma$ rises since $\lambda' > 0$. Therefore, since $\delta_{T^{\bar{\mu}}} = \frac{1-\rho}{\lambda'-\rho}\gamma$, and δ_t is strictly decreasing, $T^{\bar{\mu}}$ must rise strictly. Second, $\lambda^{\max}$ rises strictly since $\psi(t)$ rises strictly and, $\bar{\mu}$ rises weakly, and $T^* > 0$.

■

Proof of proposition 4. First, if $T < T^{\bar{\mu}}$, then $\lambda^{\max}$ is the discounted average of ψ on $[0, T]$, and is therefore strictly decreasing in T because ψ is strictly decreasing in T.

Now suppose that $T > T^{\bar{\mu}}$, in which case $\gamma > 0$, $\rho < 1$ so that $\psi(t)$ is strictly decreasing and $T^* = T^{\bar{\mu}}$. Define $\bar{\psi}$ to be the discounted average of ψ on $[0, T^{\bar{\mu}}]$:

$$\bar{\psi} = \int_0^{T^{\bar{\mu}}} \psi(t)\, e^{-rt} dt \bigg/ \int_0^{T^{\bar{\mu}}} e^{-rt} dt$$

Then

$$\begin{aligned}
\lambda^{\max} &= \left(\bar{\psi} \int_0^{T^{\bar{\mu}}} e^{-rt} dt + \bar{\mu} \int_{T^{\bar{\mu}}}^{T} e^{-rt} dt\right) \bigg/ \int_0^T e^{-rt} dt \\
&= \bar{\mu} + (\bar{\psi} - \bar{\mu}) \int_0^{T^{\bar{\mu}}} e^{-rt} dt \bigg/ \int_0^T e^{-rt} dt
\end{aligned}$$

which is strictly decreasing in T since $\bar{\psi} > \bar{\mu}$. ■

Proof of proposition 5. Consider first, as before, the negotiations between an unrelated buyer and supplier, but let us limit ourselves to the benchmark case $\gamma v = 1$ so that the prices represent fractions of value. Define Π_τ to be the negotiated payment per unit time at time $T - \tau$, so that the time remaining is τ. To solve for Π_τ, let V_τ be the surplus from agreement at time $T - \tau$. Then

$$\begin{aligned} V_\tau &= \int_0^\tau e^{-rt} - \int_0^\tau \Pi_{\tau-t} \left(1 - e^{-\phi t}\right) e^{-rt} dt \\ &= \underbrace{\int_0^\tau \left(1 - \Pi_t\right) e^{-r(\tau - t)}}_{\text{buyer's share}} + \underbrace{\int_0^\tau \Pi_t e^{-(r+\phi)(\tau-t)} dt}_{\text{supplier's share}} \end{aligned}$$

This surplus is split between buyer and supplier in a ratio $1 - \rho : \rho$. Thus

$$\begin{aligned} \rho \int_0^\tau \left(1 - \Pi_t\right) e^{-r(\tau-t)} dt &= \left(1 - \rho\right) \int_0^\tau \Pi_t e^{-(r+\phi)(\tau-t)} dt \\ \Longrightarrow \rho \int_0^\tau \left(1 - \Pi_t\right) e^{rt} dt &= \left(1 - \rho\right) e^{-\phi\tau} \int_0^\tau \Pi_t e^{(r+\phi)t} dt \end{aligned}$$

Differentiating with respect to τ and simplifying, we obtain

$$\Pi_\tau = \rho + \left(1 - \rho\right) \phi \int_0^\tau \Pi_t e^{-(r+\phi)(\tau-t)} dt \tag{7}$$

To get an exact formula for Π_τ, we need to construct an ordinary differential equation. First, if we differentiate (7) by τ (again), we obtain

$$\dot{\Pi}_\tau = \left(1 - \rho\right) \phi \left(\Pi_\tau - \left(r + \phi\right) \int_0^\tau \Pi_{\tau-t} e^{-(r+\phi)t} dt\right)$$

Substituting for $\int_0^\tau \Pi_{\tau-t} e^{-(r+\phi)t} dt = \frac{\Pi_\tau - \rho}{(1-\rho)\phi}$ and simplifying, we get the differential equation

$$\dot{\Pi}_\tau = \rho \left(r + \phi\right) - \left(r + \rho\phi\right) \Pi_\tau$$

Finally, we solve to get

$$\Pi_\tau = \rho + \left(1 - \rho\right) \frac{\phi\rho}{r + \rho\phi} \left(1 - e^{-(r+\rho\phi)\tau}\right) \tag{8}$$

Now we can use Π_τ to calculate prices for general levels of v and γ. Let P_τ be the payment per unit time at $T - \tau$. It is easy to see that the surplus at $T - \tau$ is now $\gamma v V_\tau$ and that

$$P_\tau = \gamma v \Pi_\tau \tag{9}$$

Thus, the negotiated price increases linearly in γ and v.

Now consider negotiations between a supplier and her original buyer B, who values the product at v. Let the negotiated price be $\tilde{P}_\tau$ and let the surplus from agreement be $\tilde{V}_\tau$. Recall that the surplus from unrelated buyers and suppliers is $\gamma v V_\tau$. This is now increased by $\left(1 - \gamma\right) v$ per unit time. Therefore

$$\tilde{V}_\tau = \gamma v V_\tau + \left(1 - \gamma\right) v \int_0^\tau e^{-rt}$$

Again this surplus is split in a ratio $1 - \rho : \rho$

For this to hold for all τ, we must have

$$\tilde{P}_\tau = \Pi_\tau \gamma v + \rho v \left(1 - \gamma\right) \ \forall \tau$$

As with standard asset specificity, relative to unrelated market partners, the price for related partners is higher at all times by $\rho v \left(1 - \gamma\right)$. Finally,

$$\begin{aligned} \mu\left(\tau, T\right) &= \frac{\tilde{P}_{T-\tau}}{v} = \rho\left(1 - \gamma\right) + \gamma \Pi_{T-\tau} \\ &= \rho + \gamma\left(1 - \rho\right) \frac{\phi\rho}{r + \rho\phi} \left(1 - e^{-(r+\rho\phi)(T-\tau)}\right) \end{aligned}$$

The above holds only for $\phi < \infty$.

As $\phi \to \infty$, as long as $\tau > 0$,

$$\Pi_\tau \to \rho + (1-\rho)\frac{\rho}{\rho}(1) = 1 \text{ and } \tilde{P}_\tau \to \gamma v + \rho v (1-\gamma) = \bar{\mu} v$$

Therefore, as $\phi \to \infty$, as long as $\tau < T$, $\mu(\tau, T) \to \bar{\mu}$. Thus, section II is a special case. ■

Proof of proposition 6. (1) Since δ_t is convex in t, so is ψ. The function μ is concave. Therefore μ and ψ can intersect at most twice. At $\tau = 0$ and at $\tau = T$, $\psi(\tau) > \mu(\tau)$. Suppose that ψ and μ do not intersect or are tangent at a point. Then $\psi(\tau) > \mu(\tau)$ almost everywhere (except possibly a single point of tangency) and therefore $T^* = T$. Now suppose that ψ and μ do intersect but are not tangent. Then they intersect exactly twice. The first intersection is at T^μ. Let T^{h} be the second point of intersection.

Recall that $\lambda(\tau, T)$ is a weighted average of the function

$$f(t) = \left\{ \begin{array}{c} \psi(t) \text{ if } t \leq \tau \\ \mu(t, T) \text{ if } t > \tau \end{array} \right\}$$

and $T^* = \arg\max_\tau \lambda(\tau, T)$. If $\psi(\tau) > \mu(\tau)$ then $\lambda(\tau, T)$ is increasing at τ. Therefore, we cannot have $T^* < T^\mu$ or $T^{\mathrm{h}} \leq T^* < T$. If $\psi(\tau) < \mu(\tau)$ then $\lambda(\cdot, T)$ is decreasing at τ. Therefore we cannot have $T^\mu < T^* < T^{\mathrm{h}}$. Putting this together, only $T^* \in \{T^\mu, T\}$ is possible.

(2) Suppose that $T^{\bar{\mu}} < T$. (a) When $\sigma = 1$ so that $\phi = \infty$ then the result is the same as without market-induced asset specificity, and μ and ψ intersect exactly once, at $T^{\bar{\mu}}$. Therefore $T^* = T^{\bar{\mu}}$. Now considering σ close to 1, as $\sigma \to 1$, $\mu(T^{\bar{\mu}}, T) \to \bar{\mu} = \psi(T^{\bar{\mu}})$. Therefore, as $\sigma \to 1$, $T^\mu \to T^{\bar{\mu}}$.

To see why $T^* = T^\mu$ for high σ, consider $\frac{d}{d\tau}\mu(\tau, T)$ as $\phi \to \infty$

$$\begin{aligned} \mu(\tau, T) &= \rho + (1-\rho)\frac{\phi\rho}{r+\rho\phi}\gamma\left(1 - e^{-(r+\rho\phi)(T-\tau)}\right) \\ \Longrightarrow \frac{d}{d\tau}\mu(\tau, T)\bigg|_{\tau=T} &= -(1-\rho)\frac{\phi\rho}{r+\rho\phi}\gamma(r+\rho\phi) \\ &= -(1-\rho)\phi\rho\gamma \to -\infty \text{ as } \phi \to \infty \end{aligned}$$

Therefore $T^{\mathrm{h}} \to T$ as $\phi \to \infty$. Considering the choice between T^μ and T, the only advantage of T is that it yields a higher level of f on the interval $\left(T^{\mathrm{h}}, T\right]$. As this interval is squeezed to a point as $\phi \to \infty$, for ϕ sufficiently high, λ_τ must be maximized at $\tau = T^\mu$.

(b) Compare the outcome from σ and γ to the outcome from $\hat{\sigma} \leq \sigma$ and $\hat{\gamma} \leq \gamma$ where one of these inequalities is strict. ψ is unaffected by σ. However, μ shifts to $\hat{\mu}$ such that $\hat{\mu}(\tau, T) < \mu(\tau, T)\, \forall \tau < T$.

$$T^\mu = \sup\{\tau \leq T | \psi(t) > \mu(t, T)\, \forall t \leq \tau\}$$

Since $\hat{\mu}(t, T) < \mu(t, T)\, \forall t < T$, $\forall t < T^\mu$, $\hat{\mu}(t, T) < \psi(t)$. Therefore $T^{\hat{\mu}} \geq T^\mu$. Now suppose that $T^\mu < T$. Then since $\psi(T^\mu) - \hat{\mu}(T^\mu, T) > 0$, for ε sufficiently small, $\psi(T^\mu + \varepsilon) > \hat{\mu}(T^\mu + \varepsilon, T)$. Therefore, given this ε, $T^{\hat{\mu}} \geq T^\mu + \varepsilon > T^\mu \Longrightarrow T^{\hat{\mu}} > T^\mu$. Thus, T^μ is decreasing in σ and γ, and strictly so for σ and γ such that $T^\mu < T$.

(3) Suppose that $T^{\bar{\mu}} \geq T$. Then $\bar{\mu} < \psi(t)\, \forall t$. Therefore, $\forall \sigma \leq 1, \forall t, \forall T$, $\mu(t, T) \leq \bar{\mu} < \psi(t)$. Therefore, for all σ, $T^\mu = T$.

(4) Consider the outcome when $\sigma = 0$. In this case, $\mu(\tau, T) \equiv \rho < \psi(\tau)\, \forall \tau$. Therefore $\arg\max_\tau \lambda(\tau, T) = T$. For positive σ sufficiently close to 0, we still have $\mu(\tau, T) < \psi(\tau)\, \forall \tau$, and therefore we still have $T^* = T$. ■

Proof of proposition 7. (1) First, consider optimal contracting when T is small. $\psi(\tau) = \rho + \left(\lambda' - \rho\right) e^{-u\tau} \to \lambda' > 1$ as $\tau \to 0$. By contrast, $\mu(\tau, T)$ is always < 1. Therefore, if T is sufficiently small, then $\mu(\tau, T) < \psi(\tau)\ \forall \tau \in [0, T]$ and thus $T^{\mu} = T^{*} = T$.

(2) Now consider optimal contracting when T is large. $\psi(\tau) \to \rho$ as $\tau \to \infty$. Fixing τ and varying T,

$$\begin{aligned}\mu(\tau, T) &= \rho + (1-\rho)\frac{\phi\rho}{r+\rho\phi}\gamma\left(1 - e^{-(r+\rho\phi)(T-\tau)}\right)\\ &\to \rho + (1-\rho)\frac{\phi\rho}{r+\rho\phi}\gamma > \rho \text{ as } T \to \infty\end{aligned}$$

Therefore, if T is sufficiently large, there exists τ such that $\mu(\tau, T) > \psi(\tau)$. In other words, for T sufficiently large, the functions $\mu(\cdot, T)$ and $\psi(\cdot)$ intersect (and are not tangent). If they intersect, then they intersect twice, first at T^{μ}, and second at a point we will call T^{h}.

Now consider T_1 and T_2 such that $\mu(\cdot, T_1)$ and $\mu(\cdot, T_2)$ both intersect with $\psi(\cdot)$ and such that $T_1 < T_2$. Let T_1^{μ} and T_2^{μ} be the corresponding values of T^{μ}. For all τ, $\mu(\tau, T_1) < \mu(\tau, T_2)$. Also, $\mu(\cdot, T_1)$ and $\mu(\cdot, T_2)$ are both decreasing. Therefore since $\mu(T_1^{\mu}, T_1) = \psi(T_1^{\mu})$, we know that $\mu(T_1^{\mu}, T_2) > \psi(T_1^{\mu})$. Therefore $T_1^{\mu} > T_2^{\mu}$ since for $\tau \le T_2^{\mu}$, $\mu(\tau, T_2) \le \psi(\tau)$. We have therefore shown that for levels of T such that $T^{\mu} < T$, T^{μ} falls as T rises.

(3) Finally, as in the proof of proposition 6, note that $\frac{d}{d\tau}\mu(\tau, T)\,|_{\tau=T} = -(1-\rho)\phi\rho\gamma$ and $\mu(T, T) = \rho = \lim_{\tau\to\infty} v(\tau)$. Therefore $\lim_{T\to\infty}(\mu(T) - \psi(T)) = 0$. Therefore, since $\mu'(T)$ is independent of T, $\lim_{T\to\infty}\left(T - T^{\mathrm{h}}\right) = 0$. Again, since the interval $\left(T^{\mathrm{h}}, T\right]$ is squeezed to a point as $T \to \infty$, for T sufficiently large, $T^{*} = T^{\mu}$. Putting this result together with point (2), if the investment life-span increases sufficiently, the optimal contract term becomes shorter. ■

Proof of proposition 8. Suppose that $\gamma > 0$ and $0 < \sigma < 1$. Thus $\mu(\tau, T)$ is strictly increasing in T and decreasing in τ.

Let

$$\psi_u(\tau) = \rho + \left(\lambda' - \rho\right) e^{-u\tau}$$

denote the value of our original function $\psi(\tau)$ for uncertainty level u. This function is decreasing in u and in τ. As $u \to 0$, $\psi(\tau, u) \to \lambda' > 1$. As $u \to \infty$, $\psi_u(\tau) \to \rho$.

Given any $u > 0$, let $\lambda_u(\tau, T)$ be the value of $\lambda(\tau, T)$ and let $\lambda_u^{\max}(T) = \max_{\tau}\lambda_u(\tau, T)$.

Finally, let T_u^{μ} denote the level of T^{μ} as a function of u fixing T.

(1) Let $T > 0$. We now need to show that if u is sufficiently large, $\lambda_u^{\max}$ is obtained by a contract which ends before T. To show this, it is sufficient to show that if u is sufficiently high, it is better to have no contract at all than to have a contract of term T.

Note that $\lambda_u(0, T) > \rho$ and is not affected by any increase in u since μ is independent of u. By contrast $\lambda_u(T, T) \to \rho$ as $u \to \infty$ since for all $t > 0$, $\psi_u(t) \to \rho$ as $u \to \infty$. Therefore, for u sufficiently large, $\lambda(0, T) > \lambda_u(T, T)$. Therefore, for u sufficiently large it is better to have no contract at all than to have a contract with term T so that $T^{*} < T$.

Finally, it is easy to check that for u sufficiently large, the curves $\mu(\cdot, T)$ and $\psi_u(\cdot)$ intersect so that $T_u^{\mu} < T$. Independent of u, for any $\tau < T$, $\rho < \mu(\tau, T) < 1$. By contrast, for all τ, $\psi_u(\tau) \to \rho$ as $u \to \infty$. Therefore, for any $\tau < T$, for u sufficiently large, $\psi_u(\tau) < \mu(\tau, T)$, meaning that the curves of $\mu(\cdot, T)$ and $\psi_u(\cdot)$ intersect at a point before τ so that $T_u^{\mu} < \tau < T$. We conclude that $T_u^{\mu} \to 0$ as $u \to \infty$.

(2) Let $\hat{T} > 0$. We need to show that for u and T sufficiently large, $\lambda_u^{\max}(T) > \lambda_u^{\max}\left(\hat{T}\right)$. Note that, with some rearranging of terms,

$$\lambda(0,T) = \rho + (1-\rho)\frac{\phi\rho}{r+\rho\phi}\gamma\left(1 - \frac{r}{\rho\phi}e^{-rT}\frac{1-e^{-\rho\phi T}}{1-e^{-rT}}\right)$$

Define

$$\lambda(0,\infty) = \lim_{T\to\infty}\lambda(0,T) = \rho + (1-\rho)\frac{\phi\rho}{r+\rho\phi}\gamma$$

Note that $\lambda(0,\infty) > \lambda(0,T)$ for all finite T.

Let $\hat{T}_u^\mu$ be the value of T_u^μ for $T = \hat{T}$. Thus, for any u,

$$\begin{aligned}
\lambda_u^{\max}\left(\hat{T}\right) &= \frac{\int_0^{\hat{T}_u^\mu}\psi_u(t)\,e^{-rt}dt + \int_{\hat{T}_u^\mu}^{\hat{T}}\mu\left(t,\hat{T}\right)e^{-rt}dt}{\int_0^{\hat{T}}e^{-rt}dt} \\
&< \frac{\int_0^{\hat{T}_u^\mu}\psi_u(t)\,e^{-rt}dt + \int_0^{\hat{T}}\mu\left(t,\hat{T}\right)e^{-rt}dt}{\int_0^{\hat{T}}e^{-rt}dt} \\
&= \frac{\int_0^{\hat{T}_u^\mu}\psi_u(t)\,e^{-rt}dt}{\int_0^{\hat{T}}e^{-rt}dt} + \lambda\left(0,\hat{T}\right) < \lambda'\frac{1-e^{-r\hat{T}_u^\mu}}{1-e^{-r\hat{T}}} + \lambda\left(0,\hat{T}\right)
\end{aligned}$$

Using the fact that $\lambda(0,T) \to \lambda(0,\infty) > \lambda\left(0,\hat{T}\right)$ as $T\to\infty$, choose any $\varepsilon < \lambda(0,\infty) - \lambda\left(0,\hat{T}\right)$ and then choose $\tilde{T} > \hat{T}$ such that $\lambda(0,T) > \lambda(0,\infty) - \frac{1}{2}\varepsilon$ $\forall T \geq \tilde{T}$. The result is that $\lambda(0,T) > \lambda(0,\infty) - \frac{1}{2}\varepsilon > \lambda\left(0,\hat{T}\right) + \frac{1}{2}\varepsilon$ $\forall T \geq \tilde{T}$.

Now recall that $T_u^\mu \to 0$ as $u\to\infty$. Thus, choose $\tilde{u}$ such that for $u \geq \tilde{u}$,

$$\begin{aligned}
&\hat{T}_u^\mu < -\frac{1}{r}\ln\left(1 - \frac{1}{2}\varepsilon\frac{1-e^{-r\hat{T}}}{\lambda'}\right) \quad (>0) \\
\text{i.e.}\qquad &\lambda'\frac{1-e^{-r\hat{T}_u^\mu}}{1-e^{-r\hat{T}}} < \frac{1}{2}\varepsilon
\end{aligned}$$

Then for all $T \geq \tilde{T}$ and $u \geq \tilde{u}$ we have

$$\begin{aligned}
\lambda_u^{\max}\left(\hat{T}\right) &< \lambda'\frac{1-e^{-r\hat{T}_u^\mu}}{1-e^{-r\hat{T}}} + \lambda\left(0,\hat{T}\right) \\
&< \frac{1}{2}\varepsilon + \lambda\left(0,\hat{T}\right) < \lambda(0,T) < \lambda_u^{\max}(T)
\end{aligned}$$

■