Modelling Dynamic Constraints in Electricity Markets and the Costs of Uncertain Wind Output

by

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Abstract

Building on models that represent inter-temporal constraints in the optimal production decisions for electricity generation, the paper analysis the resulting costs and their impact on prices during the day. We linearise the unit commitment problem to facilitate the interpretation of shadow prices. Analytic research gives insights for a system with one technology and numeric implementation provides results for the German power system. The model is expanded to a stochastic optimisation with recourse. The model is used to calculate the cost of wind uncertainty and the value of updating wind forecasts.

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I. INTRODUCTION

Various engineering dispatch models show the implications of intertemporal linkages for the optimal operation of a power system. In market based environments these intertemporal linkages are reflected in the prices that deviate from the variable costs of the marginal unit. We use a simplified model with one technology to explain. The costs of starting up are usually not allocated to the subsequent period but to the peak period and at the same time prices at the demand minimum are reduced to reflect the benefit of avoided shut-down and subsequent start-up decisions. Furthermore, power plants require a minimum output. This part-load constraint creates additional shifts between periods. Finally, higher variable costs, incurred if power stations are operated below their optimal rating, are allocated to the locally lowest demand.

For inflexible power stations like nuclear, combined cycle gas turbines or coal the start of the station has to be decided several hours before delivering output. At the earlier time there is still uncertainty about the future demand, possible failures of power stations and predictions for wind-output. We represent the uncertainty using stochastic programming with recourse. In combination with the linearised unit commitment representation this is a new formulation. We then represent improved wind forecasts by aggregating different wind realisations into information sets. This allows us to quantify the value of improved wind forecasts in combination with a design that makes use of this information.

The impact of inter-temporal constraints, start-up and part load costs have been frequently discussed. Schweppe et al. (1988) developed a Lagrangian formulation to calculate the impact of inter-temporal constraints on the market equilibrium and prices. Hogan and Ring (2003) discuss how to use extra payments above marginal generation costs to pay for the additional costs. Oren and Ross describe how generators can misspecify intertemporal constraints in the balancing market, in order to exercise market power (2003). Simulations by Kreuzberg (2001), Cumperayot (2004) and Müsgens (2004) indicate that the marginal value of electricity can differ significantly from the variable costs of the marginal unit producing electricity. We analyse the optimisation problem to associate an economic interpretation with the various shadow prices that arise in the formulation of the optimisation problem. Bushnell (2003) discusses the impact of intertemporal constraints on price in the context of a hydro system with market power. The scarcity value (or shadow price) of water, and not the marginal costs of running the turbine in a given hour determine the dispatch and frequently set the marginal price.

The unit commitment problem exhibits non-convexities due to the indivisibilities of power plants. To illustrate the effect assume peak demand of 50.5 GW has to be covered with 1 GW units. Then 51 units have to be started up. If demand were to be increased by 0.1 GW then no additional units have to be started up and hence the marginal demand would only pay the energy costs – and not be exposed to start up costs. From this perspective the question arises how start-up costs can be earned. Hogan and Ring (2003) suggest minimum uplift payments to dispatched units in addition to energy payments to allow them at least zero profits. O'Neil et al. (2005) discuss payment approaches to compensate individual generators for additional costs. They suggest a two-stage approach with an MIP model in the first stage and the integer solution to that problem fed as constraints into a linear model in the second step. The linear model allows an interpretation of shadow prices. Alternatively we can imagine uncertainty about demand or supply. Returning to the previous example, imagine that anticipated demand is uniformly distributed between 50 GW and 51 GW. Then the additional demand of 0.1 GW has to carry the start-up cost of an additional 1 GW unit with 10% probability or in expectation has to pay 1/10th of the start up costs of a 1 GW unit. So if uncertainty about demand and supply balance exceeds the capacity of typical units at the margin then non-convexities have limited impacts on pricing decisions.

To quantify the effect of inter-temporal constraints on generation costs a dynamic linear optimisation model is used to choose the power plant dispatch with minimal generation costs. The dynamic component is added through the simultaneous optimisation of several consecutive load levels. The initial model is then expanded to a stochastic linear program with recourse (see Carpentier et al., 1996, Takriti et al., 2000). This enables the formalisation of the uncertainty about demand, possible failure of some generation capacity or output from intermittent generation. Gröwe et al. (1995) used the same method to capture deviations of demand realisation from dispatch, though ignoring unit commitment. Hobbs et al. (1999) use a unit commitment model to calculate the optimal dispatch for each of the possible realisations. Then they choose the dispatch, which performs best when tested against all of the realisations. Their approach also allows for the use of observed errors with their intertemporal structure.

We represent the uncertainty that remains several hours before dispatch; this is the time when inflexible generating units are started up. Linear programming with recourse selects a set of realisations of, and probabilities for, the parameters that are uncertain, treats this set as a deterministic set of future outcomes, and optimises in order to minimise the expected cost function over all these realisations. The decision, according to which inflexible capacity is started up, stays fixed for all realisations of the demand and wind forecasting error, while output decisions of the started and of the flexible plants are allowed to differ between the realisations. We retain a fixed exogenously determined additional reserve quantity to compensate for power station and grid failures. This approach allows us to model the implications of uncertainty in wind predictions while retaining the linear and deterministic structure of the optimisation problem.

In a third step, we model the effect of reduced uncertainty on marginal costs. In a first set of simulations, we assume a gate closure at 2:30 p.m. on the day before delivery. At the time of gate closure, planned plant dispatch must be reported to the grid operator. All deviations from nominated schedules must be served using reserve and balancing power. Some power markets allow for changes on a shorter time scale - e.g. up to one hour before dispatch in the UK - but usually liquidity in these short-term markets is too low to allow for significant adjustments. In a second set of simulations, we calculate the value of dispatching the system using the reduced forecasting error closer to dispatch. Currently, the day-ahead market determines dispatch 24 hours before demand realisation, and therefore can only use rather inaccurate predictions. However, most power plants can be started on a shorter time frame, e.g. four hours, and allow the usage of better demand and wind predictions. We group the stochastic deviations into equal-sized information sets (Laffont, 1984). The improved information available closer to dispatch is represented by additional information specifying which information set will describe the possible deviations.

Based on the assumption that the impact of the individual units on dispatch costs is small in large markets we group units in different technologies. For every technology, the variable on start-ups is assumed to be continuous. So the model can for example start up any capacity between 0 and 21 GW of hard coal capacity available in the system. However, once the decision to start-up x GW has been made, production is restricted by that limit (and minimum production has to fulfill the partial load restriction). From the perspective of interpretation this follow the example of NYISO, where a unit commitment program initially calculates the optimal dispatch but prices are calculated in a second run allowing for start-up decisions of fractions of units. Alternatively Madrigal and Quintana (1998) suggest using the prices from the Lagrange relaxation, thereby smearing the start-up costs over larger ranges of the marginal demand. From the numerical perspective we can refer to the good match of modelled prices with observed prices in the German market that Kreuzberg (2001) obtained using this approach. Allowing for continuous start up decisions avoids the computational complexities that result from solving mixed integer problems (MIPs). The challenges and other solution approaches are described in Wood and Wollemberg (1996) and Sen and Kothari (1998). The models have been solved, initially with dynamic programming, genetic algorithms, Lagrangian relaxation and, recently, with branch and bound algorithms (Makkonen and Lahdelma, 2005).

Once the theoretical framework is established, we parameterise the model with realised data for the German market. We use the example of wind power generation to analyse the effects of uncertainty. We find that the costs of balancing wind power were relatively low in the German system in 2003. They could be reduced even further when a better forecast becomes available, either by implementing a later gate closure or by improvements in the wind forecasting model. We estimate that variable costs of conventional generation increase by approximately 1.4% if only 24 hour wind predictions are used to determine unit commitment. If improved wind forecasts are used and final dispatch is determined four hours before realisation, then variable costs only increase by 0.6%.

The paper is structured as follows. Section II introduces the formulation of the inter-temporal constraints and analytic results on how they affect prices. Section III adds uncertainty to the model using a deterministic linear equivalent of a stochastic optimisation model with recourse. Section IV presents a model to quantify the savings brought about by reduced uncertainty. In Section V, this model is then parameterised with data for the German power market in the year 2003 and applied to calculate the benefits updating wind forecasts. Section VI concludes the paper.

II. MODELLING OF INTERTEMPORAL CONSTRAINTS

We introduce three physical characteristics of power plants and the resulting intertemporal constraints. Analytic arguments are used to show how these constraints alter the marginal energy prices at different segments of the load curve.[1]

Intertemporal Constraints - Thermal System

We calculate the optimal dispatch for the operation of an electricity system. To simplify the representation in this section we only assume one technology and ignore uncertainty. We start with a model that only captures fuel and start-up costs. To ensure started capacity will subsequently be stopped, we include part-load constraints. In a second step, the model is expanded to also capture part-load costs.

The system operator determines the output choice X_t to maximise the system benefits $-TC$ over hours t of the day, given variable operational costs of c^x of unit and start up costs c^u, which are incurred when capacity U_t is started in period t. Maximize with respect to X, U, and D:

(1) $$-TC = -\sum_{t=1}^{T} \left(X_t c^X + U_t c^U \right).$$

The optimisation is subject to the energy balance for each period (shadow price λ_t^d):

(2) $$d_t - X_t = 0 \qquad \forall t.$$

The sum of capacity started in the current and preceding periods minus the sum of stopped capacity D_t must equal or exceed current production (shadow price λ_t^{su}):

(3) $$X_t - \sum_{t\_1=1}^{t} \left(U_{t\_l} - D_{t\_1} \right) \le 0 \qquad \forall t.$$

Power stations have a minimum output quantity α (with $0 \le \alpha \le 1$), below which production is not possible or only with unacceptable efficiency losses. This is represented by the part-load constraint (shadow price λ_t^{pl}):

(4) $$\alpha \cdot \sum_{t\_1=1}^{t} \left(U_{t\_1} - D_{t\_1} \right) - X_t \le 0 \qquad \forall t.$$

The Lagrange function capturing these constraints is:

(5)
$$L = -\sum_{t=1}^{T} \left(X_t c^X + U_t c^U + \lambda_t^d \left(d_t - X_t \right) + \lambda_t^{su} \left(X_t - \sum_{t\_1=1}^{t} \left(U_{t\_l} - D_{t\_1} \right) \right) + \lambda_t^{pl} \left(\alpha \sum_{t\_1=1}^{t} \left(U_{t\_1} - D_{t\_1} \right) - X_t \right) \right).$$

1 A list of symbols is shown in the appendix.

The Kuhn-Tucker condition for output choice X_t is:

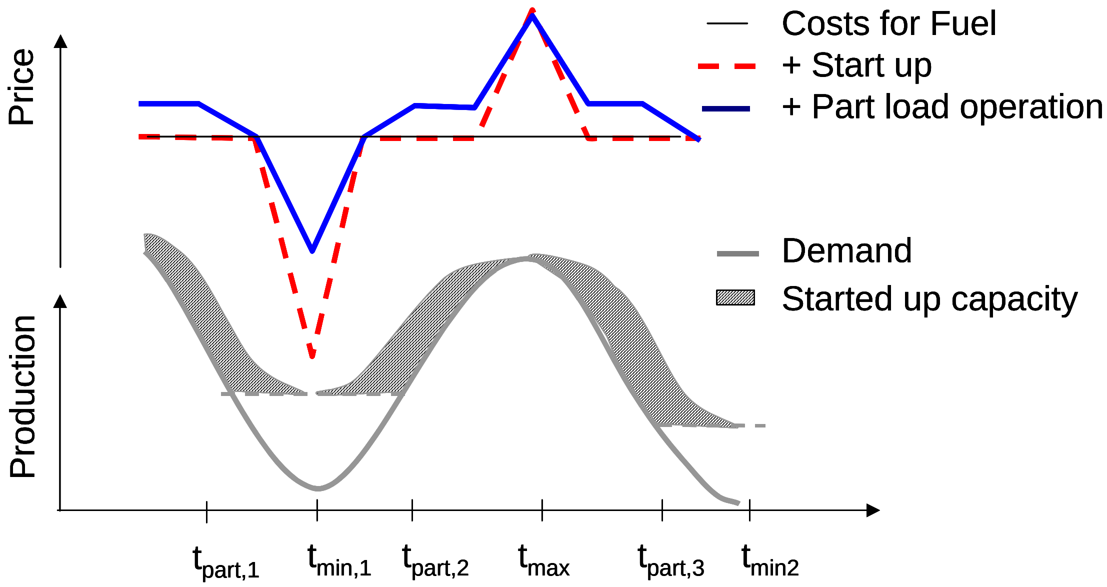
(6) $$\frac{\partial L}{\partial X_t} = -c^X + \lambda_t^d - \lambda_t^{su} + \lambda_t^{pl} = 0 .$$

This shows that the hourly energy price λ_t^d is above variable costs c^x at times when the start-up constraint is binding ($\lambda_t^{su} > 0$) and below variable costs if the part-load constraint is binding ($\lambda_t^{pl} > 0$).

The Economics of Intertemporal Dynamics in Power Plant Dispatch Without Partial Load Cost

We analyse the economic effects of these inter-temporal constraints for the case of one electricity-generation technology. Under these circumstances, we can state several properties. We use the following notation: $t_{\max}$ indicates the hour with maximum load, preceded and followed by minimum load hours $t_{\min,1}$ and $t_{\min,2}$. We assume that minimum run constraints imply that generation has to be started and stopped during the demand cycle ($\frac{1}{\alpha} d_{t_{\min}} < d_{t_{\max}}$). Figure 1 visualizes this structure.

Figure 1: Energy Prices with Inter-temporal Constraints



Note however, that we use discrete time steps both in the proof and in the simulation our problem is still discrete despite the linear form in the figure. In the model without part-load costs multiple solutions are possible, as the grey striped area for started-up capacity illustrates that this parameter is not uniquely defined. As a result, the start up or part load constraint can shift between binding and non-binding states. However, the corresponding shadow prices will always stay zero when both a constrained and unconstrained state is possible and thus all shadow prices are uniquely defined.

Appendix II contains the formal derivation of the following properties:

- *The start-up costs are not allocated to hours outside the demand peak ($\lambda_t^{su} = 0$ for $t \neq t_{\max}$)*[2] *(Proposition 2) and are fully added to the demand peak $\lambda_{t\max}^{su} = c^U$ (Proposition 3).*
- *Avoided start-up costs are not allocated to periods outside of demand minimum ($\lambda_t^{pl} = 0$ for $t_{min,1}<t<t_{min,2}$, Proposition 1) and reduce prices by $\alpha\lambda_{t\min}^{pl} = c^U$ (Proposition 4) at the demand minimum.*

Using (6), these propositions determine the value of energy through the load cycle. $\lambda_t^d = c^X + \lambda_t^{su} - \lambda_t^{pl}$ (dashed line in Figure 1). All units receive additional revenues to cover their start-up costs in the peak hour. But units that run through the entire load cycle do not incur start-up costs, and hence their revenues are reduced accordingly in the demand minimum.

The Economics of Intertemporal Dynamics in Power Plant Dispatch Including Partial Load Cost

Now we extend the model to cover the part-load costs c^{pl}, which a generator incurs for capacity that is operating but not producing electricity.[3] The maximisation problem (1) changes to the maximization of the following equations with respect to X, U and D:

(7) $$-TC = -\sum_{t=1}^{T}\left(X_t(c^X - c^{Pl}) + U_t c^U + \sum_{t\_l=1}^{t}\left(U_{t\_l} - D_{t\_l}\right)c^{Pl} \right)$$

The Lagrange function (5) is expanded with the same terms in c^{pl} and will be referred to as (5)*. The marginal value of an-extra unit o energy is given by:

(6)* $$\frac{\partial L}{\partial X_t} = -c^X + c^{pl} + \lambda_t^d - \lambda_t^{su} + \lambda_t^{pl} = 0$$

We define the boundaries $t_{part,i}$ as the points framing a minimum between which capacity is operated in partial load to avoid future start-ups. This period is restricted by the minimal demand: $d_{t_{part,i}} \leq \frac{1}{\alpha} d_{t_{\min,i-1}} \leq d_{t_{part,i+1}}$ for i=(2,3); $d_{t_{part,1}}$ accordingly. In Figure 1, started up capacity

2 The result no longer holds if we move to multiple technologies. Assume technology (a) with 10 Euro/MW start up costs and 40 Euro/MWh variable costs and technology (b) with 35 Euro/MW start up costs and 20 Euro/MWh variable costs. (a) will be started in the peak hour, setting a price of 50 Euro/MWh. If the hour adjacent to the peak exhibits the second largest demand, then some capacity of type (b) will be started for two hours. 30 Euro/MW of the start up costs will be recovered in the peak hour. The price in the adjacent hour then has to cover the remaining 5 Euro/MW start up and 20 Euro/MWh variable costs and will be 25 Euro/MWh.

3 This captures the fact that the efficiency of a plant is decreasing with the decreasing loading of the plant.

exactly tracks demand for t with $t_{part,2}<t<t_{part,3}$ ((3) holds with equality). Otherwise, if at t demand is strictly smaller than started up capacity then part-load costs could be reduced. During periods of increasing demand this would involve delaying start up decisions. During periods of falling demand this would involve shifting shut down decisions to earlier periods.

We assume start-up costs are big relative to part-load costs so that only minimum run conditions determine the amount of shut-down and start-up $\left(t_{part,2}-t_{part,1}\right)c^{pl}<c^{U}$. If we furthermore assume that demand is increasing monotonously from the demand minimum to the maximum and subsequently monotonously decreasing to the demand minimum then the following results can be derived (see appendix):

- *The start-up costs are not to hours outside the demand peak, with* $\lambda_t^{su}=0$ *for* $t_{part,1}\le t\le t_{part,2}$ *and* $\lambda_t^{su}=c^{pl}$ *otherwise (Proposition 5). Instead, they are added to the demand peak:* $\lambda_t^{su}=c^{pl}+c^{U}$ *(Proposition 6).*
- *Avoided start-up costs are not allocated to periods outside of demand minimum:* $\lambda_t^{pl}=0$ *for* $t_{min,1}<t<t_{min,2}$*. (Proposition 1 still applies) and reduce prices by* $\lambda_{t_{\min,1}}^{pl}=\left(c^{U}-(t_{part,2}-t_{part,1})c^{pl}\right)/\alpha$ *at the demand minimum (Proposition 7). The reduction is partially compensated as all part-load costs are allocated to the demand minimum.*

If we relax the strong requirement on monotony and assume that demand peaks twice, then we obtain the following additional insight

- *In system with two peaks between which capacity is operated part load but not shut down the local demand peak* t_{p1} *carries the part load costs for the time between the local peak* t_{p1} *and the time* t_{ge} *at which the global peak's demand equals the local peak (* $d_{t_{p1}}=d_{t_{ge}}$ *):* $\lambda_{p1}^{su}=\left|t_{p1}-t_{ge}\right|\cdot c^{pl}$ *. (Proposition 8)*

The results of this section are summarised in Figure 1. Start-up costs are added to the hour of peak demand. If the part-load constraint is binding, then ${}^{1}\!/_{\alpha}$ (part-load fraction) of the start-up costs will be deducted from the energy price at the lowest demand point. Part-load costs incurred during part-load operation are added to the price during the demand minimum and in double-peaking systems to the price at the lower peak.

Intertemporal Constraints – Hydro-Storage

The dispatch of hydro-storage capacity is another dynamic aspect optimised in our modelling approach. Hydro-storage plants are described by a capacity constraint restricting their maximal output at any time t and an energy constraint (posed by the amount of water stored in the basin). While these hydro-storage plants have variable generating costs of nearly zero, the energy constraint limits the time for which they can be dispatched.. Hence, storage water production is dispatched during hours where it can reduce total generation costs the most. This is usually during peak demand periods. Dispatch decisions for hydro-storage facilities are by their very nature intertemporal, as the production of hydro-storage in one hour takes up energy that would otherwise be available for production in other hours. Pump storage plants can increase the available energy budget by pumping during low demand periods.[4]

If hydro-storage is energy, and not capacity constraints, then it flattens peaks. Therefore, the start-up costs that are usually allocated to one hour are distributed over multiple hours or peaks. Each hour then only receives a fraction of the start-up costs, and prices are less volatile.

The complete set of equations describing the optimisation problem, including hydro-storage and pump-storage dispatch constraints, is described in the Appendix. Equations (17), (24) and (25) contain the endogenous optimisation of storage and pump-storage facilities.[5]

III. MODELLING OF UNCERTAINTY

It is often pointed out in the literature (e.g. E.ON wind report, 2005) that the stochastic pattern of wind power generation imposes additional costs due to an increase in the required amount of balancing power and a less favourable plant dispatch. To approximate the effects of uncertainty in our linear optimisation model, we introduce a set $r = 1,...,R$ of possible realisations of forecasting error. As we simultaneously model 24 hours of a day[6], each of the forecasting error realisations is a vector with 24 values, one for each hour of the day. An efficient dispatch of the system must take into account the distribution of forecasting errors within each hour, and their correlated between-hours. The following example illustrates the relevance of intertemporal correlation of forecasting errors. The best response for a one-hour deviation between forecast and realised demand is to start a peaking plant with low start-up and high variable costs. In contrast, if the deviation is expected to remain over several hours, then it might be worthwhile to start a plant with higher start-up costs and lower variable costs.

This increases the space from which we have to sample forecasting errors from $R \cdot 24$ to R^{24}, and makes it computationally impossible to comprehensively sample the entire space. Therefore, we must restrict ourselves to calculating dispatch situations with typical time paths of

4 Pump-storage plants consume electricity during low price periods to pump water from a lower basin up to a higher basin. Potential energy stored in the water in the higher basin can be used for electricity production during high price periods by letting it again flow into the lower basin. With an efficiency of above 75% (consume 4 MWh during low price periods to produce 3 MWh during the peak), this is a widely used way to store electricity in regions with the right landscape.

5 Optimizing ‘only’ 24 hours in our model, we make a simplification on inter-daily and long-run hydro dispatch decisions which we must treat as exogenous input. However, inter-seasonal hydro optimisation is not the focus of this article, as we concentrate on short-term dispatch decisions. In addition, we apply our methodology to the German market, which is somewhat influenced by hydro-storage facilities, but far less than other markets, e.g. Northern Europe.

6 To avoid the impact of boundary conditions, we always simulate three consecutive days and then report the results for the middle day.

forecasting error deviations. We will use observed data for the forecasting errors in our empirical simulation. We will describe the data set in section V.

We assume that each error realisation can occur, with probability θ_r. The optimal system dispatch now involves maximising the expected system benefit. This is represented by introducing the probability-weighted sum over all realisations in (7). Furthermore, we introduced additional supply technologies by including the set $s = 1,...,S$ of different technologies. We avoid the problems of indivisibilities (a non-convexity) by grouping plants in similar supply technology groups[7] and assuming infinitesimal unit size in each group. Maximize with respect to $X, U,$ and D:

(8) $$-TC = -\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{r=1}^{R}\theta_r * \left(X_{s,t,r} * \left(c_s^X - c_s^{PL}\right) + U_{s,t,r} * c_s^U + \sum_{t\_1=1}^{t}\left(U_{s,t\_1,r} - D_{s,t\_1,r}\right) * c_s^{pl} \right).$$

The demand equation in (9) must be satisfied for each realisation of the forecasting error $\rho_{t,r}$. In addition, we reduce demand by the average expected wind generation w_t^e.

(9) $$d_t - w_t^e + \rho_{r,t} + P_{r,t} - \sum_{s=1}^{S} X_{s,t,r} = 0$$

A power plant's generation is restricted by its installed available capacity. However, a plant must be started up to be able to produce. As was formalised in equation (4), plants can change both production as well as start-up and shut-down decisions. However, since the deviations brought about by the forecasting error's realisation are, by their nature, unpredicted and arising on short notice, they must be covered by reserve and balancing capacity. This brings a crucial aspect of inflexibility into the model: some technologies do not have the flexibility to start up or shut down additional capacity on short notice. Therefore, these inflexible plants' (s^{nf}) amount of capacity started up and hence ready for operation must be identical for all possible realisations of the forecasting error. (10) shows these constraints.

(10) $$U_{s,t,r} = U_{s,t}, \quad D_{s,t,r} = D_{s,t} \qquad \forall s \middle| s \le s^{nf}, \forall t, r$$

Because of computational constraints, we can only model a limited number of system realisations. Extreme deviations, occurring with low probability, are not captured by the system realisations. We add an equation for additional reserve capacity to ensure that sufficient flexible and spare operating capacity is available for these cases. We also subsume other sources of uncertainty, such as unforeseen plant outages and load deviations, in this equation. We capture this by introducing a capacity constraint for reserve and balancing power:

(11) $$d_t + rc - \sum_{s=1}^{s^{nf}}\left(UP_{s,t\_1,r} - DN_{s,t\_1,r}\right) - \sum_{s=s^{nf+1}}^{S} x_s^m \le 0$$

7 Our specific setup for the German market will be discussed in section V.

IV. UPDATING OF WIND FORECASTS

The uncertainty of wind power generation can be reduced by improving the quality of wind forecasts. This can either be achieved by getting a better 24-hour-ahead forecast or by using a more up-to-date forecast when deciding on plants' start-up and shut-down decisions. There is a limit to the second approach, as scheduling of inflexible plants requires sufficient lead-time. However, this lead-time of about four hours before production has not yet been achieved in most markets. In Germany, for example, plans for plant operation are decided and reported to the transmission grid operators at 2:30 p.m. the day before delivery, for all 24 hours of the delivery day. In theory, the British gate closure of one hour does undercut this lead-time; in practice, liquidity is too low in the intra-day market to allow for generators to reschedule efficiently. Postponing this notification, at least for wind power, to a later point in time would allow the use of a better wind forecast.[8]

We will measure the effect of the reduced uncertainty in the wind forecast, either by a later gate closure or by more advanced prediction models, by using a four-hour-ahead forecast instead of the 24-hour-ahead forecast. However, we cannot simply calculate a model run with the four-hour-ahead wind forecast instead of the 24 hour wind forecast, because the effect on system costs of individual wind forecast errors is in the same order of magnitude as the effect of improving the wind forecasts. Therefore, choosing a different set of wind forecast errors would eliminate the opportunity to compare the results of both forecast scenarios.

We therefore model this increase in information by dividing the original set for the forecasting error into different subsets. The number of possible forecasting error realisations is thus reduced in each model run. Thus, the increase in information gained by the four-hour-ahead forecast is used to decide which subset of the original set of forecasting errors is reached. This leads to a reduction in costs, as plants can operate more flexibly. Instead of one mode of operation for all possible realisations, there is now a number of different modes of operation (one for each of the newly-created subsets). An additional aspect reducing costs when moving from the 24-hour-ahead forecast to the four-hour-ahead forecast is that the remaining uncertainty in the system is also reduced. This uncertainty might be caused partly by the possibility of highly unlikely wind conditions, but also by other factors of uncertainty, such as demand forecasting errors or plant outages. We treat those aspects by introducing an additional constraint representing the reserve capacity requirement (11). The resulting effect will be analysed separately. The results can be compared to the day-ahead forecast by running separate scenarios for each information subset and averaging over these model runs.

This clear-cut way of replacing day-ahead forecasts with four-hour forecasts gives us an upper bound to the system improvements. By the very nature of a 'four-hour-ahead' wind and de-

8 However, lead time is not only limited by thermal plants' inflexibilities but also by the grid operators' responsibility to maintain a secure network, which necessitates early enough knowledge of expected power flows.

mand forecast, only the next four hours are available. For the hours five to 24 hours ahead of dispatch, we cannot expect the same forecasting accuracy as we assume in our model.

V. APPLICATION TO GERMAN POWER SECTOR

We apply the model described above to the German power market. In 2003, our reference year, Germany was the country with the largest installed wind capacity: nearly 15 GW. The costs for the integration of wind power in the German system are currently the subject of lively debate (e.g. DENA, 2005), as plans for a further doubling of wind capacity until and beyond 2010 are being discussed.

Müsgens (2004) describes a model for the entire European dispatch, at the expense of less detailed representation of intertemporal constraints, reserves and balancing requirements. This allows for the endogenous determination of interconnector flows, which are used as exogenous input in our model due to lack of empirical data with sufficient resolution. The daily energy budgets for hydro-storage and pump-storage plants are also taken from that model. This simplification reduces price elasticity, as these parameters cannot adjust price signals in the model presented in this paper.

In the following representation, we define model demand as German demand net of CHP, run off river hydro, expected wind generation and international power exchange. Hourly wind forecasts and realisations are provided by ISET e.V.

Generation plant data are taken from EWI's plant data base, as data on efficiencies and installed capacities are hardly published anymore.[9] We mentioned in section III that we subsume supply technologies in different groups. To be more precise, we distinguish 16 supply technology groups (nuclear, three lignite, four hard coal, two combined cycle gas turbine, three open cycle gas turbine, two oil-fired technologies and one storage technology). In addition, we assume a value of lost load (VOLL) of 1500 Euro/MWh, and the price for the option to call demand-side response is set at 150 Euro/MWh. This level is assumed to make it the most expensive technology and hence a 'lender of last resort'. A VOLL of 1500 Euro/MWh is significantly lower than the 2000 Pounds/MWh in the British Pool. Nonetheless, even 1500 Euro/MWh for the provision of balancing power is likely to overestimate the costs for balancing the system, given the low probability for the last MWs of the 7000 MW reserve capacity to be called.

The perfect model of an electricity market would necessitate the simultaneous optimisation of all 8760 hours of the year, but was impossible in our detailed model due to computational constraints. Therefore, we simultaneously optimise dispatch decisions for 24 hours of the day in each model run. To capture the effects of uncertainty, we allow $R = 12$ forecasting error realisations per day. This gives a total of $12 \cdot 24 = 288$ marginal cost results per model run. In addition, modelling a complete daily load cycle allows us to endogenously optimise start-up

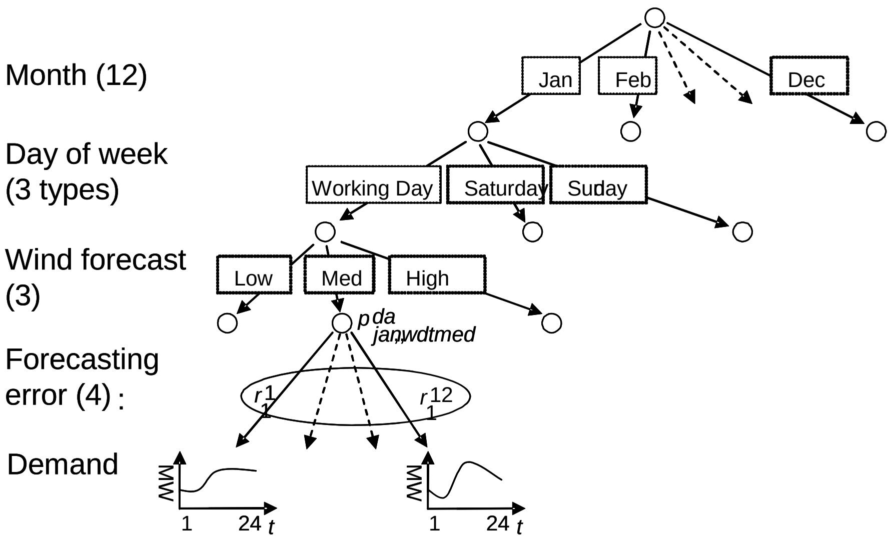
9 The last exhaustive publication, which is the foundation of the data base for Germany, was VDEW (2000).

and shut-down decisions, as well as the variation of storage and pump-storage capacity over these 24 hours (e.g. pumping at night and producing at maximum capacity during the hours of highest demand). The resulting model has 65,000 equations and 45,000 variables and was solved on a 2 GHz desktop in about five minutes.

Nonetheless, one day is obviously not representative of a whole year. Therefore, we solve the model for twelve different months per year. In each month, three different day types are analysed: a working day, a Saturday and a Sunday. We differentiate between three different wind-scenarios in each month by sorting them for strong, medium and low wind output. The total number of independent scenarios we compute for one year, as summarized in Figure 2, is 12x3x3=108. Multiplied by the 288 marginal cost results per scenario, we calculate 31104 different data points for the construction of a year.

Obviously, there are some dynamic effects which exceed the 24 hour period of one day. Of particular concern is hydro-storage; most storage facilities are not optimised on a daily basis, but on a weekly or even seasonal basis. While we chose not to account for these effects endogenously in our model, we consider them exogenously by choosing appropriate energy budgets for hydro-storage to different months and days of the week.

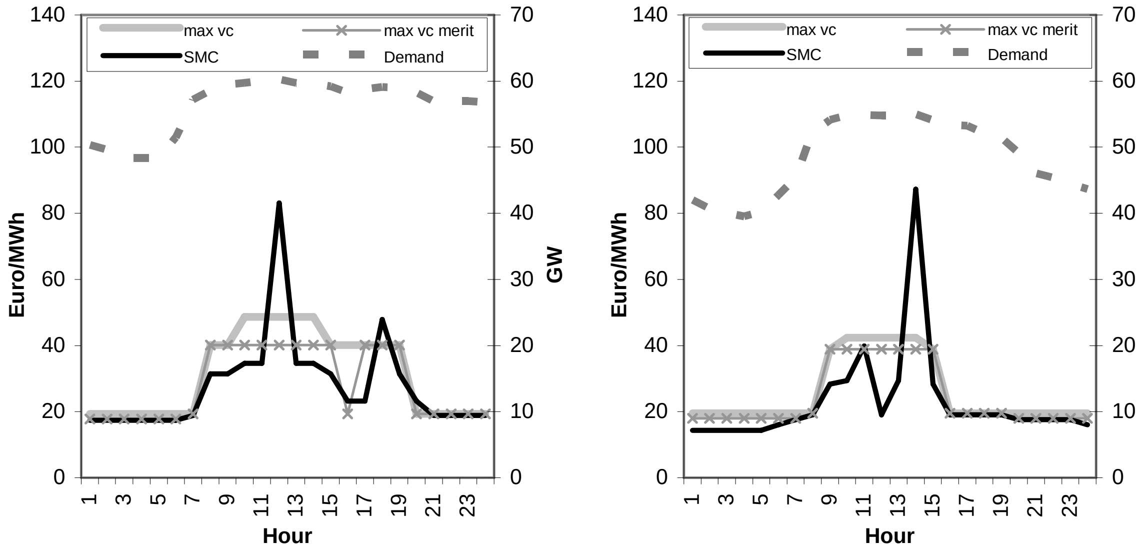
Figure 2: Total Number of Scenarios and Forecasting Error Realisations



Intertemporal Aspects

Figure 3 shows results of the model in the absence of uncertainty about wind output. System marginal costs (SMC) represent the simulated price for each hour. The grey line ‘max vc’ shows the variable generating costs of the most expensive technology producing in any hour. This line excludes the effects part-load and start-up costs have on the price. As predicted in the analytic model, the price curve is flatter with lower peak and higher off-peak prices. The analysis also illustrates the size of the errors that could result if a competitive benchmarking study were to compare observed prices with the variable costs of the most expensive unit on the system. Finally, in the curve ‘max vc merit’, the start-up and part load costs are not only ignored in the price formation but also for plant scheduling. With fewer constraints it is always possible to operate a unit with weakly lower variable costs. Hence, this line is bounded from above by ‘max vc’.

Figure 3: Costs and Demand with Hydro Storage Dispatch, January (left) and July (right), Demand [GW] and Costs [Euro/MWh]

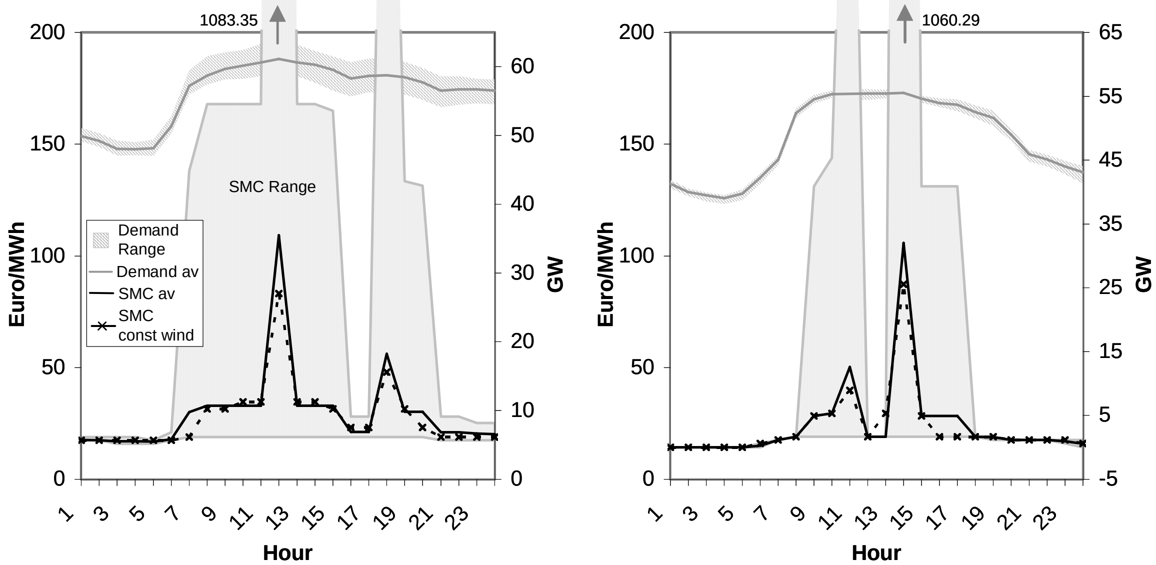


Uncertainty in the Wind Forecast

We use wind data for 2003, provided by ISET in Germany. The data set contains hourly wind generation and the forecasts from four hours and 24 hours before dispatch. Generation and forecasting error in the data set are normalised on the installed capacity of 14521 MW at the end of the year. We include in the analysis of each month some days of the following month, so that the total number of days is 36. They are then divided in three groups of twelve days with strong, medium and low wind generation. For each day, we calculate the difference between 24 hour forecast and wind realization. This gives us, for each of the strong, medium and low wind scenarios, 12 likely prediction errors. We take one additional step to make our data comparable to other studies, by scaling the prediction errors with the factor 1.04, so that the standard deviation of the prediction error over the year is 7.29% of installed wind power capacity. Thus they are compatible with the DENA-Study (2005, p. 263).

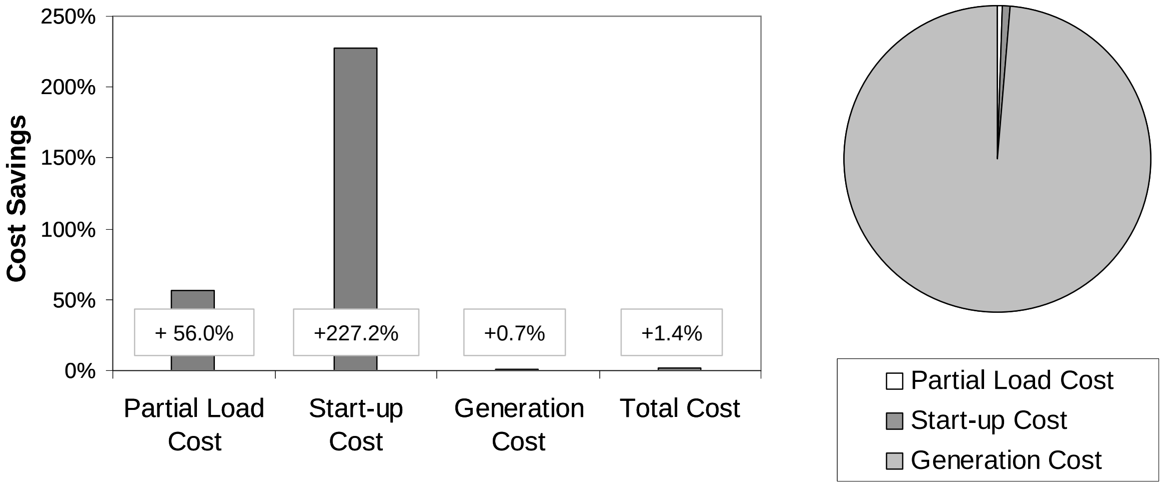
Figure 4 gives an example of the effects caused by uncertainty in the wind forecast. The demand range is determined by the maximal absolute deviations in the wind forecasting error, both upwards and downwards from the demand average. The grey-shaded SMC range is the range between minimal and maximal system marginal cost realizations. While SMC in most scenarios are grouped rather close to the average (‘SMC av’), the maximum is extremely high because it bears all the costs for the provision of reserve energy from equation (11). Comparing these results with marginal costs derived without uncertainty (included in Figure 4 in the line ‘SMC const wind’), we find that the wind power’s uncertainty adds greatly to the volatility in SMC. However, the cost influence on the average is low.

Figure 4: Uncertainty brought about by Wind Power, Medium Wind Scenario, January (left) and July (right), Demand [GW] and Costs [Euro/MWh]



The result, that the additional costs brought about by wind power's uncertainty are low, is verified when we analyze the whole year instead of just two selected days. Figure 5 shows the changes in costs when the wind generation's volatility is added to the model. We compare two model runs with identical average wind generation. Once, the wind generation is constant over all R=12 scenarios. In the alternative, the 12 forecasting errors represent the wind power's volatility as described above. We find that both costs for part-load operation, as well as start-up costs, increase significantly as the result of the increased volatility. This was to be expected, as start-up and shut-down decisions are the key variables used to balance wind power's volatility. On the other hand, we find that the increase in generation costs is marginal. This is also plausible as average wind generation is held constant and only the volatility is changed. We find that the total cost increase as a result of wind volatility is rather low. We can understand this by looking at the right part of the graph, where we see that more than 98% of total costs are coming from generation costs, even in the model run with wind volatility. Therefore, the low increase in generation costs outweighs high relative increase in start-up and part-load costs, leading to a low overall increase in total costs.

Figure 5: Annual Cost Increase due to Volatile Wind Power Generation by Component (left) and Cost Components' Share of Total Costs (right)



Moving from 24-hour to Four-hour Wind Forecasts

The costs arising from volatile wind power generation can be reduced even further when a better forecast is used. In Section IV, we gave a general description of the approach aken to include the additional information becoming available when moving from a 24-hour to a four-hour forecast. However, here we describe in greater detail how we applied this to our data set. We split the 12 forecasting error realisations into three independent scenarios, with only four realisations in each scenario (see Figure 2.

The actual determination of which forecasting error belongs in which subgroup is determined by solving another optimisation problem. This problem is non-linear with binary variables. The objective function (12) shows that it sorts the twelve realisations into three groups, minimising the total variance for all forecasting errors. The constraints ensure that

– we end up with four realisations in each subgroup (13),

– every realisation is either totally in a subgroup or not at all (14),

– and every realisation appears in exactly one subgroup (15).

(12)
$$\min_{V^1,V^2,V^3} \sum_{t=1}^{T}\sum_{r=1}^{R}\left(V_r^1 rr_{t,r}\right)^2 + \sum_{t=1}^{T}\sum_{r=1}^{R}\left(V_r^2 rr_{t,r}\right)^2 + \sum_{t=1}^{T}\sum_{r=1}^{R}\left(V_r^3 rr_{t,r}\right)^2$$
$$-\frac{1}{4}\left(\sum_{t=1}^{T}\left(\sum_{r=1}^{R} V_r^1 rr_{t,r}\right)^2 + \sum_{t=1}^{T}\left(\sum_{r=1}^{R} V_r^2 rr_{t,r}\right)^2 + \sum_{t=1}^{T}\left(\sum_{r=1}^{R} V_r^3 rr_{t,r}\right)^2\right)$$

s.t.

(13)
$$\sum_{r=1}^{R} V_r^1 = \sum_{r=1}^{R} V_r^2 = \sum_{r=1}^{R} V_r^3 = 4$$

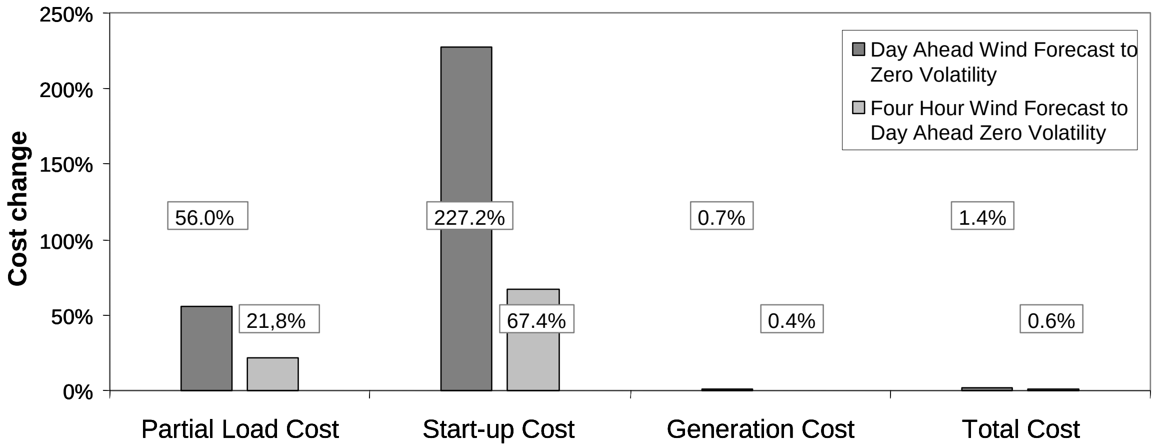
(14)
$$V_i^1 + V_i^2 + V_i^3 = 1 \qquad \forall i \in r$$

(15) $$V_i^j \in \{0,1\} \qquad \forall i \in R, j \in \{1,2,3\}$$

Grouping the realisations lowers the standard deviation for the forecasting errors, reflecting the increase in information for the four-hour forecast. Using exactly the same realisations for the forecasting errors allows a maximum of comparison between our model runs for 24-hour and four-hour-ahead forecasts. However, we want to make sure that we achieve a most realistic improvement in the forecast's accuracy. The DENA-study (2005, p. 263) names a variance of 4.92% of installed wind power generation capacity for the four-hour-ahead forecasting error. This value is again achieved by weighting the realisations in each group accordingly.

Figure 6 shows again the increase of the different cost components when moving from the model run without wind volatility to the run with the wind volatility resulting from the day-ahead forecast. In addition, the figure now also shows the increase in total costs when the lower volatility from the four-hour-ahead forecast is used. The graph shows that all costs increase by significantly less when the improved four-hour-ahead forecast is used, instead of the day-ahead forecast. Total cost increases by only 0.6% when the improved forecast's volatility is added - instead of 1.4%, when the uncertainty from the day-ahead forecast is implemented.

Figure 6: Annual Cost Increase due to Uncertainty in Wind Generation – Day-Ahead and Four-Hour-Ahead Forecast, Relative to Zero Volatility



VI. CONCLUSION

We developed a linear optimisation model to analyse electricity markets. We stayed with the linear framework, as it has many advantages for the modelling of electricity markets. Firstly, linear models find unambiguous global optimums. Secondly, they are much less burdensome on computational resources. We can therefore include many aspects relevant to the modelling of electricity markets and use the extensive amount of data that is available for these markets while still keeping the model 'tractable'.[10] However, while following the established philoso-

10 Tractable, in this context, means that we are able to run the model on a regular PC. Written in GAMS, it is able to exchange data with Excel spreadsheets. Solving one model run with the CPLEX solver takes about 10 minutes on a high-end PC. Given that one year consists of 12 months · 3 types of day per months · 3 wind scenarios in each day · 3 different groups of error realisations in the four-hour forecast, total computing time for these 324 model runs is more than one day.

phy of a linear dispatch model, we extended this framework in several important directions, to model as closely as possible many of features of electricity markets.

Our dynamic representation of the problem is able to represent the effects of start-up costs and part-load operation by optimising a whole day consisting of 24 different hourly load levels simultaneously. Improving on previous models, our setup is truly sequential. The dynamic modelling approach also enables us to endogenously optimise the dispatch of storage and pump-storage plants. We formalised this approach's effects on system marginal costs in section II.

However, we did not only implement a linear representation for the dynamic aspects of the problem, but also for uncertainty. The approach, which we chose to model uncertainty, can be referred to as stochastic programming with recourse. Some variables (in our context, start-up and shut-down decisions for inflexible plants) must be chosen before nature reveals the state of the world. However, some other variables, such as production, can be optimised after the state of the world is revealed. We illustrate this approach using wind power - a major source of uncertainty in electricity markets.

However, as the uncertainty in the wind power forecast can be reduced by either a more accurate weather forecast or a shorter time-distance between forecast and realisation, we also implement the change which an increase in information would bring. We implement this using Laffont's concept of information sets, splitting the forecasting errors' possible realisations into groups and optimising these groups separately.

In the last section of the paper, we calibrated the model developed in this article with empirical data for the German electricity market in the year 2003. We showed that following our dynamic approach, we get much more realistic marginal cost curves than with a simple static approach. System marginal costs change, especially during the very highest and lowest demand periods. Start-up costs increase prices during the hour of the highest demand only (see also section II). However, the effects of hydro-storage capacity can counter this effect. If there is enough hydro-capacity and energy, the production profile for thermal capacity can be so flat that hardly any start-ups of thermal capacity are necessary, thus bringing down the peak and distributing start-up costs over a longer period of time.

Furthermore, we use our very detailed model to quantify the effects of wind power on reserve and balancing provision. This is one important aspect in discussions of the costs and benefits of introducing a large share of wind power into an electricity system. We showed that costs for electricity generation are increased due to wind power's volatility. However, this increase can be greatly reduced if the wind forecast can be made more accurate. The increase mostly comes from increased start-up and part-load costs. Generation costs are hardly influenced. This is in accordance with expectations, as volatility does not influence the average of the demand realisations. However, as generation costs are by far the largest cost component, the total cost increase in the electricity system from wind volatility is found to be small (1.4%). This figure is reduced to 0.6% when the four-hour-ahead forecast is used. Interpreting these figures, one has to bear in mind that we are looking at data from 2003, when Germany was well endowed with generation capacity.[11] The costs of increased wind power volatility can rise significantly when the system is closer to capacity limits. On the other hand, installed capacities can adapt in the long run to achieve an optimal integration of wind power into the system, e.g. by capacity additions in less capital-intensive flexible gas-turbines. Such long-term effects are left for further research, as we concentrated on short-term dispatch in this paper.

In further research, the model could be extended to capture the effects of a continuous updating of the wind forecast, taking into account that additional data on the wind forecast are becoming available in every hour. This way, a decline in forecasting accuracy for those hours further ahead in the future than four hours can be modelled. In addition, the model can be extended to cover more than one model region and endogenously determine international power exchange. In addition, the model is directly applicable to many other empirical questions, such as the effect of CO_2emission costs on plant dispatch and costs or competitive benchmarking studies.

APPENDIX I

In the following, we give the algebra of the complete model. Following the GAMS notation, parameters (lower case letters) are exogenous and variables (capital letters) are endogenously determined as result of the optimisation process.

Indices		Unit
$t = 1,...,T$	Hour	
$s = 1,...,S$	Supply technologies	
$s = 1,...,s^{nf}$	Inflexible supply technologies (unable to balance forecasting error), e.g. nuclear, lignite, hard coal, ccgt	
$s = s^{nf+1},...,S$	Flexible supply technology (can balance forecasting error), e.g. gas turbines, hydro-storage, pump-storage	
$s = S$	Last technologies in technology set is the hydro-storage and pump-storage technology	
$r = 1,...,R$	Realisation of forecasting error	
Parameters		
θ_r	Probability of forecasting error realisation	
d_t	Expected demand	MW
rc	Reserve capacity ready to balance forecasting error	MW

11 The German market contained significant excess capacity before market liberalisation in 1998. While these capacities were reduced after liberalisation, this process was not finished before 2003.

w_t^e	Expected wind generation	MW
$\rho_{t,r}$	Realised forecasting error for wind generation	MW
c_s^X	Variable costs for production	Euro/MWh$_{el}$
c_s^U	Variable costs start-ups	Euro/MW
c_s^{PL}	Variable costs for part-load operation	Euro/MWh$_{el}$
x_s^m	Maximal capacity available for production	MW
p^m	Maximal capacity available for hydro pump storage plants' pumping	MW
η	Efficiency for hydro pumping plant operation	
e^m	Energy Budget with which hydro-storage and pump-storage plants enter the day	
Variables		
TC	Total Cost (objective)	Euro
$X_{s,t,r}$	Production	MW
$U_{s,t,r}$	Start-Up	MW
$D_{s,t,r}$	Shut-Down	MW
$P_{t,r}$	Pumping	MW

The objective function of global cost minimisation is transformed into a maximisation problem, to stick to standard OR formulation:

$$(16)\qquad \max_{X,U,D} -TC = -\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{r=1}^{R}\theta_r * \left(X_{s,t,r} * \left(c_s^X - c_s^{PL}\right) + U_{s,t,r} * c_s^U + \sum_{t\_1=1}^{t}\left(U_{s,t\_1,r} - D_{s,t\_1,r}\right) * c_s^{pl} \right)$$

s.t.

Production must cover realised demand (plus pumping):

$$(17)\qquad d_t - w_t^e + \rho_{r,t} + P_{r,t} - \sum_{s=1}^{S} X_{s,t,r} = 0 \qquad \forall r,t \qquad \lambda_t^d$$

Capacity producing must be started up:

$$(18)\qquad X_{s,t,r} - \sum_{t\_1=1}^{t}\left(U_{s,t\_1,r} - D_{s,t\_1,r}\right) \le 0 \qquad \forall r,s,t \qquad \lambda_{s,t}^{su}$$

Capacity ready for operation that is unable to provide reserve (balance $rr_{r,t}$) must be constant over all realisations of $rr_{r,t}$:

(19) $U_{s,t,r} = U_{s,t}$ $\quad \forall s \mid s \leq s^{nf}, \forall t,r \quad \lambda^{\bar{u}}_{s,t}$

(20) $D_{s,t,r} = D_{s,t}$ $\quad \forall s \mid s \leq s^{nf}, \forall t,r \quad \lambda^{\bar{d}}_{s,t}$

Minimum part-load operation:

(21) $\alpha_s * \sum_{t\_1=1}^{t} \left(U_{s,t\_1,r} - D_{s,t\_1,r} \right) - X_{s,t,r} \leq 0$ $\quad \forall s,t,r \quad \lambda^{pl}_{s,t,r}$

Installed capacity must exceed capacity started up:

(22) $\sum_{t\_1=1}^{t} \left(U_{s,t\_1,r} - D_{s,t\_1,r} \right) - x^m_s \leq 0$ $\quad \forall s,t,r \quad \lambda^{cap}_{s,t,r}$

Capacity for positive reserve provision:

(23) $d_t - w^e_t + \rho_{r,t} + rc - \sum_{s=1}^{s^{nf}} \left(U_{s,t\_1,r} - D_{s,t\_1,r} \right) - \sum_{s=s^{nf+1}}^{S} x^m_s \leq 0$ $\quad \forall s,t,r \quad \lambda^{res}_{t,r}$

Two equations determine the dispatch of hydro-storage and pump-storage capacity. Hydro-storage and pump-storage are combined to one single technology:

(24) $P_{t,r} - p^m \leq 0$ $\quad \forall t,r \quad \lambda^{pc}_{t,r}$

Hydro-storage and pump-storage budget:

(25) $\sum_{t=1}^{T} \left(X_{"S",t,r} - \eta P_{t,r} \right) - e^m \leq 0$ $\quad \forall r \quad \lambda^{p}_{t,r}$

APPENDIX II

Proposition 1: Avoided start-up costs are not allocated to periods outside of demand low: $\lambda^{pl}_t = 0 \ for \ t_{\min,1} < t < t_{\min,2}$.

Proof: Assume for any t with $t_{\min,1} < t \leq t_{\max}$ the proposition would not hold and $\lambda^{pl}_t > 0$. The constraint (4) corresponding to this Lagrange multiplier has to be binding (started up capacity runs at minimum load). This would imply one of the following:

(a) If started up capacity did not increase from the preceding hour, but load was lower in the preceding hour, then the part-load constraint was violated in the preceding hour.

(b) If started up capacity did increase relative to the preceding hour, then some of that start-up could have been delayed by one period without violating any constraints or changing the value of the objective function. (4) would not have been satisfied strictly and $\lambda^{pl}_t > 0$ would have violated the complementarity constraint.

The proof for t during decreasing load levels, $t_{max}>t>t_{min3}$ is symmetric. □

Proposition 2: (Abstracting from part load costs at this point.) Start-up costs are not allocated to hours outside the demand peak ($\lambda_t^{su} = 0$ *for* $t \neq t_{\max}$ *).*

Proof: Assume the proposition would not hold and the Lagrange multiplier $\lambda_t^{su} > 0$ for $t \neq t_{\max}$. Then the corresponding constrained (3) has to hold as equality. Demand in t-1 or t+1 is higher, hence started up capacity in t can be increased by epsilon without violating other constraints or changing the value of the objective function (e.g. start-up costs). However, (3) would now apply as inequality and therefore $\lambda_t^{su} = 0$. □

Proposition 3: (Abstracting from part load costs at this point.) Start up costs are fully allocated to the demand peak: $\lambda_{t_{\max}}^{su} = c^U$.

Proof: $\exists t_u$ with $t_{\min,1} < t_u \leq t_{\max}$ such that $U_{t_u} > 0$. For this t the Kuhn-Tucker condition of Lagrange function (5) with respect to U_{t_u} is binding with equality: $c^U = \sum_{t=t_u}^{\infty} \left(\lambda_t^{su} - \alpha \lambda_t^{pl} \right)$.

$\exists t_d$ with $t_{\max} < t_d \leq t_{\min,2}$ such that $D_{t_d} > 0$. For this t the Kuhn-Tucker condition of Lagrange function (5) with respect to D_{t_d} is binding with equality: $0 = \sum_{t=t_d}^{\infty} \left(\lambda_t^{su} - \alpha \lambda_t^{pl} \right)$.

It follows that $c^U = \sum_{t=t_u}^{t_d - 1} \left(\lambda_t^{su} - \alpha \lambda_t^{pl} \right)$. From Proposition 1 it follows $\lambda_t^{pl} = 0$ for $t_a < t \leq t_b$ and from Proposition 2 it follows $\lambda_t^{su} = 0$ for $t \neq t_{\max}$. Hence $\lambda_{t_{\max}}^{su} = c^U$. □

Proposition 4: (Abstracting from part load costs at this point.) Start up costs are deducted from the demand minimum: $\alpha \lambda_{t_{\min,1}}^{pl} = c^U$.

Proof – symmetric to 3.

If part load costs are included then (5) is expanded with the terms in c^{pl} (see (7)):

$$
(5)^* \quad L = -\sum_{t=1}^{T} \begin{pmatrix} X_t (c^X - c^{pl}) + \sum_{t\_l=1}^{t} \left(U_{t\_l} - D_{t\_l} \right) c^{pl} + U_t c^U + \lambda_t^d \left(d_t - X_t \right) \\ + \lambda_t^{su} \left(X_t - \sum_{t\_l=1}^{t} \left(U_{t\_l} - D_{t\_l} \right) \right) + \lambda_t^{pl} \left(\alpha \sum_{t\_1=1}^{t} \left(U_{t\_1} - D_{t\_1} \right) - X_t \right) \end{pmatrix} .
$$

Proposition 5: Assume $\left(t_{part,2} - t_{part,1} \right) c^{pl} < c^U$ *, then start up costs are not allocated to hours outside the demand peak.*

(a) For $t_{part,2} < t < t_{\max}$ $\lambda_t^{su} = c^{pl}$

Proof: Operating capacity exactly tracks increasing demand in these periods. The Kuhn-Tucker condition for U_t and U_{t+1} in (5)* with increasing started up capacity $U_t>0$ and $U_{t+1}>0$ implies that $dL/dU_t=0$ and $dL/dU_{t+1}=0$. The result follows from subtracting the first from the second term and using Proposition 1 ($\lambda_t^{pl} = 0$). □

(b)For $t_{\max} < t < t_{part,3}$ $\lambda_t^{su} = c^{pl}$

Proof: Operating capacity exactly tracks decreasing demand in these periods. The Kuhn-Tucker condition for D_t and D_{t+1} in (5)* with decreasing started up capacity $D_t>0$ and $D_{t+1}>0$ implies that $dL/dD_t=0$ and $dL/dD_{t+1}=0$. The result follows from subtracting the first from the second term and using Proposition 1 ($\lambda_t^{pl} = 0$) □.

(c)For $t_{part,1} \le t \le t_{part,2}$ $\lambda_t^{su} = 0$

Proof: Following the definition of the part load interval, the Kuhn-Tucker condition for U_t in (5)* is not strictly binding and hence $\lambda_t^{su} = 0$. □

Proposition 6: Start up costs at the demand peak t_{max} are $\lambda_t^{su} = c^{pl} + c^U$.

Proof: Follows directly from subtracting the binding Kuhn-Tucker condition for $D_{t_{\max}+1}$ of (5)* $dL/dD_{t_{\max}+1} = 0$ from the binding Kuhn-Tucker condition for $U_{t_{\max}}$ of (5)* $dL/dU_{t_{\max}} = 0$. □

Proposition 7: Assume $\left(t_{part,2} - t_{part,1}\right)c^{pl} < c^U$ *, then at the demand minimum prices are reduced by the avoided start up costs which additional demand would induce. This effect is partially compensated by additional part load costs:*

Proof: Follows directly from subtracting the binding Kuhn-Tucker condition for $D_{t_{part,1}-1}$ of (5)* $dL/dD_{t_{part,1}-1} = 0$ from the binding Kuhn-Tucker condition for $D_{t_{part,2}+1}$ of (5)* $dL/dU_{t_{part,2}+1} = 0$. □

Proposition 8: The local demand peak is t_{p1}*. At* t_{ge} *the global peak's demand equals the local peak* $d_{t_{p1}} = d_{t_{ge}}$*. Assume* $\left|t_{p1} - t_{ge}\right| \cdot c^{pl} < c^U$ *such that capacity is not shut down between* t_{ge} *and* t_{p1}*. It follows* $\lambda_{t_{p1}}^{su} = \left|t_{p1} - t_{ge}\right| \cdot c^{pl}$, $\lambda_{t_{\max}}^{su} = c^U + c^{pl}$ *and for t between* t_{p1} *and* t_{ge} $\lambda_t^{su} = 0$.

Proof: Assume that the local peak is reached first ($t_{p1} < t_{\max}$). For t with $t_{p1} < t < t_{ge}$ the system is running part load, hence the Kuhn-Tucker condition of (5)* with respect to

U_t requires $\lambda_t^{su} = 0$. As the part load constraint is not binding the Kuhn-Tucker condition of (5)* with respect to D_t requires $\lambda_t^{pl} = 0$.

For $t=t_{p1}$ and $t=t_{ge}+1$ the Kuhn-Tucker condition for (5)* with respect to U_t is strictly binding as additional capacity has to be started. Subtracting the two FOC with respect to U_t and using $\lambda_t^{su} = 0$ for $t_{p1} < t < t_{ge}$ and $\lambda_t^{pl} = 0$ for $t_{p1} \le t \le t_{ge}$ gives $\lambda_{t_{p1}}^{su} = \left| t_{p1} - t_{ge} \right| \cdot c^{pl}$.

Now assume $t_{\max} < t_{pl}$. The proof for $\lambda_t^{pl} = 0$ and $\lambda_t^{su} = 0$ equally applies. For $t = t_{ge}$ and $t = t_{p1} + 1$ the Kuhn-Tucker condition for (5)* with respect to D_t is strictly binding as capacity is shut down at both points. Subtracting the two FOC with respect to D_t and using $\lambda_t^{su} = 0$ for $t_{ge} < t < t_{p1}$ and $\lambda_t^{pl} = 0$ for $t_{ge} \le t \le t_{p1}$ gives $\lambda_{t_{p1}}^{su} = \left| t_{p1} - t_{ge} \right| \cdot c^{pl}$. □

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