Theory on Measuring Orientation with MEMS Accelerometers in a Centrifuge

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ABSTRACT: Microelectromechanical systems (MEMS) sensors have become a common part of everyday life and can be found in a number of consumer electronics. Specifically, MEMS accelerometers have become widespread because of their low cost, due to mass production techniques, and ability to sense constant acceleration. This ability allows devices, such as cellular phones, to measure their rotation relative to Earth's gravity. These properties also make MEMS accelerometers an option for measuring the rotation of geo-structures, such as foundations, in the field or in scale model geotechnical centrifuge tests. MEMS accelerometers appear to be especially beneficial for measuring orientation in centrifuge experiments because they are not limited by the design constraints of traditional tilt sensors: a single constant acceleration vector (Earth's gravity). This paper presents the theory behind using single-axis MEMS accelerometers to measure the orientation of an object on a plane of reactive centrifugal acceleration and Earth's gravity within a geotechnical centrifuge. The paper specifically addresses cross-axis sensitivity which can significantly impact measurements and is typically excluded from simpler theories.

INTRODUCTION

Microelctromechanical systems (MEMS) sensors are becoming a pervasive part of everyday life. Mobile phones, tablets, ink jet printers, cars, and even, as pointed out by Shaeffer (2013), power tools have them. The mass distribution of MEMS in our environment is a fundamental part of the Internet of Things, a driving force behind Big Data, and the reason personal monitoring devices award us for using the stairs. MEMS sensors are even now becoming an essential tool for civil and geotechnical engineers. A quick search of the American Society of Civil Engineers Library for “MEMS” yields 641 results (at the time of writing).

One common type of MEMS sensor is the accelerometer. Their initial growth was directly due to the automotive industry. They replaced switches as a means of triggers airbags during dangerous instances of acceleration (Spangler and Kemp 1996). MEMS accelerometers are essential spring-mass devices constructed at the micron scale using silicon fabrication techniques (Oppenheim et al. 2000, Shaeffer 2013, and Spangler and Kemp 1996). Measurements of capacitance are made relative to the mass' location, which is dictated by the applied force from the mass on the spring, which is a function of applied acceleration; for more detailed descriptions see Shaeffer (2013). Because of this design MEMS accelerometers are capable of measuring constant acceleration, such as Earth's natural gravity. This ability is the reason they are so prevalently used in mobile devices.

The adaptation of MEMS into civil engineering has been advocated since at least 2000 (Oppenheim et al. 2000). In particular MEMS accelerometers can serve two main purposes for civil engineers: dynamic measurements of sensor motion and quasi-static measurements of sensor orientation relative to gravity. It should be noted that, at its current state, the possibility of measurement of displacement over long time periods (AKA dead reckoning) with MEMS accelerometers is limited (Tanaka 2007); however, “it is 'the holy grail' for MEMS sensors,” (Shaeffer 2013). MEMS have been used both in the field and the laboratory by geotechnical engineers. In brief, examples include: measuring wave propagation with custom packaged MEMS accelerometer chips and/or circuits (Hoffman et al. 2006) and (Bhattacharya et al. 2012), the shape-acceleration array for measuring deformation (Bennett et al. 2009), and use in liquefaction field tests (Saftner et al. 2008).

An area of geotechnical testing that is beginning to explore the use of the MEMS accelerometer is geotechnical centrifuge modeling. In centrifuge modeling a model is subjected to a large reactive centrifugal acceleration (or centripetal acceleration, if viewed from outside the centrifuge) in order to develop prototype stress dependent properties. Geotechnical centrifuge modeling has a long history, (Craig 2014; Murff 1996; Schofield 1980; Scott 1977), and is currently a very active field. MEMS accelerometers have been used to measure both motions and orientation in centrifuge models, in brief, examples include: evaluation of MEMS accelerometers in dynamic centrifuge testing (Stringer et al. 2010), seismic evaluation of pile reinforced slopes (Al-Defae and Knappett 2014), and measurement of pile rotation (Lau et al. 2014). However, Stringer et al. (2010) did note problems such as the fact that residual velocities were recorded even when the sensors were static. To successfully utilize MEMS accelerometers within a geotechnical centrifuge a full and systematic methodology is needed.

This paper presents a methodology for measuring sensor orientation in a geotechnical centrifuge on the plane of reactive centrifugal acceleration and Earth's gravity with a single-axis MEMS accelerometer. This paper expands on a simplified theory for measuring rotation with MEMS accelerometers presented by Allmond et al. (2013) by addressing the of importance cross-axis sensitivity and by examining the role of centrifuge geometry and acceleration on sensor measurements.

CENTRIFUGE GEOMETRY AND ACCELERATION

This paper focuses solely on beam centrifuges with free swinging baskets, Figure 1.

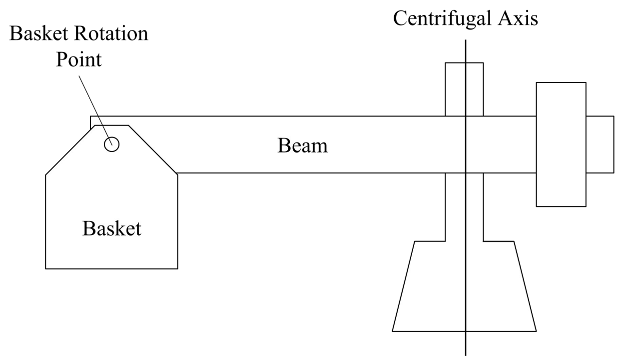


Fig. 1. Idealized beam centrifuge with free swinging basket (not to scale)

The goal of geotechnical centrifuge modeling is to utilize the fact that soil stress is proportional to both gravity (or acceleration) and depth to provide prototype stresses in small scale models. Ideally, we would like to apply an even acceleration over the model; however, this is not the case within a geotechnical centrifuge. The two main acceleration vectors relative to the model are reactive centrifugal acceleration (Equation 1) and Earth's natural gravity. Reactive centrifugal acceleration is constant along circumference of the centrifuge; Earth's gravity is perpendicular centrifugal acceleration. This paper will only address MEMS accelerometer operating on the plane perpendicular to reactive centrifugal gravity and one g.

$$g_c = \omega^2 r \qquad \text{Eq. 1}$$

where g_c is the reactive centrifugal acceleration, ω is rotation velocity, and r is radius.

Basket Orientation.

A free swing basket significantly simplifies the acceleration applied to the model because the basket will tend to an angle, from vertical, with side proportional to centrifugal acceleration and one g, as described in Figure 2 and Equation 2. This is due to the balance of forces from the applied accelerations; as force is mass multiplied by acceleration, Figure 2.

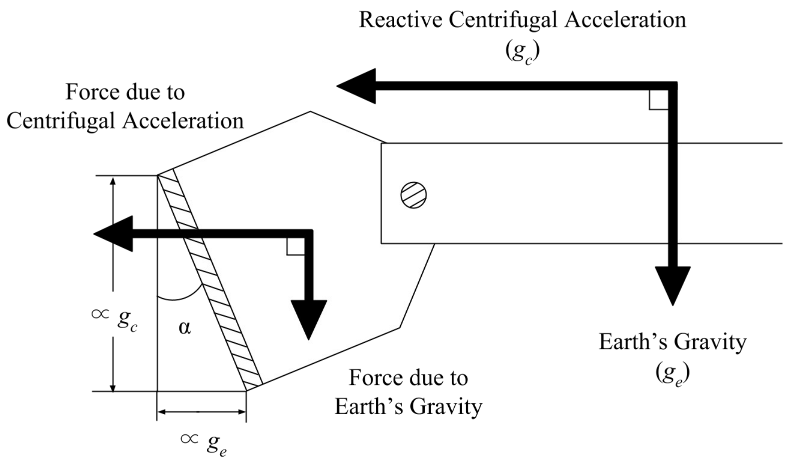


Fig. 2: Free swing basket orientation (not to scale).

$$\alpha = \arctan\left(\frac{g_e}{g_c}\right) \qquad \text{Eq. 2}$$

where α is the angle from vertical as defined in Figure 2 and g_e is Earth's gravity.

Relative Centrifuge Gravity

As outlined in Allmond et al. (2013) the acceleration relative to the basket (or centrifuge gravity) is the resultant of the reactive centrifugal acceleration and Earth's gravity, Equation 3. The gravity will be perpendicular to the basket floor and resultant accelerations parallel to the basket will be equal and opposite; this can be verified by Equations 4 and 5. Significant acceleration vectors relative to the floor of the basket and at its center are diagrammed in Figure 3.

$$g = \sqrt{\left(\omega^2 r\right)^2 + {g_e}^2} \qquad \text{Eq. 3}$$

$$g_{e\alpha} = \cos\left(\alpha\right) \cdot g_e \qquad \text{Eq. 4}$$

$$g_{c\alpha} = \sin\left(\alpha\right) \cdot g_c \qquad \text{Eq. 5}$$

where g is centrifuge gravity relative to the basket, $g_{e\alpha}$ is component of Earth's gravity parallel to the basket floor, and $g_{c\alpha}$ is component of reactive centrifugal acceleration parallel to the basket floor.

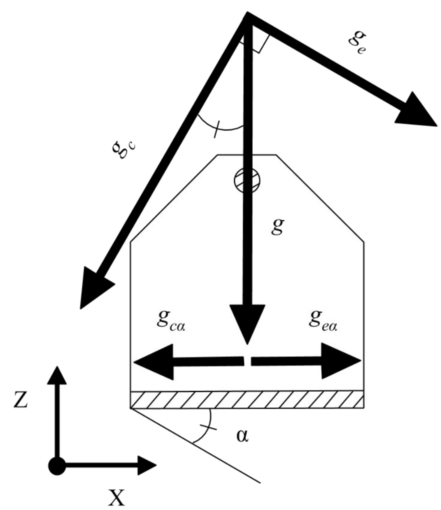


Fig. 3. Relative centrifuge gravity on the basket floor at its center and other acceleration vectors of significance (not to scale).

A final consideration on centrifuge gravity is that because the basket is not parallel to the centrifuge's axis, any movements in the models x and z directions will result in a change in radius (Equations 6 and 7) and therefore a change in centrifuge gravity (Equation 3). This means that centrifuge gravity is dependent on model horizontal and vertical location. Figure 4 provides an example of variation in centrifuge gravity within a 1 m by 1 m test area where distance to the center of the basket from the axis at 25 g is 5 m.

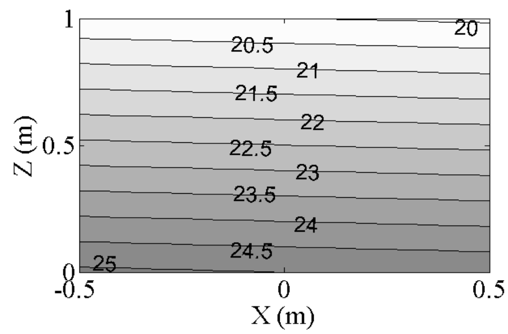
$$\Delta r = \sin(\alpha) \cdot \Delta x \qquad \text{Eq. 6}$$

$$\Delta r = \cos(\alpha) \cdot \Delta z \qquad \text{Eq. 7}$$

where Δr is change in radius, Δx is displacement in x-direction, and Δz is displacement in z-direction, as defined by Figure 3.

MEMS ACCELEROMETERS

Measurements made by a single-axis MEMS accelerometer in a geotechnical centrifuge model can be described relative to its angle while rotating into centrifuge gravity, Equation 8 and defined in Figure 5. It is assumed the sensor's measurement direction is along the sensor's x-axis.



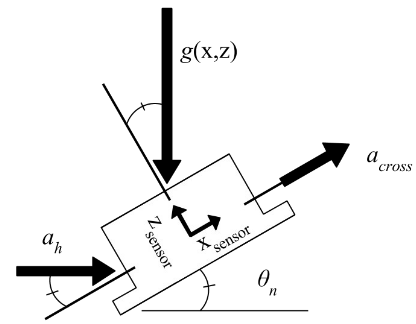


Fig. 4. Example variation in centrifuge gravity at 25 g.

Fig. 5. Acceleration measured by MEMS accelerometer (not to scale).

$$a_{meas} = g(\mathrm{x},\mathrm{z})\sin(\theta_n) + a_h \cos(\theta_n) + a_{cross} \qquad \text{Eq. 8}$$

where a_{meas} is the measured acceleration, g(x,z) is centrifuge gravity as a function of sensor location, θ_n is rotation (Figure 5), a_h is horizontal acceleration, and a_{cross} is the component of measured acceleration due to cross-axis sensitivity. This is similar to Allmond et al. (2013), but with centrifuge gravity dependent on model coordinates. Additionally, we will further expand the cross-axis sensitivity be composed of portions due to centrifuge gravity and Coriolis acceleration:

$$a_{cross} = a_{xg} + a_{xc} \qquad \text{Eq. 9}$$

where a_{xg} is the component measured due to centrifuge gravity acting in the sensors z-direction and a_{xc} is the component due to Coriolis acceleration acting in the sensors y-direction. For more on Coriolis acceleration in geotechnical centrifuge see Schofield (1980). Both a_{xg} and a_{xc} are a function of applied acceleration in the non-measurement directions of the sensor and can be defined by Equations 10 and 11 respectively.

$$a_{xg} = C_g \cdot g(x,z)\cos(\theta_n) \qquad \text{Eq. 10}$$

$$a_{xc} = C_{Cr} \cdot a_c \qquad \text{Eq. 11}$$

where a_c is the Coriolis acceleration and C_g and C_{Cr} are correlation factors determined from MEMS accelerometer calibration.

QUASI-STATIC THEORY

For the quasi-static condition Equation 8 can be simplified: a_{xc} is zero since it is dependent on the sensor motion towards or away from the centrifuge axis and a_h can also assumed to be zero in quasi-static conditions. This results in:

$$a_n = g(\mathrm{x,z})\sin\left(\theta_n\right) + a_{xg} \qquad \text{Eq. 12}$$

where a_n is the assumed measured acceleration. Rotation of the MEMS accelerometer can then be define as:

$$\theta_n = \arcsin\left(\frac{a_n - a_{xg}}{g(x,z)}\right) \qquad \text{Eq. 13}$$

Since the a_{cross} is dependent on θ_n and θ_n is dependent on a_{cross} an iterative process is required for calculating sensor orientation.

DISCUSSION

Basket Orientation

It is possible for the basket to rotate and not be at α from vertical, for various reasons. This will result in Equations 4 and 5 not being equal and opposite, but as noted by Allmond et al. (2013). The difference between α and the actual angle can be taken directly from θ_n and can even be zeroed out if constant through the experiment.

Sensor Range and Initial Orientation

For optimal use it is recommended that the MEMS accelerometer is mounted so that it will rotate into the centrifuge gravity rather than away. This will allow the most accuracy as the sine function is more variable than the cosine at small angles. Additionally, this allows more accurate low g MEMS accelerometers to be used in the high g environment. For example the angular range of a 10 g sensor is eight degrees in 70 g of centrifuge gravity.

Cross-Axis Sensitivity

Cross-axis sensitivity can have a significant impact on measurement of angle. Some reported sensitivities include: ±5 % of the sensor's span (MEMSIC Inc. n.d.) and 2 – 3 % of measurement (Silicon Design Inc. 2013). In some cases it may be possible to zero out the cross-axis sensitivity if its value is constant over the targeted range of measurements, but doing so would also zero any information about the initial orientation of the system. However, it appears that cross-axis sensitivity neither constant nor even linear.

Initial calibration of MEMS accelerometer with a ± 10 g range (MEMSIC CXL10GP1) does indicate that the cross-axis sensitivity is nonlinear Figure 6. However, this may not be the case for larger values of a_g, like 70 or 100 g. Therefore, it is recommended that the cross-axis sensitivity be calibrated for the range of acceleration that will be exerted in the applicable transverse direction during testing.

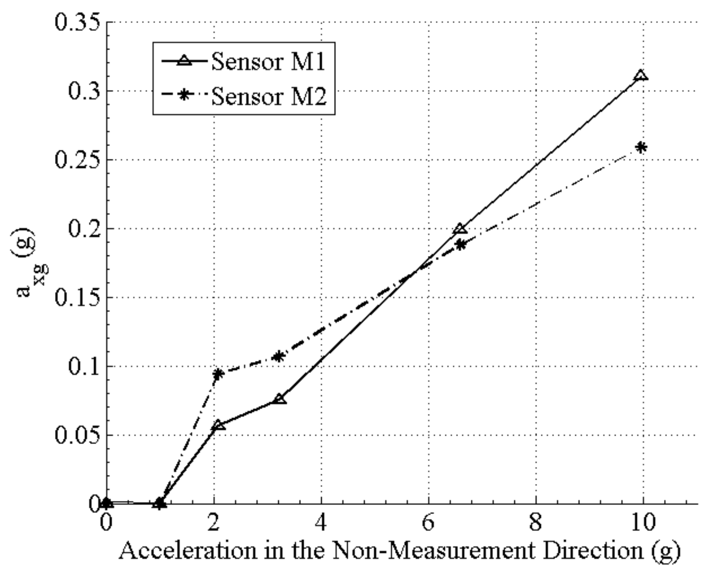


Fig. 6. Initial cross-axis sensitivity calibration results.

Sensor Translation and Rotation with Large Eccentricity

As noted previously, centrifuge gravity within the testing area of a free swinging basket is a function of local coordinates, Equations 6 and 7. This must be kept in mind when using MEMS accelerometers to measure tilt or rotation since θ_n is dependent on $g(x,z)$. Two situations where this could be a concern include modeling plastic deformation, such as the failure of a slope, and measuring rotation at a large eccentricity. Additionally, it can be seen from Equations 2 and 6 that horizontal translation becomes less significant at high g.

CONCLUSION AND FUTURE WORK

MEMS accelerometers are becoming a prolific and inexpensive means to measure acceleration, including constant acceleration. These sensors appear to be a useful tool with many applications within the area of geotechnical centrifuge testing. However, when using MEMS accelerometers in the centrifuge there are a number of items to consider: MEMS accelerometers can be used to measure vibrations and orientation, centrifuge gravity varies relative to the model coordinate, translation or rotation at large eccentricities can impact measurements, cross-axis sensitivity can have a significant effect on measurements, and calibration of cross axis sensitivity at the experimental g is recommended.

This paper covers the use of single-axis MEMS accelerometer on the plane of reactive centrifugal acceleration and Earth's gravity. Future work will be needed to address: multi-axis MEMS sensors in a geotechnical centrifuge, Coriolis effects in the y-direction due to movement on the plane of centrifugal gravity and Earth's gravity for multi-axis MEMS accelerometers, and use of MEMS accelerometers outside the plane of centrifugal gravity and Earth's gravity.

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