# Gains and Losses from Tax Competition with Migration

Seppo Honkapohja and Arja Turunen-Red

February 2004

CWPE 0416

Not to be quoted without permission

#### Abstract

We consider international labor (entrepreneur) mobility in a two-country overlapping-generations model. Interactions of decreasing and increasing returns in production yield multiple equilibria that are stable under adaptive learning. Governments have an unilateral incentive to reduce income taxes at the joint optimum. We compare the Nash equilibrium in taxes under full labor mobility to the closed economy with no mobility. Despite strategic tax setting, the free mobility outcome is often better in welfare terms. Large, discrete gains in welfare may be attained because of the tax competition. Expectational barriers for discrete welfare improvements can be overcome through tax competition.

Key words: tax policy, mobility of labor, multiple equilibria, expectation traps

JEL classification: H87, F22, H21

## Gains and Losses from Tax Competition with Migration <sup>1</sup>

Seppo Honkapohja, University of Cambridge Arja Turunen-Red, University of New Orleans February 2004

#### 1 Introduction

International tax competition has attracted much interest in recent literature. At issue is the allocation of mobile tax bases (typically capital), the location of which may be affected by strategic policy choices (in particular, tax reductions) of governments eager to attract them. The fear is that such unilateral and aggressive tax policies could prove harmful to aggregate welfare since public services might have to be cut as tax revenues dwindle.<sup>2</sup>

Theoretical work has largely supported the above viewpoint. The Nash equilibrium of the tax competition game has been shown to be inferior to the hypothetical joint optimum that could be attained if tax policy toward mobile activities were cooperatively determined. The Nash solution involves a supply of public goods that is lower than jointly optimal and, because all nations are compelled to select low taxes on mobile activities, no nation is able to reap asymmetric, compensating gains for its sacrifice in tax revenue. Explicit coordination of national tax policies is usually suggested as a remedy for the potential welfare loss from tax competition.<sup>3</sup>

<sup>&</sup>lt;sup>1</sup>We gratefully acknowledge financial support from grants by the Academy of Finland, Bank of Finland, Yrjö Jahnsson Foundation and Nokia Group. This research was partly conducted while the first author was affiliated with the Research Unit on Economic Structures and Growth, University of Helsinki.

<sup>&</sup>lt;sup>2</sup>Sinn (2000, 2003) among others has highlighted the problems that tax competition may pose for the European-style welfare state.

<sup>&</sup>lt;sup>3</sup>The voluminous literature on tax competition is surveyed in Wilson (1999). See Sorensen (2000), Wildasin (2000), and Devereux, Lockwood and Redoano (2002) for recent empirical evidence. Persson and Tabellini (1992) suggested that consequences of tax competition may be countered by adaptation of countries' internal political systems. Edwards and Keen (1996) show that tax competition may provide a useful constraint to self-serving motivations of public decision makers.

The difficulty with the tax coordination solution is, however, that such cooperation seems to be very difficult to achieve in practice. For example, the EU has tried for many years to reach agreement on common levels and other aspects of corporate taxation. Hence, comparing equilibria with tax competition with a coordinated social optimum may not be the most natural way to assess the role of globalization and tax competition. It would also seem natural to compare equilibria with free mobility and tax competition with what closed economies can achieve at best, i.e., country-wise social optima when there is no international mobility of productive factors.

In this paper, we make the latter comparison and argue that there are circumstances in which the opening of national borders and the resulting tax competition can be positively helpful. possibility that strategic tax setting may actually improve welfare centers on two related phenomena: (i) the large, discrete jumps in employment, output, and welfare after comparative-dynamic changes that can take place when there are multiple equilibria and (ii) the fact that low expectations, accompanied by adaptive learning dynamics, can block potential welfare improvements from We show first that, by simply expanding factor being realized. flows, tax competition can sometimes yield significant (discrete) welfare gains over socially optimal equilibria in which individuals are internationally immobile (due to closed borders) but are optimally taxed. Secondly, when multiple potential equilibria exist and a low equilibrium is realized, tax competition can serve as an international coordination mechanism that breaks the low expectations that support the low output trap, thus creating a positive jump in output and welfare. Furthermore, once a high output steady state has been reached, we show that carefully chosen cooperative tax increases may be instituted without disturbing the newly attained high equilibrium. Thus, while expectational dynamics may cause stagnation at a low equilibrium trap, stability of equilibrium under learning can also support cooperative taxation of mobile factors.

We employ a symmetric two-country version of the simple overlapping-generations model of social increasing returns due to Evans and Honkapohja (1995) to derive these results.<sup>4</sup> (For a summary discussion of the model see Section 4.6 of Evans and Honkapohja (2001).) The model and the symmetry assumption are naturally restrictive but they allow us to streamline our analysis. We assume that all individuals born in either country are potentially mobile in their first period of life. Income earned is taxed according to the source principle, and the tax revenues are spent to supply publicly provided goods and services for individuals in their second period of life when they no longer work. We deviate from the standard tax competition models by treating the mobile individuals as household-producers or entrepreneurs. Accordingly, they do not exchange labor for a market wage but are, instead, free to set up shop and offer their services (measured in aggregate consumption) in either country depending on the return to the service being offered. Individuals of this type comprise skilled professions (IT services, consulting, entertainment, design, arts, etc.) and their services reflect the implicit stock of human capital in existence.<sup>5</sup> In our model lower taxes and international mobility of entrepreneurship can provide strong incentives to increase productive effort and to expand aggregate output. This increase can, moreover, be magnified by increasing social returns in a certain range of aggregate effort.<sup>6</sup>

The possibility of large jumps in equilibria is due to the social increasing returns of the model. In particular, we assume that while individual (entrepreneurial) effort is subject to decreasing returns in each location, external gains in productivity are reaped if the aggregate labor supply in a location exceeds a minimum threshold level. The interaction of the positive productivity externalities with

<sup>&</sup>lt;sup>4</sup>Wildasin and Wilson (1996) have developed an overlapping-generations model to analyze land-value maximizing taxation under imperfect resident mobility.

<sup>&</sup>lt;sup>5</sup>Devereux, Lockwood and Redoano (2002) have analyzed international mobility of firms when financial capital is also internationally mobile. Our model does not include financial capital, but the individual entrepreneurs in our model could be interpreted as representing mobile entrepreneurial human capital ("firms").

<sup>&</sup>lt;sup>6</sup>The benefits from tax competition due to increased effort are different from benefits that can emerge from agglomeration effects and core-periphery equilibria, which have recently been considered by Baldwin and Krugman (2004). They argue that the core country can act as a Stackelberg leader and use a "limit taxation" strategy towards the periphery.

the decreasing individual returns to effort yields the potential for multiple steady state solutions. Given such multiplicities, changes in tax policy can induce endogenous jumps from one steady state to another; we are particularly interested in determining the direction of changes associated with tax competition.

We apply the concept of stability under adaptive learning so as to classify steady state equilibria. Under this criterion, stable equilibria will be approached via an expectational adjustment process in which individuals observe the outcome of the economic process at each point in time, adjust their forecast functions, and learn about the equilibrium values of the model variables. Near stable equilibria, expectational errors diminish directing the economy toward a particular stable solution; in contrast, unstable steady states cannot be reached because behavior will be altered to reflect the ever-growing forecast errors near these equilibria.

Since equilibria that are unstable under learning cannot be approached by small, gradual steps, an unstable steady state that separates a high output equilibrium from an initial low output state forms an expectational barrier that cannot be easily overcome. Only discrete changes in policy or other exogenous disturbances of sufficient size can cause an upward jump in expectations and the performance of the economy. We show that the seemingly destructive tax competition can serve in this welfare improving role.

The contents of the paper are as follows. Section 2 describes the model and characterizes the feasible steady states. Section 3 illustrates the gains arising from international tax competition, and section 4 analyzes the role of expectations in the maximization of such gains. Section 5 concludes.

#### 2 Model

In this section, we expand the Evans and Honkapohja (1995) overlapping-generations model to include two symmetric countries,

<sup>&</sup>lt;sup>7</sup>For an exhaustive discussion of adaptive learning, see Evans and Honkapohja (2001).

H(ome) and F(oreign), that interact freely in commodity and factor markets. Autarky equilibria are defined later by suitably adjusting this open economy model.

#### 2.1 Production Technology

At any point in time, both countries H and F are the birthplace of a fixed number (K) individuals who live for two time periods. In their first period of life, the young work and sell aggregate consumption to the retired. Labor is the only variable input. Individual production functions, identical to all workers, are specified by

$$f(n_j, N_j) = n_i^{\alpha} \psi(N_j), \quad f(n_i^*, N_j) = (n_i^*)^{\alpha} \psi(N_j), \quad j = H, F, \quad (1)$$

where the  $n_j$  (resp.  $n_j^*$ ) denote work performed in the two countries by those born in H (resp. F). Thus, the superscript \* indicates the location of birth in F, whereas the subscripts j = H, F indicate the place of employment. The production parameter  $\alpha$  is assumed to be less than one so that decreasing returns to individual effort prevail.<sup>8</sup>

Increasing external returns to labor are represented by the function  $\psi$  in (1). We assume that the external returns depend on the aggregate employment in a given location, i.e., in (1),

$$N_H \equiv K(n_H + n_H^*), \quad N_F = K(n_F^* + n_F).$$
 (2)

The function  $\psi$  is taken to be increasing in  $N_j$ ; thus, the larger the total supply of labor in country j, the higher the productivity of each worker in that country.

A particular functional form for social returns has been suggested by Evans and Honkapohja (1995). According to this specification,

$$\psi(N_j) = \max\left[\widehat{I}, I_j\right]^{\beta}, \quad \widehat{I} > 0, \quad \beta \ge 1,$$
(3)

$$I_j \equiv \frac{\lambda N_j}{1 + a\lambda N_j}, \quad \lambda \in (0, 1), \quad a > 0.$$
 (4)

<sup>&</sup>lt;sup>8</sup>Individuals in their first period of life can be interpreted as entrepreneurs for whom human capital is the fixed factor. While this human capital is not directly mobile, entrepreneurial effort, measured by the variables  $n_j$  and  $n_j^*$ , is taken to be freely mobile.

where j = H, F. By (3), aggregate employment must exceed an exogenous threshold value,  $\hat{I}$ , before external productivity gains can be felt. If the labor supply  $N_j$  is sufficiently large, the multiplier  $I_j^{\beta}(=\psi(N_j))$  obtained from (3) is substituted into the individual production functions (1). Otherwise, there are no social returns to labor and the production functions include the constant multiplier  $\hat{I}^{\beta}$ .

We imagine that the social gains in productivity reflect the sharing of experiences and ideas that naturally takes place when individuals (entrepreneurs) operate in some proximity to each other. The expression  $I_j$  in (4) represents this sharing of ideas. In particular, we posit that new ideas are broadcast at a uniform rate by all active workers and that, for any individual, the fraction  $\lambda$  of ideas is suitable to be applied. Assuming that each worker needs a time units to absorb and understand a new idea once it has arrived, the total time required to receive and apply an usable idea equals  $a + (\lambda N_j)^{-1}$ . Per fixed unit time period, therefore, the total number of usable ideas that any active individual receives is  $I_j = (a + (\lambda N_j)^{-1})^{-1}$  as specified in (4). This  $I_j$  is increasing in the total labor supply,  $N_j$ , whereby external productivity gains increase with aggregate employment. There is, however, an upper bound for the gains; by (4),  $I_j$  approaches 1/a as  $N_j$  becomes very large.

#### 2.2 Overlapping Generations

Welfare is derived from private consumption and public services. In the following utility function, which applies to a representative individual born in country H in the beginning of period t,

$$W_H = U(c_{H,t+1}) - V(n_{Ht} + n_{Ft}) + \mu \mathcal{U}(G_{H,t+1}), \tag{5}$$

 $c_{H,t+1}$  denotes private consumption and the parameter  $\mu$  gives the welfare weight of publicly provided consumption,  $G_{H,t+1}$ , both of which are enjoyed in retirement in time period (t+1) (the second period of life). Disutility of labor is reflected in the second term on the right-hand-side of (5). We treat work performed in both locations symmetrically, i.e., we exclude any psychological as well

as monetary costs that may be associated with labor mobility. The functions U and U are assumed to be increasing and concave, while V is increasing and convex.

National governments independently provide benefits to the retired. In order to supply such public consumption, the governments appropriate a fraction  $\tau_j$ , j = H, F, of national output in each period. Accordingly, we have

$$\tau_j Y_{jt} = G_{jt}, \quad Y_{jt} \equiv f(n_{jt}, N_{jt}) + f(n_{jt}^*, N_{jt}), \quad j = H, F,$$
 (6)

where  $Y_{jt}$  gives the total per capita output of domestic and foreign workers in country j in period t. The variables  $\tau_j$  define the national tax rates by which individual income is taxed in each country.

The subsequent (per capita) budget constraints of a representative individual who is born in H in period t and retires in period t+1 are

$$(1 - \tau_H)p_t f(n_{Ht}, N_{Ht}) + (1 - \tau_F)p_t f(n_{Ft}, N_{Ft}) = M_t,$$
 (7)

$$p_{t+1}^e c_{H,t+1} = M_t. (8)$$

In (7) and (8),  $M_t$  stands for the net income, measured in money, that the young in H plan to spend in retirement, and  $p_{t+1}^e$  is the price forecast for future consumption by the workers in period t. (Identical forecasts for different individuals are assumed for simplicity.) Analogous budget constraints apply to individuals born in country F. We assume common currency for the two countries, and the stock of money is taken to be constant.

Labor supply is chosen so as to maximize life-time welfare (5) subject to the constraints (7)-(8). Accordingly, when making their decisions, workers consider the current price of consumption, the expected cost of consumption in the future, the domestic and foreign tax rates on their earnings, the productivity of labor in each country, and their personal disutility from having to earn their

<sup>&</sup>lt;sup>9</sup>This formulation is an extension of the Samuelson (1958) overlapping-generations model. By (5), access to public services is contingent on nationality, meaning that only public services of the home country can be used when retired. This eliminates any incentive to migrate for social assistance purposes considered, e.g., by Sinn (2000, 2003) and Breyer and Kolmar (2002)).

living. Aggregate employment  $(N_H, N_F)$  and public services  $(G_H, G_F)$  are taken as given in individuals' optimization.

Optimal labor supply is implicitly characterized by the first order conditions

$$U'(c_{H,t+1})\frac{(1-\tau_j)p_t}{p_{t+1}^e}f'_n(n_{jt},N_{jt}) = V'(n_{Ht}+n_{Ft}), \ j=H,F, \quad (9)$$

$$U'(c_{F,t+1}^*) \frac{(1-\tau_j)p_t}{p_{t+1}^e} f_n'(n_{jt}^*, N_{jt}) = V'(n_{Ht}^* + n_{Ft}^*), \ j = H, F. \quad (10)$$

By (9) and (10), the optimal  $n_j$  and  $n_j^*$ , j = H, F, equalize the marginal expected returns to labor in the two markets and both are equal to the marginal disutility of labor.<sup>10</sup>

The first order conditions (9)-(10) can be simplified by observing that along a perfect foresight equilibrium path price forecasts are correct, i.e.,  $p_{t+1}^e = p_{t+1}$ , and the world market for private consumption (per capita) clears in all periods, i.e.,

$$C_t^W \equiv c_{Ht} + c_{Ft}^* = (1 - \tau_H)Y_{Ht} + (1 - \tau_F)Y_{Ft}$$
 (11)

for all t. Given the constant world supply of money (equal to M), market clearing for the world requires that total nominal savings by the young equal nominal money supply. Equivalently,  $C_t^W = M/p_t$ , which yields

$$\frac{p_t}{p_{t+1}} = \frac{(1 - \tau_H)Y_{H,t+1} + (1 - \tau_F)Y_{F,t+1}}{(1 - \tau_H)Y_{Ht} + (1 - \tau_F)Y_{Ft}} = \frac{C_{t+1}^W}{C_t^W}$$
(12)

in any symmetric equilibrium.

When substituted into the first order conditions (9)-(10) this price ratio yields the following (per capita) labor offer curves:

$$(1 - \tau_j)U'(c_{H,t+1})C_{t+1}^W = \frac{C_t^W V'(n_{Ht} + n_{Ft})}{f_n'(n_{jt}, N_{jt})}, \ j = H, F,$$
 (13)

$$(1 - \tau_j)U'(c_{F,t+1}^*)C_{t+1}^W = \frac{C_t^W V'(n_{Ht}^* + n_{Ft}^*)}{f_n'(n_{jt}^*, N_{jt})}, \ j = H, F,$$
 (14)

Notation:  $f'_n$  denotes the partial derivative of the production function f with respect to its first argument.

where

$$c_{H,t+1} = \frac{\left[ (1 - \tau_H) f(n_{Ht}, N_{Ht}) + (1 - \tau_F) f(n_{Ft}, N_{Ft}) \right] C_{t+1}^W}{C_t^W}, \quad (15)$$

$$c_{F,t+1}^* = \frac{\left[ (1 - \tau_H) f(n_{Ht}^*, N_{Ht}) + (1 - \tau_F) f(n_{Ft}^*, N_{Ft}) \right] C_{t+1}^W}{C_t^W}. \quad (16)$$

Equations (13)-(16), together with definitions (2) and (11), determine the evolution of the four labor supply variables,  $(n_j, n_j^*)$ , j = H, F, and thus the evolution of aggregate employment in each country,  $(N_H, N_F)$ , is determined as well. The time paths of the remaining endogenous variables can subsequently be solved by applying the previous definitions.

#### 2.3 Symmetric Equilibria and Learning Dynamics

When the tax rates are the same in both countries ( $\tau_H = \tau_F = \tau$ ) the equilibrium values of all labor supply variables are equal implying no net mobility of labor - and the common steady-state value, denoted by n, can be solved using one of the offer curves (13)-(14).

For the purposes of illustration and simplification, we often adopt the isoelastic utility functions

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma}, \quad V(n) = \frac{n^{1+\epsilon}}{1+\epsilon}, \quad \mathcal{U}(G) = \frac{G^{1-\sigma}}{1-\sigma}, \quad 0 < \sigma, \varepsilon < 1.$$

$$\tag{17}$$

For these utility functions and assuming symmetry, equations (13)-(14) can be reduced to<sup>12</sup>

$$n_t^{\frac{1+\varepsilon}{1-\sigma}} = 2^{-\frac{(\varepsilon+\sigma)}{1-\sigma}} \alpha^{\frac{1}{1-\sigma}} (1-\tau) n_{t+1}^{\alpha} \max \left[ \widehat{I}, \frac{\lambda N_{t+1}}{1+a\lambda N_{t+1}} \right]^{\beta}, N_t = 2K n_t.$$
(18)

When  $\tau_H = \tau_F = \tau$ , we have  $p_t/p_{t+1} = Y_{t+1}^W/Y_t^W = C_{t+1}^W/C_t^W$  in (13)-(16), where  $Y_t^W = Y_{Ht} + Y_{Ft}$ .

<sup>&</sup>lt;sup>12</sup>Equation (18) is obtained using (1)-(4) and (17), and by noting that, due to symmetry,  $Y_t^W = 2Y_t = 4f(n_t, N_t)$  and  $n_{Ht} + n_{Ft} = 2n_t$ .

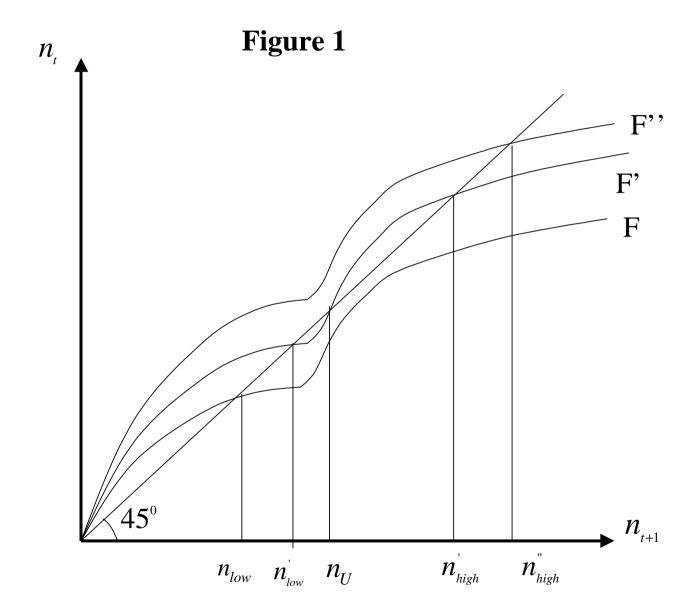
Typical offer curves in  $(n_{t+1}, n_t)$ -space are depicted in Fig. 1.<sup>13</sup> We remark that in order to depict the total per capita labor supply (including both domestic work effort and any labor performed in the other country) the offer curves in Fig. 1 should be interpreted as twice the solution for  $n_t$  in (18).

Offer curves F, F' and F'' in Fig. 1 comprise two concave segments, separated by a kink. Immediately to the right of the kink, there may be a convex segment as shown in Fig. 1.

The concave portions of the offer curves are observed when individual production technology (1) is subject to overall decreasing returns. Along the first concave segment, to the left of the kink, per capita labor supply is too low for external returns to labor to be felt and so decreasing returns must prevail. 14 The convex region to the right of the kink appears when external productivity gains are sufficiently strong to overcome individual decreasing returns. <sup>15</sup> But, since the positive externality effect is bounded from above, all offer curves eventually turn concave as per capital labor supply increases; this yields the second concave region on the offer curves F, F' and F''. (See the Appendix, part 1), for a detailed qualitative derivation of the offer curves in Fig. 1.)

<sup>&</sup>lt;sup>13</sup>For a discussion of the shape of the offer curves in a closed economy we refer to Evans and Honkapohja (1995, 2001). The functional form of the open economy offer curve (18) is similar to that of the corresponding autarky curve, the differences being (i) the coefficient  $2^{-\frac{(\varepsilon+\sigma)}{1-\sigma}}$  that appears on the right-hand side of (18) and (ii) the definition of the aggregate labor supply that equals  $N_t = Kn_t$  in autarky. See Section 3 below for a brief derivation of the autarky offer curve.

<sup>&</sup>lt;sup>14</sup>In this region,  $I_j < \widehat{I}$  and thus  $\psi(N_j) = \widehat{I}^{\beta}$  in (1) is a constant. <sup>15</sup>In this region,  $I_j > \widehat{I}$  whereby  $\psi(N_j) = I^{\beta}$  in (1) is increasing in  $N_j$ .



Three types of interior equilibria may be realized depending on the shape of the offer curve (18). (In addition, the equilibrium at the origin  $(n_t = 0)$  always exists.) First, there may be a unique interior steady state to the left of the kink such as  $n_{Low}$  on the offer curve F in Fig. 1. At this equilibrium, young generations work relatively little and output and consumption are low. Second, a unique equilibrium may occur to the right of the kink, as is the case at  $n_{High}$  on the offer curve F''. At this steady state productivity of labor is enhanced by external gains from worker interactions. The third possibility is that there are multiple interior equilibria such as illustrated by the steady states  $n'_{Low}$ ,  $n_U$ , and  $n'_{High}$  along F'. At  $n'_{Low}$  output and consumption are much below those realized at the high equilibrium  $n'_{High}$ . Welfare is predictably affected: for any given level of labor taxation, steady state welfare is an increasing function of n across all steady states so that all individuals must be better off at  $n'_{High}$  than at  $n'_{Low}$ . <sup>16</sup>

We introduce dynamic adjustment paths toward equilibria using the adaptive learning approach. Accordingly, individuals in time period t are assumed to base their decisions about how much and where to work on their expectations regarding future values of economic variables. Expectations for time period (t+1) are formed by adjusting the formerly held expectations regarding time period t; the manner of the adjustment reflects the discrepancy between the observed, realized, economy and the former expectations. Through a process of ever more accurate forecasts and adjustments, some steady states will eventually be approached; these equilibria are stable under adaptive learning. Equilibria that cannot be reached by this sort of dynamic learning process are unstable under adaptive learning.

The first order conditions (9)-(10) identify the quantity, the marginal utility of consumption deflated by the forecast relative price of consumption  $(U'(c_{t+1})p_t/p_{t+1})$ , that the individuals are

<sup>&</sup>lt;sup>16</sup>Compare two steady states, indexed by 1 and 2, where  $n_1 \leq n_2$ . Then, since the production function f is increasing in N and the tax rate  $\tau$  is fixed, welfare (as defined in (5)) satisfies  $W(n_1, N_1) < W(n_1, N_2)$ . Furthermore, since  $n_2 \geq n_1$  maximizes welfare given  $N_2$  and  $\tau$ ,  $W(n_1, N_2) \leq W(n_2, N_2)$ , and thus  $W(n_1, N_1) < W(n_2, N_2)$ .

learning about. Given (12), however, the price forecast may be replaced by predictions regarding labor supply in the next time period,  $n_{t+1}^e$ .<sup>17</sup> This is also sufficient to forecast future consumption,  $c_{t+1}$ , as indicated by (15) and (16).

The following simple learning rule formalizes the adaptive learning process that yields  $n_{t+1}^e$ . Given an expected level of employment,  $n_{t+1}^e$ , the optimal per capita labor supply for the current time period,  $n_t$ , is determined from the labor offer curve, i.e., using the notation of Fig. 1,  $n_t = F(n_{t+1}^e)$ . The fixed points of this mapping are the steady state equilibria of the model. If a steady state is not realized in time period t, offer curve  $F(n_{t+1}^e)$  yields a temporary equilibrium relative to the given expectation,  $n_{t+1}^e$ . Then, being outside an equilibrium, workers adjusts their expectations toward the currently realized level of employment and output. Formally, <sup>18</sup>

$$n_{t+1}^e = n_t^e + \frac{\kappa}{t} (F(n_t^e) - n_t^e), \quad \kappa > 0.$$
 (19)

The quantity  $\kappa/t$  is known as the *gain parameter*, and it determines the degree of adjustment of expectations to forecast errors. (For  $\kappa = 1$  and appropriate initial condition,  $n_{t+1}^e$  is equal to the average of past values of  $n_t$ .)

The stability properties of the equilibria in Fig. 1 are determined by Proposition 1 of Evans and Honkapohja (1995). Specifically, the interior equilibria at which an offer curve cuts the 45°-line from above  $(n'_{Low}$  and  $n'_{High})$  are stable under learning, whereas equilibria at which the 45°-line cuts the offer curve from above are unstable  $(n_U)$ . (For all  $0 < n^e_{t+1} < n_U$ , the economy converges to  $n'_{Low}$  and, for all  $n^e_{t+1} > n_U$ , the economy converges to  $n'_{High}$ .) All unique interior equilibria  $(n_{Low}$  and  $n_{High})$  are necessarily stable. It is clear from Fig. 1 that, excluding unusual circumstances, unstable

This is equivalent to forecast  $C_{j,(t+1)}$ , j = H, F, instead of  $p_{t+1}$ . Given (1), this is equivalent to forecasting  $n_{j,(t+1)}$ . Since the two economies are symmetric and all individuals are identical (including the learning rules), it suffices to forecast the common value  $n_{t+1}$ . More elaborate formulations with explicit price forecasts could be applied, but the stability properties of the equilibria would be unaffected. See Evans and Honkapohja (2001: pp. 50-52) for the closed economy version of these arguments.

<sup>&</sup>lt;sup>18</sup>This formulation for learning about steady states is common in the recent literature. See Chapter 11 of Evans and Honkapohja (2001).

equilibria, when they exist, will be located between two steady states that are stable under adaptive learning.

#### 3 Gains from Tax Competition

We now turn to comparisons of three particular equilibria: the Nash equilibrium in taxes that results from unilateral policy making, the joint optimum obtained if labor taxes are set cooperatively, and autarky (where we assume that taxes are set optimally). Our goal is to show that, even when accompanied by international tax competition, free mobility of entrepreneurial labor can yield significant (discrete) welfare gains over autarky.

Figures 2 compactly makes our point. In Fig. 2a we show three labor offer curves, labeled  $F_a$ ,  $F_m$  and  $F_{NE}$ , respectively. Of these, let  $F_m$  yield the current (period t) per capita supply of services in an open economy at some fixed level of income taxation, denoted by  $\hat{\tau}$ . Given  $F_m$ , the interior steady state at  $n_m$  is observed.

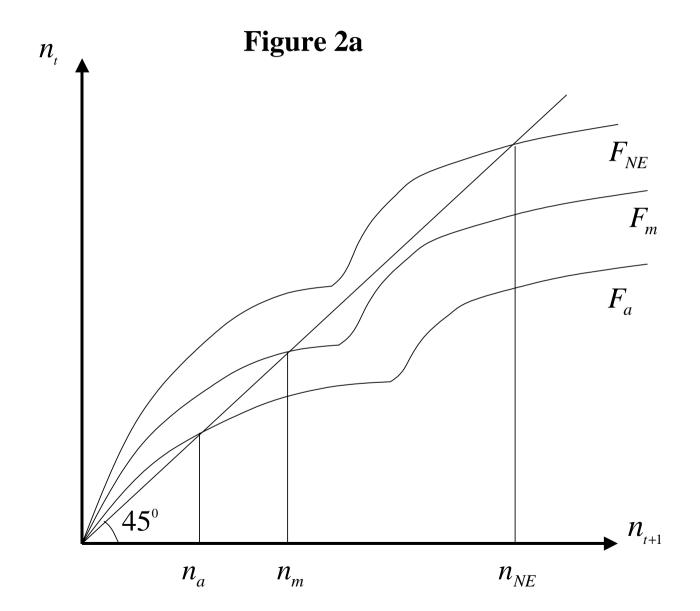
Offer curve  $F_a$  in Fig. 2a represents labor supply given  $\hat{\tau}$  but in autarky. In order to obtain curve  $F_a$ , we adjust the production functions (1), the utility function (5), and the individuals' budget constraints (7) so as to include only domestic variables. Given these adjustments, equations (13)-(14) are replaced by their autarky equivalents

$$(1-\tau)U'((1-\tau)Y_{j,(t+1)})Y_{j,(t+1)} = \frac{Y_{jt}V'(n_{jt})}{f'_n(n_{jt}, N_{jt})}, \quad j = H, F, \quad (20)$$

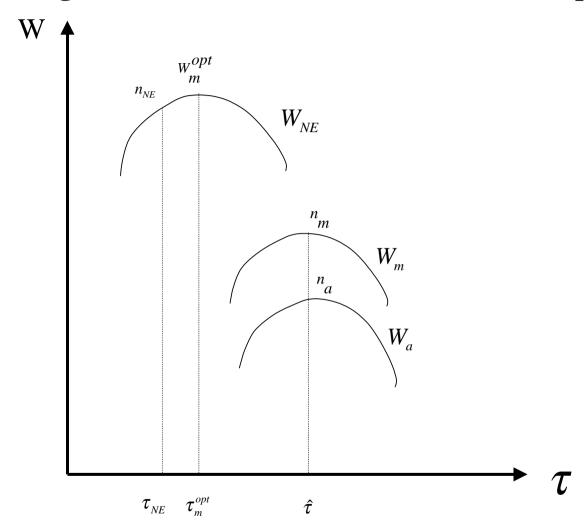
where  $Y_{jt} \equiv f(n_{jt}, N_{jt})$ ,  $N_j = Kn_j$ . When utility functions are isoelastic, equations (20) simplify and yield the autarky offer curve

$$n_t^{\frac{1+\varepsilon}{1-\sigma}} = \alpha^{\frac{1}{1-\sigma}} (1-\tau) n_{t+1}^{\alpha} \max \left[ \widehat{I}, \frac{\lambda N_{t+1}}{1+a\lambda N_{t+1}} \right]^{\beta}, \quad N_t = K n_t, \quad (21)$$

that is common for individuals in both countries.



## **Figure 2b: Bifurcation Gain from Tax Competition**



Curve  $F_a$  in Fig. 2a depicts equation (21) when  $\tau = \hat{\tau}$ ; autarky steady state is found at  $n_a$ . That  $F_a$  must be located below  $F_m$  can be seen by observing that the solutions for  $n_t$  obtained using the two offer curves, (18) and (21), differ by the multiplier  $2^{\frac{1-\sigma}{1+\varepsilon}}(>1)$ , given any  $\tau$ .<sup>19</sup> This implies that, at all levels of taxation and expected future employment  $(n_{t+1}^e)$ , all individuals work harder in an open economy than in autarky. (The interpretation of the offer curve  $F_{NE}$  will be discussed shortly.)

Welfare around equilibria is illustrated in Fig. 2b. Curve  $W_m$ depicts per capita well-being near the initial equilibrium  $n_m$  and curve  $W_a$  applies in autarky. Welfare as a function of the tax rate has an inverted U-shape as shown in Fig. 2b if public consumption has a positive weight in the utility function (5). (This condition guarantees that all individuals prefer some positive tax rate to zero taxation and no provision of public consumption. Appendix, part 2) for formal arguments.) The positive distance between  $W_m$  and  $W_a$  in Fig. 2b reflects the gains from international factor mobility that are obtained keeping the income tax rate fixed These gains are analogous to the well-known production gains from trade and are due to the increase and reallocation of labor supply that accompany the dynamic adjustment from autarky to free resource mobility. Since no commodity trade takes place in symmetric equilibria, we can identify all gains with the factor mobility.

In the comparison between  $W_m$  and  $W_a$ , the gains from labor mobility are local, i.e., both the autarky equilibrium at  $n_a$  and the open economy steady state  $n_m$  are low productivity steady states and in neither are positive externalities observed. We call this sort of a shift in an equilibrium of a given type a local change (even when the change may involve a discrete perturbation in the model variables as between  $n_a$  and  $n_m$ ). We have assumed in Fig. 2b that the common tax rate  $\hat{\tau}$  maximizes welfare in autarky (along  $W_a$ ).

<sup>&</sup>lt;sup>19</sup>When external productivity gains are present, the open economy solution for  $n_t$  also differs from its autarky counterpart in that the aggregate labor supply  $N_t$  equals  $2Kn_t$  and not  $Kn_t$  as in autarky. This difference, when present, shifts the open economy offer curve further above the autarky  $F_a$ .

That this same tax rate maximizes welfare when labor is mobile as well (along  $W_m$ ) is due to the fact that productive externalities are not present near either steady state.<sup>20</sup> In general, we have that  $\tau_{high}^{opt} \leq \tau_{low}^{opt}$ , where  $\tau_{high}^{opt}$  is the tax rate jointly optimal when labor productivity is subject to externalities and  $\tau_{low}^{opt}$  is optimal when this is not the case (near  $n_a$  and  $n_m$ ). The strict inequality applies when a reduction in the income tax results in external productivity gains.

When the initial income tax is jointly optimal and labor is mobile, each of the two countries usually has a unilateral incentive to undertake an asymmetric tax reduction (see the Appendix, part 4), for formal arguments). Thus, if there are no productive externalities, a leftward movement from  $\hat{\tau}$  along curve  $W_m$  is observed. That the resulting outcome of tax competition is worse in welfare terms than the joint optimum is the *standard argument against international tax competition* and, as shown in Fig. 2b, this argument is replicated in our model. Additional results follow, however, when we allow for the possibility that tax competition may cause a bifurcation in the set of equilibria.

In order to illustrate this possibility, we return to Fig. 2a. According to the following lemma, labor offer curves shift up (while leaving the origin unchanged) if income taxes are symmetrically reduced. (See the Appendix, part 5), for a proof.)

**Lemma 1** Near a symmetric equilibrium (where  $\tau_H = \tau_F = \tau$ ) and given any expected level of future employment,  $n_{t+1}^e$ , a common reduction in the income taxes  $\tau_H$  and  $\tau_F$  is followed by an expansion in the per capita labor supply,  $n_t$ .

Accordingly, an offer curve such as  $F_{NE}$ , entirely above  $F_m$ , applies when taxes are Nash. A possible Nash (steady state) equilibrium is identified by point  $n_{NE}$  on  $F_{NE}$ .

The adjustment from an initial steady state such as  $n_a$  (or  $n_m$ ) to the Nash equilibrium at  $n_{NE}$  is not a local change but

<sup>&</sup>lt;sup>20</sup>For brevity, our discussion here focuses on the straightforward case, where (i) increasing returns are not reachable under autarky at any positive tax rates and (ii) increasing returns are not reached at tax rate  $\hat{\tau}$  under entrepreneurial mobility. We will relax (ii) below.

a large discrete jump (bifurcation) in all economic variables. Due to the large expansion in labor supply that follows the competitive tax reduction in Fig. 2a, a new, higher productivity, regime of production is established.<sup>21</sup> New steady state equilibria corresponding to this high output regime are located on the second concave segment of the offer curve  $F_{NE}$  near  $n_{NE}$ , and these equilibria appear only when the labor taxes are sufficiently low. At these low levels of taxation, the low output steady states near  $n_m$  are eliminated (equilibria along the first concave segment of  $F_{NE}$  do not exist).

The discrete welfare gain that is correspondingly experienced is illustrated by the jump from an initial equilibrium on  $W_a$  (or on  $W_m$ ) to  $n_{NE}$  on  $W_{NE}$  in Fig. 2b. (Curve  $W_{NE}$ , which depicts individual well-being near the high output equilibria, does not coincide with  $W_m$  because the two welfare curves correspond to different production regimes.) The fact that nonlocal welfare improvements of this type are feasible in our model shows that tax competition can play a surprising positive role: by amplifying the output expansion that follows the liberalization of resource flows, a noncooperative tax reduction may push competing economies much beyond their customary levels of performance. If this happens, it is likely that the realized growth in output and consumption dwarfs the still unrealized potential gain that could be attained from cooperative taxation. In other words, even though, in Fig. 2b, the Nash equilibrium in taxes,  $n_{NE}$ , is worse than the cooperative overall optimum, indicated by point  $W_m^{opt}$  on  $W_{NE}$ , it is nevertheless significantly better than any initial equilibrium on  $W_m$  or  $W_a$ .

The following proposition identifies a sufficient condition under which the movement from autarky to the Nash equilibrium in taxes is, in our context, an overall welfare improvement, despite the fact that taxes are noncooperatively set (the proof is given in the

<sup>&</sup>lt;sup>21</sup>Here bifurcations are assumed to occur after the move from  $n_m$  to the Nash equilibrium  $n_{NE}$ , so that the Nash equilibrium is associated with a unique high employment steady state. Subsequent discussion will also incorporate other possibilities.

<sup>&</sup>lt;sup>22</sup>In Fig. 2b, the tax rate  $\tau_m^{opt}$  is jointly optimal. The Nash tax  $\tau_{NE}$  is generally lower than  $\tau_m^{opt}$ . See the Appendix, part 4), for formal details.

Appendix, part 6)).

**Proposition 2** Let utility functions be isoelastic and assume that the model parameters satisfy the inequality

$$(1 + \mu^{\frac{1}{\sigma}})^{\frac{\sigma}{1 - \sigma}} < 2^{1 - \alpha}. \tag{22}$$

Then, all individuals strictly prefer free labor mobility and the Nash equilibrium in taxes to national social optimum under autarky.

Proposition 2 shows that labor mobility and tax competition can produce gains such that the open economy, even under strategic taxation, is strictly better than autarky. In this sense, even if the Nash equilibrium in taxes may still be worse than the hypothetical joint optimum, tax competition, when compared to autarky, is not necessarily a race to the bottom; outcomes that are much worse could be the result of restricting individual freedom of movement. This observation offers some theoretical support to Bhagwati (2002) who has argued that standard criticisms of tax competition (that such competition inevitably leads to a race to the bottom) are little supported by empirical evidence.

Condition (22), which involves the technology and preference parameters,  $\alpha$  and  $\sigma$ , and the welfare weight for public goods,  $\mu$ , is used to ensure that international mobility or workers induces a sufficient increase in labor supply and output for all to benefit from such mobility. The term on the right-hand side of (22) determines the scope for production gains from openness (as  $\alpha$ approaches one, technology approaches constant returns and the right-hand side term declines; thus, opportunities for welfare gains from international mobility diminish). On the left-hand side, as  $\sigma$  increases (i.e., the demand elasticity  $(1-\sigma)$  declines), there is less scope for utility gains from any increase in output and, as the preference for public goods increases ( $\mu$  grows), the left-hand side of (22) similarly increases. In extreme cases, if  $\mu$  is very high, factor mobility and tax competition may reduce welfare. Given (22), however, an equilibrium such as  $n_{NE}$  strictly dominates autarky.<sup>23</sup>

<sup>&</sup>lt;sup>23</sup>We emphasize that since condition (22) is only a sufficient (but not necessary) condition for a welfare improvement, the Nash equilibrium may dominate autarky even when (22) fails.

Proposition 2 is not conditional on the existence of favorable bifurcations. Accordingly, if condition (22) is satisfied, even when there are no productive externalities the Nash equilibrium (on  $W_m$ ) must still be a welfare improvement compared to autarky (on  $W_a$ ). In Figs. 2, overall welfare gains are enhanced by the discrete jump in the production regime that creates the high output equilibria near  $n_{NE}$  (on  $W_{NE}$ ) but while such large improvements may arise from tax competition, gains attributed to bifurcations in equilibria are by no means necessary for openness to be desirable.

Bifurcational gains cannot exist unless there are multiple production regimes. In the present simple model, production externalities are the source of potential multiplicities. Technological complementarities can also serve in this role; see Honkapohja and Turunen-Red (2002). Fig. 2a suggests that favorable bifurcations are more likely when (i) autarky taxes are high (so that the initial equilibrium is a low output state, analogous to  $n_a$ ), (ii) tax competition involves a large reduction in taxes (so that individual incentives are significantly altered), and (iii) the high production regime can be easily accessed (the kink in offer curve  $F_m$  occurs at a low value of  $n_{t+1}$ ). Beyond the current model is the possibility that there may be several production regimes, each separated by a productivity threshold, some of which may occur at relatively low levels of employment and output (so that the externality gains can reasonably be accessed).

While we have emphasized the role of tax competition in generating favorable bifurcations, upward jumps in equilibria may arise solely because individuals become internationally mobile and even when taxation remains unchanged. This possibility can be illustrated in Fig. 2a by shifting offer curve  $F_m$  up until a high productivity equilibrium (such as  $n_{NE}$  on  $F_{NE}$ ) appears and the low steady state  $(n_m)$  is eliminated. In such a case, tax competition merely yields an additional local improvement in productivity (a further shift up in the offer curve) and welfare but it does not further change the production regime. While this latter possibility

<sup>&</sup>lt;sup>24</sup>The kink shifts to the left in Fig. 1 if population expands (K increases), the fraction of useful ideas among ideas broadcast ( $\lambda$ ) increases, the time it takes an individual to absorb an idea (a) decreases, or if the exogenous employment treshold  $\widehat{I}$  gets smaller.

of bifurcational welfare gains arising from international openness alone may appear more intuitively obvious<sup>25</sup>, we have attempted to show that tax competition, through its expansionary effects on labor supply, works in the same direction as any liberalization of factor flows. Accordingly, even when not being the cause of bifurcational jumps in equilibria, tax competition may still significantly increase the positive welfare impact of free factor mobility.

#### 4 Tax Competition and Expectational Barriers

Expectations have a significant role to play when there are multiple equilibria. In order to illustrate the interplay of expectations and changes in policy we apply our previous Fig. 1.

Let us assume that offer curve F in Fig. 1 applies in autarky (when  $\tau = \hat{\tau}$ ) and F' depicts per capita labor supply when individuals are mobile but taxes remain fixed at  $\hat{\tau}$ . Then, as we move from autarky to complete openness, the set of equilibria expands from a single low output state  $(n_{Low})$  to a set of three potential steady states along F'. Of these,  $n'_{High}$  yields welfare much higher than  $n'_{Low}$  and a seemingly average equilibrium exists at  $n_U$ . Which of these three equilibria is actually realized when individuals are free to work anywhere they choose? The answer is:  $n'_{Low}$ .

Why is  $n'_{High}$  unattainable? The reason revolves around the expectational dynamics that we have assumed govern individual reactions to changes in the economic environment. In order to reach the high output steady state at  $n'_{High}$  all individuals must work much harder so as to overcome the employment threshold that restricts the functioning of productive externalities. While per capita labor supply does respond positively as offer curve F' is reached and expectations are accordingly adjusted upwards, there is along F' an expectational barrier that cannot easily be overcome: as employment and output approach the unstable equilibrium  $n_U$ 

<sup>&</sup>lt;sup>25</sup>Honkapohja and Turunen-Red (2002) demonstrate that bifurcational gains may arise from international trade in capital goods.

from below, errors in individual expectations grow and, following the learning rule (19), all respond by moving back toward the lower, but expectationally stable, steady state at  $n'_{Low}$ . Essentially,  $n'_{High}$  is unattainable because all believe so.

Our Fig. 1 suggests a possible remedy for this expectational impasse. A further upward shift in offer curve F' can, when the shift is sufficiently large, eliminate both the low output equilibrium near  $n'_{Low}$  and the unstable steady state  $n_U$ . And such a shift in the offer curve may take place when the countries compete in taxes. In other words, vigorous tax competition may end up selecting among feasible equilibria the one at which welfare is highest. Compared to autarky, such an outcome must be welfare improving.

**Proposition 3** If tax competition selects among multiple (stable) equilibria, then all individuals prefer free labor mobility and the Nash equilibrium in taxes to autarky.

Proposition 3 brings to the fore the expectational dynamics that guide individuals and the economy from one (stable) equilibrium to another. Since expectations are adjusted so as to reduce forecast errors, only equilibria that are stable with respect to this learning behavior can be approached. If individuals are habitually expecting a low steady state (such as  $n'_{Low}$ ) to be realized, they will not find their way to a higher, yet feasible, steady state (e.g.,  $n'_{High}$ ) if that state is separated by an expectational barrier from the current equilibrium. To overcome the expectational hurdle, some further change is required. The above discussion suggests that tax competition for mobile resources can assume this constructive policy role.

Of course, one may argue that, in a situation such as depicted in Fig. 1, governments have a strong incentive to cooperate as by doing so they may be able to attain a high output state near or above  $n''_{High}$ . Yet, the point of Proposition 3 is that, even if such cooperation were eventually feasible, cooperation may not be necessary because noncooperative policies can yield a nearly equivalent outcome. From this point of view, tax competition may at times be a reasonable substitute for tax cooperation.

There is an additional observation regarding cooperative tax policy that can be obtained using Fig. 1 and Proposition 3. One may imagine two scenarios for the factor market liberalization to be played out. First, countries may agree on a gradual approach in which factor flows are slowly freed and taxes are adjusted in small cooperative moves. Or, cooperation may fail, leading to temporary tax competition before some cooperative tax agreement is eventually reached. The second alternative may appear inferior of the two.

Fig. 1 suggests there are circumstances where the opposite is the case. If the gradual approach is taken, the low output steady state  $(n'_{Low})$  will be observed as long as that equilibrium continues to exist and gains from openness remain only local. However, tax competition, even when temporary, may cause a large shift in work habits and expectations so that a high welfare state  $(n''_{High})$  becomes established. And what is most important, once individuals have learned the existence of the high equilibrium, gradual (cooperative) tax increases can be undertaken and these will not cause a backward reversal to the low output state (near  $n'_{Low}$ ).

The last conclusion follows because the same expectational inertia that can make it difficult for an economy to learn its way to a new and better equilibrium tends to maintain the high output state (near  $n''_{High}$ ) once it has been realized. Only a very large discrete increase in taxation could cut expectations sufficiently for the low output state  $(n'_{Low})$  to be re-established (the tax increase would have to be large enough to eliminate both  $n''_{High}$  and the unstable equilibrium near  $n_U$  that will appear as  $\tau$  increases). Thus, there clearly can be circumstances in which the seemingly prudent, careful and gradual (cooperative) policy path is worse than the path of liberalization, tax competition, and possible cooperative adjustments only later. These conclusions apply whenever there is reason to believe that an economy is capable of sudden productivity and growth spurts.

#### 5 Conclusions

In this paper, we have analyzed consequences of factor mobility and tax competition in a simple two-country overlapping generations model. A special feature of the model is the multiplicity of equilibria that reflects the interactions of increasing and decreasing returns to scale in production.

When productive resources are internationally mobile, countries have an incentive to attract them by cutting taxes. Such tax competition can reduce gains from factor trade by forcing governments to curtail public services if tax revenue falls. Literature on tax competition has duly observed that the Nash equilibrium in taxes when resources are mobile is worse in welfare terms than the joint optimum that could be attained if countries were able to agree on coordinated tax policies. But, since tax coordination is always politically and practically difficult, in this paper we have compared the Nash equilibrium in taxes to the other alternative: autarky (no factor mobility) and optimal national taxation.<sup>26</sup> Our comparison may well be the realistic one between possible alternatives that do not require international political cooperation.

We have been particularly interested in the ability of tax competition enhance (not reduce) the gains from factor mobility. These gains come from two sources. First, lower taxes expand the supply of the mobile factors and, when there are externalities, there may be additional productivity benefits that can be reaped. In Section 3, we showed that, even without externalities, the Nash equilibrium in taxes can be better than the autarky alternative. Whether this is so depends on the parameters that determine the magnitude of the production gains from trade and the degree to which these gains are transformed into welfare improvements.

We also demonstrated that tax competition can be the source of new, large, bifurcational gains in welfare. Such gains are realized when the lowering of taxes causes a discrete shift in the supply of the

<sup>&</sup>lt;sup>26</sup>We consider autarky but our results could be generalized to allow for limited factor mobility as well. This could be done by augmenting the model by a parameter that represents costs of factor mobility.

mobile factors and this change is sufficiently large to significantly alter individual expectations. Then, it may be possible for the economy to reach a new high-employment, high-output, steady state.

Finally, we observed that since expectations impact the current factor supply, the large potential gains from mobility that potentially exist may not be reaped if individual expectations remain persistently low. In such circumstances, tax competition may be helpful because it perturbs the status quo and encourages individuals to work much harder. Most significantly, once the new reality of a much higher equilibrium has been learned be all, the high welfare steady state can still be sustained even if some cooperative tax increases were to be undertaken. In other words, a seemingly radical policy choice of significant change, followed by cooperation at a later stage, may sometimes be better than the gradual and cooperative approach that never shocks the economy out of its present path.

In the current model, taxation finances intergenerational transfers from the young who work to the retired who consume. These social benefits are distributed according to the nationality principle according to which the old generations enjoy only the public goods that their native countries provide. By this device we have excluded the possibility that social benefits themselves may encourage factor flows. We also do not allow for individual heterogeneity; this rules out intragenerational transfers for redistribution purposes that, again, can motivate decisions to migrate. Since our main goal has been to point out the likelihood of overall gains from factor mobility and the positive role that tax competition may play by creating additional discrete gains in welfare, we have chosen a simple framework in which the motivation for international mobility is, as in traditional tax competition literature, the net return to services. Nevertheless, future work in relaxing some of our assumptions remains desirable.

#### References

- [1] Baldwin, R.E. and P. Krugman (2004), "Agglomeration, Integration and Tax Harmonization", European Economic Review, 48, 1-23.
- [2] Bhagwati, J. (2002), Free Trade Today, Princeton University Press, Princeton.
- [3] Breyer, F. and M. Kolmar (2002), "Are National Pension Systems Efficient if Labor is (Im)perfectly Mobile?", *Journal of Public Economics*, 83, 347-374.
- [4] Devereux, M., B. Lockwood and M. Redoano (2002), Do Countries Compete over Corporate Tax Rates, CEPR Discussion Paper 3400.
- [5] Edwards, S. and M. Keen (1996), "Tax Competition and Leviathan", European Economic Review, 40, 113-134.
- [6] Evans, G. and S. Honkapohja (1995), "Increasing Social Returns, Learning, and Bifurcation Phenomena", in *Learning and Rationality in Economics*, A. Kirman and M. Salmon (eds.), Basil Blackwell Ltd., Oxford.
- [7] Evans, G. and S. Honkapohja (2001), Learning and Expectations in Macroeconomics, Princeton University Press.
- [8] Honkapohja, S. and A. Turunen-Red (2002), "Complementarity, Growth and Trade", Canadian Journal of Economics, 35, 517-530.
- [9] Persson, T. and G. Tabellini (1992), "The Politics of 1992: Fiscal Policy and European Integration", Review of Economic Studies, 59, 689-701.
- [10] Samuelson, P. (1958), "An Exact Consumption Loan Model of Interest with or without the Social Contrivance of Money", Journal of Political Economy, 66, 467-482.

- [11] Sinn, H.-W., "EU Enlargement and the Future of the Welfare State", Working Paper No.307, CESifo, June 2000.
- [12] Sinn, H.-W. (2003), The New Systems Competition, Basil Blackwell, Oxford.
- [13] Sorensen, P. (2000), "The Case for International Coordination Reconsidered", *Economic Policy*, October.
- [14] Wildasin, D. (2000), "Factor Mobility and Fiscal Policy in the EU: Policy Issues and Analytical Approaches", *Economic Policy*, October.
- [15] Wilson, J.D. (1999), "Theories of Tax Competition", National Tax Journal 52, 269-304.

### Appendix:

1) Qualitative derivation of the offer curves in Fig. 1: By (3)-(4), when  $\lambda N_{t+1}(1+a\lambda N_{t+1}) < \widehat{I}$  (to the left of the kink on the offer curves in Fig 1), we have  $\psi(N_j) = \widehat{I}^{\beta}$  and equation (18) yields

$$n_t = \left[ (1 - \tau) \widehat{I}^{\beta} \alpha^{\frac{1}{1 - \sigma}} 2^{\frac{\varepsilon + \sigma}{1 - \sigma}} \right]^{\frac{1 - \sigma}{1 + \varepsilon}} n_{t+1}^{\frac{\alpha(1 - \sigma)}{1 + \varepsilon}}. \tag{23}$$

Since the parameters  $\alpha$ ,  $\sigma$ , and  $\varepsilon$  are positive and less than one,  $n_t$  is a concave function of  $n_{t+1}$ .

When  $\lambda N_{t+1}(1 + a\lambda N_{t+1}) > \widehat{I}$  (to the right of the kink on the offer curves in Fig. 1), we have

$$n_{t} = 2^{\frac{\beta(1-\sigma)-(\varepsilon+\sigma)}{1+\varepsilon}} n_{t+1}^{\frac{(\alpha+\beta)(1-\sigma)}{1+\varepsilon}} \left[ (1-\tau) \widehat{I}^{\beta} \alpha^{\frac{1}{1-\sigma}} \right]^{\frac{1-\sigma}{1+\varepsilon}} \left[ \frac{K\lambda}{(1+2a\lambda K n_{t+1})} \right]^{\frac{\beta(1-\sigma)}{1+\varepsilon}}$$

$$(24)$$

It can be shown (see Evans and Honkapohja (1995), p. 220) that when  $\sigma < 1$ , there can be a convex segment on offer curve (24), as long as  $n_{t+1} > (\widehat{I}\lambda^{-1}(1-a\widehat{I})^{-1})K^{-1}$ , i.e., we are on the right-hand side of the kink (the kink is defined by the equation  $n_{t+1} = (\widehat{I}\lambda^{-1}(1-a\widehat{I})^{-1})K^{-1}$ ). As  $n_{t+1}$  increases, offer curve (24) eventually becomes concave.

2) Welfare has an inverted U-shape as a function of the tax rate in Fig. 2b: First, consider autarky (curve  $W_a$ ). In autarky and given (5), welfare equals

$$W_a(\tau, n_a) = U((1 - \tau)Y(n_a)) - V(n_a) + \mu \mathcal{U}(\tau Y(n_a)), \qquad (25)$$

where  $Y(n_a) = f(n_a, N_a) \equiv Y_a$ . Thus,

$$\frac{\partial W_a}{\partial \tau} = U' \left[ (1 - \tau) \frac{\partial Y_a}{\partial \tau} - Y_a \right] - V' \frac{\partial n_a}{\partial \tau} + \mu \mathcal{U}' \left[ \tau \frac{\partial Y_a}{\partial \tau} + Y_a \right], \quad (26)$$

and

$$\frac{\partial Y_a}{\partial \tau} = (f_1' + K f_N') \frac{\partial n_a}{\partial \tau}.$$
 (27)

Given the autarky first order condition (20), (28) gives

$$\frac{\partial W_a}{\partial \tau} = \left[\mu \mathcal{U}' - U'\right] Y_a + \left[U'(1-\tau)Kf_N' + \mu \mathcal{U}'\tau(f_1' + Kf_N')\right] \frac{\partial n_a}{\partial \tau}.$$
(28)

Since  $\partial n_a/\partial \tau$  is negative (see the derivation of the employment derivatives below in part 4) and -U'Y < 0, the sign of the total welfare derivative is determined by the relative magnitude of the term  $\mu \mathcal{U}'$  (> 0) in (28).<sup>27</sup>

Assuming the isoelastic utility functions (17), however, we obtain that the first bracketed term of (28) equals

$$[\mu \mathcal{U}' - U'] Y_a = [\mu(\tau Y_a(t))^{-\sigma} - ((1 - \tau)Y_a(\tau))^{-\sigma}] Y_a$$
 (29)  
= 
$$[\mu \tau^{-\sigma} - (1 - \tau)^{-\sigma}] Y_a^{1-\sigma}.$$

As  $\tau$  approaches zero, this term grows arbitrarily large but the second (negative) bracketed term in (28) has a finite limit. Thus, when  $\tau$  is near zero, the welfare derivative (28) is positive. Accordingly, when taxes and publicly provided consumption are low, total welfare is increasing with respect to  $\tau$ . When  $\tau$  increases, however, steady state production  $Y_a$  converges towards zero and will reach zero at some value of  $\tau$ . (In Fig. 2a, the offer curve rotates downwards around the origin as  $\tau$  increases.) Thus, when

<sup>&</sup>lt;sup>27</sup>For brevity, we assume that the interior steady state is unique and of low type for the considered domain of values of  $\tau$ .

the tax rate increases the second bracketed term in (28) is likely to dominate. Then, total welfare in decreasing in  $\tau$ .

In the open economy, we evaluate the sum of the derivatives  $\partial W_m^H/\partial \tau_H$  and  $\partial W_m^H/\partial \tau_F$  at a symmetric equilibrium where  $\tau_H = \tau_F = \tau$ . (Here subscript m refers to "mobility".) With respect to changes in the domestic tax in Home, the welfare derivative equals

$$\frac{\partial W_m^H}{\partial \tau_H} = U' \left[ \frac{\partial c_H}{\partial \tau_H} \right] - V' \left[ \frac{\partial n_H}{\partial \tau_H} + \frac{\partial n_F}{\partial \tau_H} \right] + \mu \mathcal{U}' \left[ \tau_H \frac{\partial Y_H}{\partial \tau_H} + Y_H \right], \quad (30)$$

where  $Y_H = f(n_H, N_H) + f(n_H^*, N_H)$  and, with respect to changes in the foreign tax,

$$\frac{\partial W_m^H}{\partial \tau_F} = U' \left[ \frac{\partial c_H}{\partial \tau_F} \right] - V' \left[ \frac{\partial n_H}{\partial \tau_F} + \frac{\partial n_F}{\partial \tau_F} \right] + \mu \mathcal{U}' \left[ \tau_H \frac{\partial Y_H}{\partial \tau_F} \right]. \tag{31}$$

Furthermore, since  $c_H = (1 - \tau_H) f(n_{Ht}, N_{Ht}) + (1 - \tau_F) f(n_{Ft}, N_{Ft})$  at a steady state,

$$\frac{\partial c_H}{\partial \tau_H} = -f(n_H, N_H) - (1 - \tau) f_1' \left[ \frac{\partial n_H}{\partial \tau_H} + \frac{\partial n_F}{\partial \tau_H} \right] + (32)$$

$$K(1 - \tau) f_N' \left[ \frac{\partial n_H}{\partial \tau_H} + \frac{\partial n_F}{\partial \tau_H} + \frac{\partial n_H^*}{\partial \tau_H} + \frac{\partial n_F^*}{\partial \tau_H} \right],$$

$$\frac{\partial c_H}{\partial \tau_F} = -f(n_F, N_F) - (1 - \tau) f_1' \left[ \frac{\partial n_H}{\partial \tau_F} + \frac{\partial n_F}{\partial \tau_F} \right] +$$

$$K(1 - \tau) f_N' \left[ \frac{\partial n_H}{\partial \tau_F} + \frac{\partial n_F}{\partial \tau_F} + \frac{\partial n_H^*}{\partial \tau_F} + \frac{\partial n_F^*}{\partial \tau_F} \right]$$
(33)

$$\frac{\partial Y_H}{\partial \tau_H} = (f_1' + 2K f_N') (\frac{\partial n_H}{\partial \tau_H} + \frac{\partial n_H^*}{\partial \tau_H}), \quad \frac{\partial Y_H}{\partial \tau_F} = (f_1' + 2K f_N') (\frac{\partial n_H}{\partial \tau_F} + \frac{\partial n_H^*}{\partial \tau_F}). \tag{34}$$

The employment derivatives are computed below in part 3) of the Appendix.

Substituting (32)-(34) into (30)-(31) and applying the first order

conditions (13)-(14) yields

$$\frac{\partial W_m^H}{\partial \tau_H} = -U' f(n_H, N_H) + \mu \mathcal{U}' Y_H 
+ \mu \mathcal{U}' \tau (f_1' + 2K f_N') \left[ \frac{\partial n_H}{\partial \tau_H} + \frac{\partial n_H^*}{\partial \tau_H} \right] 
+ U' K (1 - \tau) f_N' \left[ \frac{\partial n_H}{\partial \tau_H} + \frac{\partial n_F}{\partial \tau_H} + \frac{\partial n_H^*}{\partial \tau_H} + \frac{\partial n_F^*}{\partial \tau_H} \right],$$
(35)

$$\frac{\partial W_m^H}{\partial \tau_F} = -U'f(n_F, N_F) + \mu \mathcal{U}'\tau(f_1' + 2Kf_N') \left[ \frac{\partial n_H}{\partial \tau_F} + \frac{\partial n_H^*}{\partial \tau_F} \right] (36)$$
$$+ U'K(1 - \tau)f_N' \left[ \frac{\partial n_H}{\partial \tau_F} + \frac{\partial n_F}{\partial \tau_F} + \frac{\partial n_H^*}{\partial \tau_F} + \frac{\partial n_F^*}{\partial \tau_F} \right].$$

Hence,

$$\frac{\partial W_m^H}{\partial \tau_H} + \frac{\partial W_m^H}{\partial \tau_F} = \mu \mathcal{U}' Y_H - U' \left[ f(n_H, N_H) + f(n_F, N_F) \right]$$

$$+ \mu \mathcal{U}' \tau (f_1' + 2K f_N') \left[ \frac{\partial n_H}{\partial \tau_H} + \frac{\partial n_H^*}{\partial \tau_H} + \frac{\partial n_H^*}{\partial \tau_F} + \frac{\partial n_H^*}{\partial \tau_F} \right]$$

$$+ 2U' K (1 - \tau) f_N' \left[ \frac{\partial n_H}{\partial \tau_H} + \frac{\partial n_F}{\partial \tau_H} + \frac{\partial n_H^*}{\partial \tau_H} + \frac{\partial n_F^*}{\partial \tau_H} \right].$$
(37)

The first term in the above welfare derivative is positive and the other terms are negative. However, due to symmetry, we obtain that

$$\mu \mathcal{U}' Y_H - U' [f(n_H, N_H) + f(n_F, N_F)] = [\mu \mathcal{U}' - U'] Y_H,$$

in (37); further,  $G_H = \tau Y_H$  and, by symmetry,  $c_H = (1 - \tau) Y_H$ . Thus, arguments analogous to the autarky case apply in the open economy as well, and welfare as a function of the tax rate has an inverted U-shape as claimed.

3) Labor derivatives: For the isoelastic utility functions and taking into account production technology (1) equations (9)-(10) yield the following characterization of the steady state:

$$\alpha(1-\tau_H)c_H^{-\sigma}n_H^{\alpha-1}\Psi(N_H) = (n_H + n_F)^{\varepsilon}, \tag{38}$$

$$\alpha(1 - \tau_F)c_H^{-\sigma} n_F^{\alpha - 1} \Psi(N_F) = (n_H + n_F)^{\varepsilon}, \tag{39}$$

$$\alpha(1 - \tau_F)c_F^{*(-\sigma)} n_F^{*(\alpha - 1)} \Psi(N_F) = (n_H^* + n_F^*)^{\varepsilon}, \tag{40}$$

$$\alpha(1 - \tau_H)c_H^{*(-\sigma)} n_H^{*(\alpha - 1)} \Psi(N_H) = (n_H^* + n_F^*)^{\varepsilon}. \tag{41}$$

Furthermore, in (38)-(41), the consumption terms equal

$$c_H = (1 - \tau_H)f(n_H, N_H) + (1 - \tau_F)f(n_F, N_F), \tag{42}$$

$$c_F^* = (1 - \tau_H)f(n_H^*, N_H) + (1 - \tau_F)f(n_F^*, N_F). \tag{43}$$

Equations (38)-(41) can also be written in the form

$$\alpha(1 - \tau_H)c_H^{-\sigma} n_H^{\alpha - 1} \Psi(N_H) = (1 + M^{\frac{1}{1 - \alpha}})^{\varepsilon} n_H^{\varepsilon}, \tag{44}$$

$$\alpha(1-\tau_F)c_F^{*(-\sigma)}n_F^{*(\alpha-1)}\Psi(N_F) = \left[\frac{1+M^{\frac{1}{1-\alpha}}}{M^{\frac{1}{1-\alpha}}}\right]^{\varepsilon}n_F^{*\varepsilon},\tag{45}$$

$$R \equiv \frac{\Psi(N_F)}{\Psi(N_H)} \frac{(1 - \tau_F)}{(1 - \tau_H)},\tag{46}$$

$$c_H = n_H^{\alpha} \left[ (1 - \tau_H) \Psi(N_H) + (1 - \tau_F) R^{\frac{\alpha}{1 - \alpha}} \Psi(N_F) \right],$$
 (47)

$$c_F^* = n_F^{*(\alpha)} \left[ (1 - \tau_F) \Psi(N_F) + \frac{(1 - \tau_H) \Psi(N_H)}{R^{\frac{\alpha}{1 - \alpha}}} \right]. \tag{48}$$

If we solve the above equations at a symmetric equilibrium, we obtain (as  $\tau_H = \tau_F = \tau$ ) that

$$n_{H} = \alpha^{\frac{1}{z}} \Psi(N)^{\frac{1-\sigma}{z}} \frac{(1-\tau_{H})^{\frac{1-\alpha+\varepsilon}{z(1-\alpha)}} \left[2-\tau_{H}-\tau_{F}\right]^{-\frac{\sigma}{z}}}{\left[(1-\tau_{H})^{\frac{1}{1-\alpha}} + (1-\tau_{F})^{\frac{1}{1-\alpha}}\right]^{\frac{\varepsilon}{z}}}.$$

$$= \frac{\alpha^{\frac{1}{z}} (1-\tau)^{\frac{1-\sigma}{z}} \Psi(N)^{\frac{1-\sigma}{z}}}{2^{\frac{\varepsilon+\sigma}{z}}}.$$

$$(49)$$

Differentiation of (44) yields

$$Ddn_H = T_1 d\tau_H + T_2 d\tau_F, (50)$$

$$D = -\alpha(1 - \tau_H)\Psi(N_H)n_H^{\alpha - 2}c_H^{-(1 + \sigma)}\left[(1 - \alpha)c_H + \sigma n_H \frac{\partial c_H}{\partial n_H}\right] - \varepsilon n_H^{\varepsilon - 1}2^{\varepsilon} < 0,$$

$$T_1 = \alpha n_H^{\alpha - 1}\Psi(N_H)c_H^{-\sigma - 1}\left[c_H + \sigma(1 - \tau_H)\frac{\partial c_H}{\partial \tau_H}\right] + n_H^{\varepsilon}\frac{\varepsilon 2^{\varepsilon - 1}}{(1 - \alpha)(1 - \tau_H)} > 0,$$

$$T_2 = \sigma \alpha(1 - \tau_H)n_H^{\alpha - 1}\Psi(N_H)c_H^{-\sigma - 1}\frac{\partial c_H}{\partial \tau_F} - n_H^{\varepsilon}\frac{\varepsilon 2^{\varepsilon - 1}}{(1 - \alpha)(1 - \tau_H)} < 0,$$

$$\frac{\partial c_H}{\partial n_H} = 2\alpha n_H^{\alpha - 1}(1 - \tau)\Psi(N_H) > 0,$$

$$\frac{\partial c_H}{\partial \tau_H} = -n_H^{\alpha}\Psi(N_H) < 0, \quad \frac{\partial c_H}{\partial \tau_F} = -n_F^{\alpha}\Psi(N_F) < 0.$$

Therefore, we obtain

$$\frac{\partial n_H}{\partial \tau_H} = \frac{\partial n_F^*}{\partial \tau_F} = \frac{T_1}{D} < 0, \tag{51}$$

$$\frac{\partial n_H}{\partial \tau_F} = \frac{\partial n_F^*}{\partial \tau_H} = \frac{T_2}{D} > 0. \tag{52}$$

In addition, because  $n_F = R^{\frac{1}{1-\alpha}} n_{H_i}$ , we have that

$$\frac{\partial n_F}{\partial \tau_H} = \frac{\partial n_H^*}{\partial \tau_F} = \frac{\partial n_H}{\partial \tau_H} + \frac{n_H}{1 - \alpha} \frac{\partial R}{\partial \tau_H} = \frac{T_1}{D} + \frac{n_H}{(1 - \alpha)(1 - \tau_H)},$$

$$\frac{\partial n_F}{\partial \tau_F} = \frac{\partial n_H^*}{\partial \tau_H} = \frac{dn_H}{\partial \tau_F} + \frac{n_H}{1 - \alpha} \frac{\partial R}{\partial \tau_F} = \frac{T_2}{D} - \frac{n_H}{(1 - \alpha)(1 - \tau_H)}.$$

The labor supply derivatives imply: (i) the world labor supply declines if one country increases its tax, i.e.,

$$\frac{\partial n_F}{\partial \tau_H} + \frac{\partial n_F^*}{\partial \tau_H} + \frac{\partial n_H}{\partial \tau_H} + \frac{\partial n_H^*}{\partial \tau_H} = \frac{2(T_1 + T_2)}{D} < 0;$$

(ii) a common change of taxes reduces each country's domestic (internal) labor supply, i.e.,

$$\frac{\partial n_H}{\partial \tau_H} + \frac{\partial n_H}{\partial \tau_F} = \frac{T_1 + T_2}{D} < 0;$$

(iii) in each country, a domestic tax increase reduces total labor supply, i.e.,

$$\frac{\partial n_H}{\partial \tau_H} + \frac{\partial n_H^*}{\partial \tau_H} = \frac{T_1 + T_2}{D} - \frac{n_H}{(1 - \alpha)(1 - \tau_H)} < 0.$$

4) Each country unilaterally lowers its tax when the symmetric tax is jointly optimal: At a symmetric equilibrium, the jointly optimal (common) tax rate,  $\tau^{opt}$ , is characterized by the equations

$$\left[\frac{\partial W_m^j}{\partial \tau_H} + \frac{\partial W_m^j}{\partial \tau_F}\right] \mid_{\tau_H = \tau_F = \tau^{opt}} = 0, \quad j = H, F,$$
 (53)

where the welfare derivatives are as in (35)-(36). We want to argue that, at the joint optimum,  $\left[\partial W_m^H/\partial \tau_H\right]$   $|_{optimum} < 0$ , i.e., each country has a unilateral incentive to lower its own tax.

Given (35), equation (53) yields for (36) that

$$\frac{\partial W_m^H}{\partial \tau_F} |_{optimum} = -\mu \mathcal{U}' Y_H + U' f(n_F, N_F) - \mu \mathcal{U}' \tau(f_n' + 2K f_N') \left[ \frac{\partial n_H}{\partial \tau_H} + \frac{\partial n_H^*}{\partial \tau_H} \right]$$

$$-U' K (1 - \tau) f_N' \left[ \frac{\partial n_H}{\partial \tau_H} + \frac{\partial n_F}{\partial \tau_H} + \frac{\partial n_H^*}{\partial \tau_H} + \frac{\partial n_F^*}{\partial \tau_H} \right].$$
(54)

If  $\partial W_m^H/\partial \tau_F$  is positive at the joint optimum, then  $\partial W_m^H/\partial \tau_H < 0$  at this optimum as we require.

The first term on the right-hand side of (54) is negative but the other terms are positive. In particular, if we consider only the first two terms of (54), we obtain (using the symmetry of the equilibrium and the isoelastic utility functions) that

$$-\mu \mathcal{U}'(G_H)Y_H + U'(c_H)f(n_F, N_F) = f(n_H, N_H) \left[ U'(c_H) - 2\mu U'(G_H) \right],$$

$$(55)$$

$$c_H = 2(1 - \tau^{opt})f(n_H), \quad G_H = 2\tau^{opt}f(n_H).$$

Thus,  $\partial W_m^H/\partial \tau_F > 0$  if

$$U'(c_H) - 2\mu U'(G_H) > 0,$$

or equivalently,

$$MRS_{cG} \equiv \frac{U'(c_H)}{\mu U'(G_H)} = \frac{1}{\mu} \left[ \frac{\tau^{opt}}{1 - \tau^{opt}} \right]^{\sigma} > 2.$$

Accordingly, each country wishes to unilaterally lower its tax rate if the jointly optimal tax is sufficiently large compared to the weight of public services in individual welfare. Certainly, when the weight of public services in welfare is not very high ( $\mu$  tends to zero), this sufficient (but not necessary) condition is satisfied.

5) Proof of Lemma 1: Given that  $\tau_H = \tau_F = \tau$ , the price ratio  $p_t/p_{t+1}$  in (12) equals  $Y_{t+1}^W/Y_t^W$ . Thus, equations (13)-(14) yield

$$(1 - \tau)U'(c_{t+1})Y_{t+1}^W = \frac{Y_t^W V'(2n_t)}{f_1'(n_t, N_t)}.$$
 (56)

Given  $n_{t+1}$ , the left-hand side of (56) is decreasing in  $\tau$ . Consequently, the right-hand side of (56) is decreasing in  $\tau$  as well. But since the term  $Y_t^W V'(2n_t)/f'_1(n_t)$  is increasing in  $n_t$  (since V is convex and f is concave in n), the  $n_t$  determined by (56) must increase as the tax rate  $\tau$  declines.

6) Proof of Proposition 2: Consider first the case without productive externalities. Then, as a function of  $n_a$ , autarky welfare (for either country) satisfies the inequality

$$W_{a}(\tau, n_{a}) = U((1 - \tau)Y(n_{a})) - V(n_{a}) + \mu \mathcal{U}(\tau Y(n_{a}))$$

$$= \frac{Y(n_{a})^{1-\sigma}}{1-\sigma} \left[ (1 - \tau)^{1-\sigma} + \mu \tau^{1-\sigma} \right] - \frac{n_{a}^{1+\varepsilon}}{1+\varepsilon}$$

$$\leq W_{a}^{upper}(n_{a}) \equiv \frac{(1 + \mu^{\frac{1}{\sigma}})^{\sigma}}{(1-\sigma)} n_{a}^{\alpha(1-\sigma)} \widehat{I}^{\beta(1-\sigma)} - \frac{n_{a}^{1+\varepsilon}}{1+\varepsilon}.$$
(57)

Inequality (57) is obtained by replacing the term  $[(1-\tau)^{1-\sigma} + \mu\tau^{1-\sigma}]$  in the welfare expression by its maximum value with respect to  $\tau$  (the maximum occurs at  $\tau = \mu^{\frac{1}{\sigma}}/(1+\mu^{\frac{1}{\sigma}})$ ). Further, maximizing  $W_a^{upper}(n_a)$  with respect to  $n_a$  yields

$$W_a(\tau, n_a) \le S\alpha^{\frac{1+\varepsilon}{z}} \widehat{I}^{\frac{\beta(1-\sigma)(1+\varepsilon)}{z}} \left[ 1 + \mu^{\frac{1}{\sigma}} \right]^{\frac{\sigma(1+\varepsilon)}{z}}, \tag{58}$$

$$S \equiv \frac{z}{\alpha(1-\sigma)(1+\varepsilon)}, \quad z \equiv 1+\varepsilon - \alpha(1-\sigma). \tag{59}$$

When individuals are freely mobile and taxes are Nash, welfare is at least as high as when taxes are equal to zero (this is because welfare has an inverted U-shape as a function of  $\tau$ ). When taxes are zero, welfare is derived solely from private market activities and equals

$$W_{\tau=0}^{mob} = S \left[ 2n \right]^{1+\varepsilon}, \tag{60}$$

where  $n = \alpha^{\frac{1}{z}} 2^{-\frac{(\varepsilon+\sigma)}{z}} \hat{I}^{\frac{\beta(1-\sigma)}{z}}$  as can be determined using (18). Substituting this expression into (60) yields

$$W_{\tau=0}^{mob} = S\alpha^{\frac{1+\varepsilon}{z}} 2^{\frac{(1+\varepsilon)(1-\sigma)(1-\alpha)}{z}} \widehat{I}^{\frac{\beta(1-\sigma)(1+\varepsilon)}{z}}.$$
 (61)

Comparison of (61) and (58) yields  $W_{\tau=0}^{mob} \geq W_a^{upper}$  if (22) is satisfied.

If external productivity gains exist in autarky, they will also be observed when labor is mobile (labor supply in each country is larger when workers are mobile). The output expansion caused by the externality is therefore larger in the open economy than in autarky. Thus, Proposition 2 still holds.

If there are no productive externalities in autarky but such gains appear when individuals are mobile, Proposition 2 is guaranteed to hold given (22).