Modelling Volatilities and Conditional Correlations in Futures Markets with a Multivariate t Distribution*

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Abstract

This paper considers a multivariate t version of the Gaussian dynamic conditional correlation (DCC) model proposed by Engle (2002), and suggests the use of devolatized returns computed as returns standardized by realized volatilities rather than by GARCH type volatility estimates. The t-DCC estimation procedure is applied to a portfolio of daily returns on currency futures, government bonds and equity index futures. The results strongly reject the normal-DCC model in favour of a t-DCC specification. The t-DCC model also passes a number of VaR diagnostic tests over an evaluation sample. The estimation results suggest a general trend towards a lower level of return volatility, accompanied by a rising trend in conditional cross correlations in most markets; possibly reflecting the advent of euro in 1999 and increased interdependence of financial markets.

JEL Classifications: C51, C52, G11
Key Words: Volatilities and Correlations, Futures Market, Multivariate t, Financial Interdependence, VaR diagnostics.

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1   Introduction

Modelling of conditional volatilities and correlations across asset returns is an integral part of portfolio decision making and risk management. In risk management the value at risk (VaR) of a given portfolio can be computed using univariate volatility models, but a multivariate model is needed for portfolio decisions. Even in risk management the use of a multivariate model would be desirable when a number of alternative portfolios of the same universe of m assets are under consideration. By using the same multivariate volatility model marginal contributions of different assets towards the overall portfolio risk can be computed in a consistent manner. Multivariate volatility models are also needed for determination of hedge ratios and leverage factors.

The literature on multivariate volatility modelling is large and expanding. Bauwens, Laurent, and Rombouts (2006) provide a recent review. A general class of such models is the multivariate generalized autoregressive conditional heteroscedastic (MGARCH) specification. (Engle and Kroner (1995)). However, the number of unknown parameters of the unrestricted MGARCH model rises exponentially with m and its estimation will not be possible even for a modest number of assets. The diagonal-VEC version of the MGARCH model is more parsimonious, but still contains too many parameters in most applications. To deal with the curse of dimensionality the dynamic conditional correlations (DCC) model is proposed by Engle (2002) which generalizes an earlier specification by Bollerslev (1990) by allowing for time variations in the correlation matrix. This is achieved parsimoniously by separating the specification of the conditional volatilities from that of the conditional correlations. The latter are then modelled in terms of a small number of unknown parameters, which avoids the curse of the dimensionality. With Gaussian standardized innovations Engle (2002) shows that the log-likelihood function of the DCC model can be maximized using a two step procedure. In the first step, m univariate GARCH models are estimated separately. In the second step using standardized residuals, computed from the estimated volatilities from the first stage, the parameters of the conditional correlations are then estimated. The two step procedure can then be iterated if desired for full maximum likelihood estimation.

DCC is an attractive estimation procedure which is reasonably flexible in modeling individual volatilities and can be applied to portfolios with a large number of assets. However, in most applications in finance the Gaussian assumption that underlies the two step procedure is likely to be violated. To capture the fat-tailed nature of the distribution of asset returns, it is more appropriate if the DCC model is combined with a multivariate t distribution, particularly for risk analysis where the tail properties of return distributions are of primary concern. But Engle's two-step procedure will no longer be applicable to such a t-DCC specification and a simultaneous approach to the estimation of the parameters of the model, including the degree-of-freedom parameter of the multivariate t distribution would be needed. This paper develops such an estimation procedure and proposes the use of devolatized returns computed as returns standardized by realized volatilities rather than by GARCH type volatil-

ity estimates. Devolatized returns are likely to be approximately Gaussian although the same can not be said about the standardized returns. (Andersen, Bollerslev, Diebold, and Ebens (2001), and Andersen, Bollerslev, Diebold and Labys (2001)). In the absence of intradaily observations the paper proposes an approximate measure based on contemporaneous daily returns and their lagged values.

The t-DCC estimation procedure is applied to a portfolio composed of six currency futures, four 10 year government bonds and five equity index futures over the period 02 January 1995 to 31 December 2006, split into an estimation sample (1995 to 2004) and an evaluation sample (2005 to 2006). The results strongly reject the normal-DCC model in favour of a t-DCC specification. The t-DCC model also passes a number of VaR diagnostic tests over the evaluation sample.

The estimates over the full sample show a number of interesting patterns: there has been a general trend towards a lower level of volatility in all markets, with currency futures leading the way. In contrast, conditional correlations across currencies and equity returns have been rising. Only the conditional correlations of bonds and equities seem to have been declining. Some of these patterns might be reflecting the advent of euro and the increased interdependence of financial markets particularly over the past decade. A detailed analysis of these trends and their possible explanations is beyond the scope of the present paper.

The plan of the paper is follows. Section 2 introduces the t-DCC model and discussed the devolatized returns and the rational behind their construction. Section 3 considers recursive relations for real time analysis. The maximum likelihood estimation of the t-DCC model is set out in Section 4, followed by a review of diagnostics in Section 5. The empirical application to return futures is discussed in Section 6, followed by some concluding remarks in Section 7.

2   Modelling Conditional Correlation Matrix of Asset Returns

Let $\mathbf{r}_t$ be an $m \times 1$ vector of asset returns at close day t assumed to have a conditional multivariate t distribution with means, $\boldsymbol{\mu}_{t-1}$, and the non-singular variance-covariance matrix $\boldsymbol{\Sigma}_{t-1}$, and $v_{t-1} > 2$ degrees of freedom. Here we are not concerned with how mean returns are predicted and take $\boldsymbol{\mu}_{t-1}$ as given.[1] For specification of $\boldsymbol{\Sigma}_{t-1}$ we follow Bollerslev (1990) and Engle (2002) consider the decomposition

$$\boldsymbol{\Sigma}_{t-1} = \mathbf{D}_{t-1} \mathbf{R}_{t-1} \mathbf{D}_{t-1}, \tag{1}$$

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[1] Although, the estimation of $\boldsymbol{\mu}_{t-1}$ and $\boldsymbol{\Sigma}_{t-1}$ are inter-related, in practice mean returns are predicted by least squares techniques (such as recursive estimation or recursive modelling) which do not take account of the conditional volatility. This might involve some loss in efficiency of estimating $\boldsymbol{\mu}_{t-1}$, but considerably simplifies the estimation of the return distribution needed in portfolio decisions and risk management.

where

$$\mathbf{D}_{t-1} = \begin{pmatrix} \sigma_{1,t-1} & & & \\ & \sigma_{2,t-1} & 0 & \\ & 0 & \ddots & \\ & & & \sigma_{m,t-1} \end{pmatrix},$$

$$\mathbf{R}_{t-1} = \begin{pmatrix} 1 & \rho_{12,t-1} & \rho_{13,t-1} & \cdots & & \rho_{1m,t-1} \\ \rho_{21,t-1} & 1 & \rho_{23,t-1} & \cdots & & \rho_{2m,t-1} \\ \vdots & & \ddots & & & \vdots \\ \vdots & & & & \rho_{m-1,m,t-1} \\ \rho_{m1,t-1} & \cdots & \cdots & \rho_{m,m-1,t-1} & 1 \end{pmatrix},$$

$\mathbf{R}_{t-1} = (\rho_{ij,t-1}) = (\rho_{ji,t-1})$ is the symmetric $m \times m$ correlation matrix , and $\mathbf{D}_{t-1}$ is the $m \times m$ diagonal matrix with $\sigma_{i,t-1}, i = 1, 2, \ldots, m$ denoting the conditional volatility of the i-th asset return. More specifically

$$\sigma_{i,t-1}^2 = V\left(r_{it} \mid \Omega_{t-1}\right),$$

and $\rho_{ij,t-1}$ are conditional pair-wise return correlations defined by

$$\rho_{ij,t-1} = \frac{Cov\left(r_{it}, r_{jt} \mid \Omega_{t-1}\right)}{\sigma_{i,t-1}\sigma_{j,t-1}},$$

where Ω_{t-1} is the information set available at close of day $t - 1$. Clearly, $\rho_{ij,t-1} = 1$, for $i = j$.

Bollerslev (1990) considers (1) with a constant correlation matrix $\mathbf{R}_{t-1} = \mathbf{R}$. Engle (2002) allows for $\mathbf{R}_{t-1}$ to be time-varying and proposes a class of multivariate GARCH models labeled as dynamic conditional correlation (DCC) models. An alternative approach would be to use the conditionally heteroskedastic factor model discussed, for example, in Sentana (2000) where the vector of unobserved common factors are assumed to be conditionally heteroskedastic. Parsimony is achieved by assuming that the number of the common factors is much less than the number of assets under considerations.

The decomposition of $\mathbf{\Sigma}_{t-1}$ in (1) allows separate specification of the conditional volatilities and conditional cross-asset returns correlations. For example, one can utilize the GARCH (1,1) model for $\sigma_{i,t-1}^2$, namely

$$V\left(r_{it} \mid \Omega_{t-1}\right) = \sigma_{i,t-1}^2 = \bar{\sigma}_i^2\left(1 - \lambda_{1i} - \lambda_{2i}\right) + \lambda_{1i}\sigma_{i,t-2}^2 + \lambda_{2i}r_{i,t-1}^2, \quad (2)$$

where $\bar{\sigma}_i^2$ is the unconditional variance of the i-th asset return. Under the restriction $\lambda_{1i} + \lambda_{2i} = 1$, the unconditional variance does not exist and we have the integrated GARCH (IGARCH) model used extensively in the professional financial community, which is mathematically equivalent to the "exponential smoother" applied to the r_{it}^2's[2]

$$\sigma_{i,t-1}^2\left(\lambda_i\right) = \left(1 - \lambda_i\right)\sum_{s=1}^{\infty}\lambda_i^{s-1}r_{i,t-s}^2 \qquad 0 < \lambda_i < 1, \quad (3)$$

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[2] See, for example, Litterman and Winkelmann (1998).

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or written recursively

$$\sigma_{i,t-1}^2 \left(\lambda_i\right) = \lambda_i \sigma_{i,t-2}^2 + \left(1 - \lambda_i\right) r_{i,t-1}^2. \tag{4}$$

For cross-asset correlations Engle proposes the use of the following exponential smoother applied to the "standardized returns"

$$\hat{\rho}_{ij,t-1} \left(\phi\right) = \frac{\sum_{s=1}^{\infty} \phi^{s-1} z_{i,t-s} z_{j,t-s}}{\sqrt{\sum_{s=1}^{\infty} \phi^{s-1} z_{i,t-s}^2} \sqrt{\sum_{s=1}^{\infty} \phi^{s-1} z_{j,t-s}^2}}, \tag{5}$$

where the standardized returns are defined by

$$z_{it} = \frac{r_{it}}{\sigma_{i,t-1} \left(\lambda_i\right)}. \tag{6}$$

For estimation of the unknown parameters, $\lambda_1, \lambda_2, ..., \lambda_m$, and ϕ, Engle (2002) proposes a two-step procedure whereby in the first step individual GARCH(1,1) models are fitted to the m asset returns separately, and then the coefficient of the conditional correlations, ϕ, is estimated by the Maximum Likelihood method assuming that asset returns are conditionally Gaussian. This procedure has two main drawbacks. First, the Gaussianity assumption does not hold for daily returns and its use can under-estimate the portfolio risk. Second, the two-stage approach is likely to be inefficient even under Gaussianity.

2.1 Pair-wise correlations based on realized volatilities

In this paper we consider an alternative formulation of $\rho_{ij,t-1}$ that makes use of realized volatilities, or their approximations based on daily observations when realized volatility measures are not available. In a series of papers Andersen, Bollerslev and Diebold show that daily returns on foreign exchange and stock returns standardized by realized volatility are approximately Gaussian. See, for example, Andersen, Bollerslev, Diebold, and Ebens (2001), and Andersen, Bollerslev, Diebold and Labys (2001). The transformation of returns to Gaussianity is important since as recently shown by Embrechts et al. (2003), the use of correlation as a measure of dependence can be misdealing in the case of (conditionally) non-Gaussian returns. In contrast, estimation of correlations based on devolatized returns that are nearly Gaussian is likely to be more generally meaningful. Denote the realized volatility of ith return in day t by $\sigma_{it}^{realized}$ and standardize the returns by the realized volatilities to obtain

$$\tilde{r}_{it} = \frac{r_{it}}{\sigma_{it}^{realized}}. \tag{7}$$

To avoid confusions we refer to $\tilde{r}_{it}$ as the "devolatized returns", and refer to z_{it} defined by (6) as the standardized returns. The conditional pair-wise return correlations based on $\tilde{r}_{it}$ are now given by

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$$\tilde{\rho}_{ij,t-1}\left(\phi\right) = \frac{\sum_{s=1}^{\infty} \phi^{s-1} \tilde{r}_{i,t-s} \tilde{r}_{j,t-s}}{\sqrt{\sum_{s=1}^{\infty} \phi^{s-1} \tilde{r}_{i,t-s}^2} \sqrt{\sum_{s=1}^{\infty} \phi^{s-1} \tilde{r}_{j,t-s}^2}}, \qquad (8)$$

where $-1 < \tilde{\rho}_{ij,t-1}\left(\phi\right) < 1$ for all values of $|\phi| < 1$.

As compared to z_{it}, the use of $\tilde{r}_{it}$ is more data intensive and requires intradaily observations. Although, intradaily observations are becoming increasingly available across a large number of assets, it would still be desirable to work with a version of $\tilde{r}_{it}$ that does not require intradaily observations, but is nevertheless capable of rendering the devolatized returns approximately Gaussian. One of the main reasons for the non-Gaussian behavior daily returns is presence of jumps in the return process as documented for a number of markets in the literature (see, for example, Barndorff-Nielsen and Shephard (2002)). The standardized return, z_{it}, used by Engle does not deal with such jumps, since the jump process that affects the numerator of z_{it} in day t does not enter the denominator of z_{it} which is based on past returns and exclude the current return, r_t. The problem is accentuated due to the facts that jumps are typically independently distributed over time. The use of realized volatility ensures that the numerator and the denominator of the devolatized returns, $\tilde{r}_{it}$, are both affected by the same jumps in day t.

In the absence of intradaily observations the following simple estimate of σ_{it} based on daily returns, *inclusive* of the contemporaneous value of r_{it}, seem to work well in practice

$$\tilde{\sigma}_{it}^2(p) = \frac{\sum_{s=0}^{p-1} r_{i,t-s}^2}{p}. \qquad (9)$$

The lag-order, p, needs to chosen carefully. We have found that for daily returns a value of $p = 20$ tends to render the devolatized returns, $\tilde{r}_{it} \approx r_{it}/\tilde{\sigma}_{it}(p)$, nearly Gaussian, with approximately unit variances, for all asset classes foreign exchange, equities, bonds or commodities.[3] Note that $\tilde{\sigma}_{it}^2(p)$ is not the same of the rolling historical estimate of σ_{it} defined by

$$\hat{\sigma}_{it}^2(p) = \frac{\sum_{s=1}^{p} r_{i,t-s}^2}{p}.$$

Specifically

$$\tilde{\sigma}_{it}^2(p) - \hat{\sigma}_{it}^2(p) = \frac{r_{it}^2 - r_{i,t-p}^2}{p}.$$

It is the inclusion of the current squared returns, r_{it}^2, in the estimation of $\tilde{\sigma}_{it}^2$ that seems to be critical in transformation of r_{it} (which is non-Gaussian) into $\tilde{r}_{it}$ which seems to be approximately Gaussian.

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[3] For some empirical evidence in support of this claim see Section 6.

3 Real Time Risk Analysis and Updates

In financial analysis estimation and evaluation are in general recursive and the unknown parameters need to be updated over time.[4] The frequency by which parameters are updated depends on the processing costs and the expected benefit from the updates. When processing costs are negligible parameter updates are carried out on the arrival of new data or shortly thereafter. For daily observations (the focus of the present paper) weekly or even monthly updates are recommended. Daily updates can be quite time consuming for large portfolios, and the expected benefit of the more frequent (daily) updates unclear. For model evaluation, however, a daily frequency seems desirable. Clearly, model evaluation need not be carried out at the same frequency with which parameters are updated. In analysis of market risk where daily or even intradaily observations are available evaluation is typically carried out on a daily basis.

The implementation of the real time analysis is very much facilitated using recursive formulae in the estimation and the evaluation process. For computational of $\rho_{ij,t-1}$, given by (5) and (8), as noted by Engle (2002) we have

$$\tilde{\rho}_{ij,t-1}(\phi) = \frac{q_{ij,t-1}}{\sqrt{q_{ii,t-1}q_{jj,t-1}}} \tag{10}$$

where

$$q_{ij,t-1} = \phi q_{ij,t-2} + (1-\phi)\tilde{r}_{i,t-1}\tilde{r}_{j,t-1}. \tag{11}$$

The recursive expression for $\hat{\rho}_{ij,t-1}(\phi)$ is identical except that instead of devolatized returns the standardized returns, z_{it}, given by (6) are used.

The above models for $\rho_{ij,t-1}$ are non-mean reverting. A more general mean-reverting specification is given by

$$q_{ij,t-1} = \bar{\rho}_{ij}(1-\phi_1-\phi_2) + \phi_1 q_{ij,t-2} + \phi_2 \tilde{r}_{i,t-1}\tilde{r}_{j,t-1}, \tag{12}$$

where $\bar{\rho}_{ij}$ is the unconditional correlation of r_{it} and r_{jt} and $\phi_1 + \phi_2 < 1$. One would expect $\phi_1 + \phi_2$ to be close to unity. The non-mean reverting case can be obtained as a special case by setting $\phi_1 + \phi_2 = 1$. In practice it is impossible to be sure if $\phi_1 + \phi_2 < 1$ or not. The unconditional correlations, $\bar{\rho}_{ij}$, can be estimated using an expanding window. In the empirical applications we shall consider the mean reverting as well as the non-mean reverting specifications, and experiment with the two specifications of the conditional correlations that are based on standardized and devolatized returns.

3.1 Initialization, Estimation and Evaluation Samples

Suppose daily observations are available on m daily returns in the $m \times 1$ vector $\mathbf{r}_t$ over the period $t = 1, 2, ..., T, T+1, ..., T+N$. The first T_0 observations are used for computation of (9), the initialization of the recursions (12), and the

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[4]A general discussion of real time econometric analysis is provided in Pesaran and Timmermann (2005).

estimation of sample variances and correlations, namely $\bar{\sigma}_i^2$ and $\bar{\rho}_{ij}$, used in (2) and (12), respectively. Let s denote the starting point of the most recent sample of observations to be used in estimation. Clearly, we must have $T > s > T_0 > p$. The size of the estimation window will then be given by $T_e = T - s + 1$. The remaining N observations can then be used for evaluation purposes. More specifically, the initialization sample will be given by $\mathcal{S}_0 = \{\mathbf{r}_t,\, t = 1, 2, ..., T_0\}$, the estimation sample by $\mathcal{S}_e = \{\mathbf{r}_t,\, t = s, s + 1, ..., T\}$, and the evaluation sample, $\mathcal{S}_{\text{eval}} = \{\mathbf{r}_t,\, t = T + 1, T + 2, ..., T + N\}$. This decomposition allows us to vary the size of the estimation window $(T_e = T - s + 1)$ by moving the index s along the time axis in order to accommodate estimation of the unknown parameters using expanding or rolling observation windows, with different estimation update frequencies. For example, for an expanding estimation window we set $s = T_0 + 1$. For a rolling window of size W we need to set $s = T + 1 - W$. The whole estimation process can then be rolled into the future with an update frequency of h by carrying the estimations at $T + h, T + 2h$, ..., using either expanding or rolling estimation samples from $t = s$. Note that model (risk) evaluation can be carried out using observations $t = T + 1, T + 2, ...$, irrespective of the update frequency parameter h.

3.2 Mean Reverting Conditional Correlations

In the mean reverting case we also need the estimates of the unconditional volatilities and the correlation coefficients. These can be estimated by

$$\bar{\sigma}_{i,t}^2 \;\; = \;\; \frac{\sum_{\tau=1}^{t} r_{i\tau}^2}{t}, \tag{13}$$

$$\bar{\rho}_{ij,t} \;\; = \;\; \frac{\sum_{\tau=1}^{t} r_{i\tau} r_{j\tau}}{\sqrt{\sum_{\tau=1}^{t} r_{i\tau}^2}\sqrt{\sum_{\tau=1}^{t} r_{j\tau}^2}}. \tag{14}$$

The index t refers to the end of the available estimation sample which in real time will be recursively rolling or expanding, namely $t = T, T + h, T + 2h, ...$

4 Maximum Likelihood Estimation of the t-DCC Model

In its most general formulation (the non-mean reverting specifications given by (2) and (12)) the DCC(1,1) model contains $2m + 3$ unknown parameters; $2m$ coefficients $\boldsymbol{\lambda}_1 = (\lambda_{11}, \lambda_{12}, \ldots, \lambda_{1m})'$ and $\boldsymbol{\lambda}_2 = (\lambda_{21}, \lambda_{22}, \ldots, \lambda_{2m})'$ that enter the individual asset returns volatilities, the 2 coefficients ϕ_1 and ϕ_2 that enter the conditional correlations, and the degrees of freedom of the multivariate t distribution, v. The parameters $\bar{\sigma}_i^2$ and $\bar{\rho}_{ij}$ in (2) and (12) refer to the unconditional volatilities and return correlations and can be estimated using the estimation sample or the estimation plus initialization sample. See (13) and (14). In the non-mean reverting case these intercept coefficients disappear, but for the

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initialization of the recursive relations (2) and (12) it is still advisable to use unconditional estimates of the correlation matrix and asset returns volatilities.

Denote the unknown coefficients by

$$\boldsymbol{\theta} = (\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \phi_1, \phi_2, v)'.$$

Then based on a sample of observations on returns, $\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_t$, available at time t, the time t log-likelihood function based on the decomposition (1) is given by

$$l_t(\boldsymbol{\theta}) = \sum_{\tau=s}^{t} f_\tau(\boldsymbol{\theta}), \qquad (15)$$

where $s < t$ is the start date of the estimation window (see above). Under t-DCC specification $f_\tau(\boldsymbol{\theta})$ refers to the density of the multivariate distribution with v degrees of freedom which can be written in terms of the $\boldsymbol{\Sigma}_{t-1} = \mathbf{D}_{t-1}\mathbf{R}_{t-1}\mathbf{D}_{t-1}$ as[5]

$$
\begin{aligned}
f_\tau(\boldsymbol{\theta}) &= -\frac{m}{2}\ln(\pi) - \frac{1}{2}\ln|R_{\tau-1}(\boldsymbol{\theta})| - \ln|D_{\tau-1}(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2)| \\
&\quad + \ln\left[\Gamma\left(\frac{m+v}{2}\right)/\Gamma\left(\frac{v}{2}\right)\right] - \frac{m}{2}\ln(v-2) \\
&\quad - \left(\frac{m+v}{2}\right)\ln\left[1 + \frac{\mathbf{e}'_\tau D_{\tau-1}^{-1}(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) R_{\tau-1}^{-1}(\boldsymbol{\theta}) D_{\tau-1}^{-1}(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2)\mathbf{e}_\tau}{v-2}\right],
\end{aligned}
\qquad (16)
$$

where

$$\mathbf{e}_\tau = \mathbf{r}_\tau - \boldsymbol{\mu}_{\tau-1},$$

and

$$\ln|D_{\tau-1}(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2)| = \sum_{i=1}^{m}\ln[\sigma_{i,\tau-1}(\lambda_{1i}, \lambda_{2i})]. \qquad (17)$$

It is worth noting that under Engle's specification $\mathbf{R}_{t-1}$ depends on $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$ as well as on ϕ_1 and ϕ_2. Under the alternative specification advanced here (based on devolatized returns) $\mathbf{R}_{t-1}$ does not depend on $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$, but depends on ϕ_1 and ϕ_2, and p, the lag order used in the devolatization process.

The ML estimate of $\boldsymbol{\theta}$ based on the sample observations, $\mathbf{r}_s, \mathbf{r}_2, ..., \mathbf{r}_T$, can now be computed by maximization of $l_t(\boldsymbol{\theta})$ with respect to $\boldsymbol{\theta}$, which we denote by $\hat{\boldsymbol{\theta}}_t$. More specifically

$$\hat{\boldsymbol{\theta}}_t = Arg\max_{\boldsymbol{\theta}}\{l_t(\boldsymbol{\theta})\}, \text{ for } t = T, T+h, T+2h, ..., T+N, \qquad (18)$$

where h is the (estimation) update frequency, and as before N refers to the length of the evaluation sample. The standard errors of the ML estimates are

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[5] Typically the multivariate t density is written in terms of a scale matrix. But assuming $v > 2$ ensures that $\boldsymbol{\Sigma}_{t-1}$ exists and therefore the scale matrix of the multivariate t distribution can be written in terms of $\boldsymbol{\Sigma}_{t-1}$, which is more convenient for the analysis of multivariate volatility models. See, for example, Bauwens and Laurent (2005).

computed using the asymptotic formulae[6]

$$\widehat{Cov}(\hat{\boldsymbol{\theta}}_t) = \left\{ \sum_{\tau=s}^{t} \left[\frac{-\partial^2 f_\tau(\boldsymbol{\theta})}{\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}'} \right]_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}_t} \right\}^{-1}.$$

In comparison with general specifications of multivariate GARCH model, the model set out in this paper is quite parsimonious. The number of unknown coefficients of the general MGARCH model rises as a quadratic function of m, while the parameters of the DCC model rises linearly with m. Nevertheless, in practice the simultaneous estimation of all the parameters of the DCC model could be problematic, namely can encounter convergence problems, or could lead to a local maxima of the likelihood function. When the returns are conditionally Gaussian one could simplify (at the expense of some loss of estimation efficiency) the computations by adopting Engle's two-stage estimation procedure. But for our preferred distributional assumption the use of such a two-stage procedure does not seem possible and can lead to contradictions. For example, estimation of separate $t - GARCH(1,1)$ models for individual asset returns can lead to different estimates of v, while the multi-variate t distribution requires v to be the same across all assets.[7]

5   Simple Diagnostic Tests of the t-DCC Model

Consider a portfolio based on the m assets with the return vector $\mathbf{r}_t$ using the $m \times 1$ vector of pre-determined weights, $\mathbf{w}_{t-1}$. The return on this portfolio is given by

$$\rho_t = \mathbf{w}'_{t-1} \mathbf{r}_t. \tag{19}$$

Suppose that we are interested in computing the capital Value at Risk (VaR) of this portfolio expected at the close of business on day $t - 1$ with probability $1 - \alpha$, which we denote by $VaR(\mathbf{w}_{t-1}, \alpha)$. For this purpose we require that

$$\Pr\left[\mathbf{w}'_{t-1} \mathbf{r}_t < -VaR(\mathbf{w}_{t-1}, \alpha) \, |\Omega_{t-1} \right] \leq \alpha.$$

Under our assumptions, conditional on Ω_{t-1}, $\mathbf{w}'_{t-1}\mathbf{r}_t$ has a Student t distribution with mean $\mathbf{w}'_{t-1}\boldsymbol{\mu}_{t-1}$, the variance $\mathbf{w}'_{t-1}\boldsymbol{\Sigma}_{t-1}\mathbf{w}_{t-1}$, and the degrees of freedom v. Hence

$$z_t = \sqrt{\frac{v}{v-2}} \left(\frac{\mathbf{w}'_{t-1}\mathbf{r}_t - \mathbf{w}'_{t-1}\boldsymbol{\mu}_{t-1}}{\sqrt{\mathbf{w}'_{t-1}\boldsymbol{\Sigma}_{t-1}\mathbf{w}_{t-1}}} \right),$$

conditional on Ω_{t-1} will also have a t distribution with v degrees of freedom. It is easily verified that $E(z_t|\Omega_{t-1}) = 0$, and $V(z_t|\Omega_{t-1}) = v/(v-2)$. Denoting

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[6] An analytical expression for the information matrix for the multivariate t-GARCH model is provided by Florentini, Sentana, and Calzolari (2003). But in the applications considered in this paper we did not encounter any problems using numerical derivatives to compute the information matrix.

[7] Marginal distributions associated with a multi-variate t-distribution with v degrees of freedom are also t-distributed with the same degrees of freedom.

the cumulative distribution function of a Student t with v degrees of freedom by $F_v(z)$, $VaR(\mathbf{w}_{t-1},\alpha)$ will be given as the solution to

$$F_v\left(\frac{-VaR(\mathbf{w}_{t-1},\alpha) - \mathbf{w}'_{t-1}\boldsymbol{\mu}_{t-1}}{\sqrt{\frac{v-2}{v}\left(\mathbf{w}'_{t-1}\boldsymbol{\Sigma}_{t-1}\mathbf{w}_{t-1}\right)}}\right) \leq \alpha.$$

But since $F_v(z)$ is a continuous and monotonic function of z we have

$$\frac{-VaR(\mathbf{w}_{t-1},\alpha) - \mathbf{w}'_{t-1}\boldsymbol{\mu}_{t-1}}{\sqrt{\frac{v-2}{v}\left(\mathbf{w}'_{t-1}\boldsymbol{\Sigma}_{t-1}\mathbf{w}_{t-1}\right)}} = F_v^{-1}(\alpha) = -c_\alpha,$$

where c_α is the $\alpha\%$ critical value of a Student t distribution with v degrees of freedom. Therefore,

$$VaR(\mathbf{w}_{t-1},\alpha) = \tilde{c}_\alpha\sqrt{\left(\mathbf{w}'_{t-1}\boldsymbol{\Sigma}_{t-1}\mathbf{w}_{t-1}\right)} - \mathbf{w}'_{t-1}\boldsymbol{\mu}_{t-1}, \tag{20}$$

where $\tilde{c}_\alpha = c_\alpha\sqrt{\frac{v-2}{v}}$.

Following Christoffersen (1998) and Engle and Manganelli (2004), a simple test of the validity of t-DCC model can be computed recursively using the VaR indicators

$$d_t = I\left(\mathbf{w}'_{t-1}\mathbf{r}_t + VaR(\mathbf{w}_{t-1},\alpha)\right) \tag{21}$$

where $I(A)$ is an indicator function which is equal to unity if $A > 0$ and zero otherwise. These indicator statistics can be computed in-sample or preferably can be based on recursive out-of-sample one-step ahead forecast of $\boldsymbol{\Sigma}_{t-1}$ and $\boldsymbol{\mu}_{t-1}$, for a given (pre-determined set of portfolio weights, $\mathbf{w}_{t-1}$). In such an out–of-sample exercise the parameters of the mean returns and the volatility variables ($\boldsymbol{\beta}$ and $\boldsymbol{\theta}$, respectively) could be either kept fixed at the start of the evaluation sample or changed with an update frequency of h periods (for example with $h = 5$ for weekly updates, or $h = 20$ for monthly updates). For the evaluation sample, $\mathcal{S}_{\text{eval}} = \{\mathbf{r}_t,\ t = T + 1, T + 2, ..., T + N\}$, the mean hit rate is given by

$$\hat{\pi}_N = \frac{1}{N}\sum_{t=T+1}^{T+N} d_t. \tag{22}$$

Under the t-DCC specification, $\hat{\pi}_N$ will have mean $1 - \alpha$ and variance $\alpha(1 - \alpha)/N$. The standardized statistic,

$$z_\pi = \frac{\sqrt{N}\left[\hat{\pi}_N - (1 - \alpha)\right]}{\sqrt{\alpha(1 - \alpha)}}, \tag{23}$$

will have a standard normal distribution for a sufficiently large evaluation sample size, N. This result holds irrespective of whether the unknown parameters are estimated recursively or fixed at the start of the evaluation sample. In the

case of the latter the validity of the test procedure requires that $N/T \to 0$ as $(N, T) \to \infty$. For a proof see Pesaran and Zaffaroni (2007).

The z_π statistic provides evidence on the performance of Σ_{t-1} and μ_{t-1} in an average (unconditional) sense. (Lopez (1999)). An alternative conditional evaluation procedure, proposed by Berkowitz (2001), can be based on probability integral transforms[8]

$$\hat{U}_t = F_v \left(\frac{\mathbf{w}'_{t-1}\mathbf{r}_t - \mathbf{w}'_{t-1}\hat{\boldsymbol{\mu}}_{t-1}}{\sqrt{\frac{v-2}{v}\mathbf{w}'_{t-1}\hat{\boldsymbol{\Sigma}}_{t-1}\mathbf{w}_{t-1}}} \right), \ t = T+1, T+2, ..., T+N. \qquad (24)$$

Under the null hypothesis of correct specification of the t-DCC model, the probability transform estimates, $\hat{U}_t$, are serially uncorrelated and uniformly distributed over the range $(0, 1)$. Both of these properties can be readily tested. The serial correlation property of $\hat{U}_t$ can be tested by Lagrange multiplier tests using OLS regressions of Z_t on an intercept and the lagged values $\hat{U}_{t-1}, \hat{U}_{t-2}, ..., \hat{U}_{t-s}$. The maximum lag length, s, can be selected by the application of the AIC criteria, for example. The uniformity of the distribution of $\hat{U}_t$ over t can be tested using the Kolmogorov-Smirnov statistic defined by, $KS_N = \sup_x \left| F_{\hat{U}}(x) - U(x) \right|$, where $F_{\hat{U}}(x)$ is the empirical cumulative distribution function (CDF) of the $\hat{U}_t$, for $t = T+1, T+2, ..., T+N$, and $U(x) = x$ is the CDF of iid $U[0, 1]$. Large values of the Kolmogorov-Smirnov statistic, KS_N, indicate that the sample CDF is not similar to the hypothesized uniform CDF.[9]

6   Volatilities and Conditional Correlations in Futures Markets

We estimated alternative versions of the t-DCC model for a portfolio composed of returns on six currency futures: Japanese yen, euro, British pound, Swiss franc, Canadian and Australian dollars, denoted by JY, EU, BP, CH, CD, AD; four government bond futures: US ten year Treasury Note, 10 year government bonds issued by Germany, UK and Japan, denoted by $TNote, Bund, Gilt$, and JGB; and five equity index futures in US, UK: Germany, France and Japan, namely S&P 500, FTSE, DAX, CAC and Nikkei, denoted by $SP, FTSE, DAX, CAC$, and, NK. The daily futures prices are obtained from Datastream and cover the twelve years from 02-Jan-95 to 31-Dec-2006.

Table 1 provides summary statistics for the daily returns (r_{it}, in percent) and the devolatized daily returns $\tilde{r}_{it} = r_{it}/\tilde{\sigma}_{it}(p)$, where in the absence of intradaily observations $\tilde{\sigma}_{it}^2(p)$ is defined by (9), with $p = 20$. The choice of $p = 20$ was guided by some experimentation with pre-1995 returns with the aim of transforming r_{it} into an approximately Gaussian process. A choice of p well above

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[8] See also Christoffersen (1998) for a related test that applied to the VaR indicators, d_t, defined by (21).

[9] For details of the Kolmogorov-Smirnov test and its critical values see, for example, Massey (1951), and Neave and Worthington (1992, pp.89-93).

20 does now allow the (possible) jumps in r_{it} to become adequately reflected in $\tilde{\sigma}_{it}(p)$, and a value of p well below 20 transforms r_{it} to an indicator looking function. In the extreme case where $p = 1$ we have $\tilde{r}_{it} = 1$, if $r_{it} > 0$, and $\tilde{r}_{it} = -1$, if $r_{it} < 0$, and $\tilde{r}_{it} = 0$, if $r_{it} = 0$. We did not experiment with other values of p for the sample under consideration and set $p = 20$ for all the 15 assets. For the non-devolatized returns the results are as to be expected from previous studies. The future returns seem to be symmetrically distributed with kurtosis in some cases well in excess of 3 (the value for the Gaussian distribution). The excess kurtosis is particularly large for equities, JY, AD, and JGB. In contrast, the devolatized returns do not show any excess kurtosis. For example, for equities the excess kurtosis of the devolatized returns is below 0.11 (for SP), and the excess kurtosis of JY, DAX, and JGB, have fallen from $9.83, 4.56$ and 4.18 to 0.70, -0.07, and 0.38, respectively. The means and standard deviations of the devolatized returns are also very close to $(0, 1)$.

The extent to which the devolatization has been effective in transforming the returns into Gaussian variates can be seen in Figures 1-15. The left panel of each figure gives the histograms, a kernel density fitted to the returns together with the normal density and the normal QQ-plots. These plots graphically compare the distribution of returns to the normal distribution (represented by a straight line in the case of the QQ-plots). The figures on the right panel display the same graphs for the devolatized returns. These figures clearly show that devolatization has been quite effective in achieving Gaussianity to a high degree of approximation. This can be seen particularly if one compares QQ-plots of returns and their devolatized counterparts. For the devolatized returns the QQ-plots generally lie on the straight-line with a few exceptions. But for the raw returns there are important departures from normality, particularly in tails of the return distributions.

Since we are primarily interested in volatility modelling and VaR diagnostics we set $\boldsymbol{\mu}_{t-1} = 0$, and estimate the DCC models on daily returns (close on close) over the period 01-Jan-95 to 31-Dec-2004 (2610 observations), and use the observations January 2, 2005 to December 31, 2006 for the evaluation of estimated volatility models using the VaR and distribution free diagnostics.[10] We also estimated *separate* t-DCC models for currencies, bonds and equities for purposes of comparisons. All estimations are carried out for the unrestricted versions of the DCC(1,1) model with asset-specific volatility parameters $\boldsymbol{\lambda}_1 = (\lambda_{11}, \lambda_{12}, \ldots, \lambda_{1m})'$, $\boldsymbol{\lambda}_2 = (\lambda_{21}, \lambda_{22}, \ldots, \lambda_{2m})'$, and common conditional correlation parameters, ϕ_1 and ϕ_2, and the degrees-of-freedom parameter, v, under conditionally t distributed returns. We did not encounter even a single case of non-convergence, and furthermore obtained the same ML estimates when starting from different initial parameter values.

To evaluate the statistical significance of the multivariate t distribution for the analysis of return volatilities, in Table 2 we first provide the maximized log-likelihood values under multivariate normal and t distributions for currencies,

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[10] The ML estimation and the computation of the diagnostic statistics are carried using Microfit 5. See Pesaran and Pesaran (2007).

bonds and equities separately, as well as for all the 15 assets jointly. We report these results both for standardized and devolatized returns. It is firstly clear from these results that the normal-DCC specifications are strongly rejected relative to the t-DCC models for all asset categories. The maximized log-likelihood values for the t-DCC models are significantly larger than the ones for the normal-DCC models. The estimated degrees of freedom are also in the range 5.91 (for currencies) to 10.16 (for equities), all well below the values of 30 and above that one would expect for a multivariate normal distribution. These conclusions are robust to the way returns are standardized for computation of cross asset return correlations. The maximized log-likelihoods for the standardized and devolatized returns are very close, although due to the non-nested nature of the two return transformations no definite conclusions can be reached as to their relative merits here we adopt the devolatized returns in the estimation of correlations on the grounds of their approximate Gaussianity.

6.1 Testing for Integrated GARCH Effects

Table 3 presents the detailed estimation results of the t-DCC model for all the 15 assets using devolatization results over the period January, 1995 to December 2004. The asset-specific estimates of the volatility decay parameters are all highly significant, with the estimates of λ_{i1}, $i = 1, 2, ..., 15$ falling in the range 0.9097 (for Nikkei) to 0.9687 (for Canadian dollar). The average estimate of $\boldsymbol{\lambda}_1$ across assets is 0.9521 which is just inside the range $(0.95 - 0.97)$ of values recommended by Riskmetrics for their exponential smoothing estimates of volatilities. There are, however, notable differences across asset groups with λ_{i1} estimated to be larger for currencies as compared to the estimates for bonds and equities. The sum of the estimates of λ_{i1} and λ_{i2} are very close to unity, but the hypothesis that $\lambda_{i1} + \lambda_{i2} = 1$ is statistically rejected for 13 out of the 15 assets; the exceptions being Canadian dollar and US Treasury Note. The correlation parameters, ϕ_1 and ϕ_2 are also very precisely estimated with $\hat{\phi}_1 = 0.9810(0.0012)$, $\hat{\phi}_2 = 0.0107(0.0005)$, and $1 - \hat{\phi}_1 - \hat{\phi}_2 = 0.0083(0.0008)$.[11] These estimates suggest very slow but statistically significant mean reverting volatilities and conditional correlations. There are also statistically significant evidence of parameter heterogeneity across assets, although these differences may not be important in practice as their differences (although statistically significant) are quantitatively rather small.

6.2 Diagnostics

For an equal-weighted portfolio, namely setting all elements of $\mathbf{w}$ in (19) equal to unity, and $\alpha = 1\%$ one would expect $\hat{\pi}$ defined by (22) to be around 0.99. For the t-DCC estimates reported in Table 3, with the estimates *fixed* at the end of 2004, we obtain $\hat{\pi} = 0.9904$, $z_\pi = 0.0882$ over the evaluation sample January 2, 2005 to December 31, 2006 (520 daily observations), and the null hypothesis

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[11] The standard errors are given in brackets.

that $\pi = 0.99$ can not be rejected.[12] We also find no statistically significant evidence of serial correlation in the estimates of $\hat{U}_t$, $t = T+1, T+2, ..., T+520$, defined by (24).[13] The value of the Kolmogorov-Smirnov statistic computed using $\hat{U}_t$, $t = T+1, T+2, ..., T+520$, turned out to be $KS_N = 0.0404$, which is below 0.0596, the 5% critical value of the KS test with $N = 520$, and does not indicate any major departures of $\hat{U}_t$ from uniformity.[14]

6.3 Changing Interdependence in Financial Markets

The t-DCC model can provide important insights into the changing volatilities and correlations over the past two decades. To this end we re-estimated the model over the full sample period, January 2, 1995 to December 31, 2006 and obtained very similar results as those reported in Table 3. The time series plots of volatilities are displayed in Figures 16-18 for currencies, equities and bond futures, respectively. Conditional correlations of Euro with other currencies, S&P futures with other equity future indices, US 10 year bond futures with other bond futures are shown in Figures 19 to 21, respectively. To reduce the impact of the initialization on the plots of volatilities and conditional correlations initial estimates for 1995 are not shown. These figures show a declining trends in volatilities over the 1996-2006 period, most pronounced in the case of currency futures, and a rising trend in correlations most notable in the case of equity futures. These trend could reflect the advent of Euro and a closer integration of the world economy, particularly in the euro area. In contrast, there are no clear trends in the cross market correlations, correlation between bonds and equities, currencies and equities, or bonds and currencies. See Figures 22-24.

The above conclusions concerning the trends in daily volatilities and correlations ought to be viewed with caution given the relatively short span of years that they are covering. Unfortunately, futures market do not go back far enough to enable us to arrive at a more definite conclusion. Only for the main currencies (yen, euro and British pound) longer spans of futures data are available. For these currencies fitting a t-DCC model to the daily observations over the period

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[12]Similar results are also obtained when the parameters of the t-DCC model are updated at the end of 2005.

[13]Recall that these estimates are obtained with the value of $\boldsymbol{\theta}$ estimated over the sample ending in $t = T$.

[14]See Table 1 in Massey (1951).

2 January 1985 to May 1, 2007 yields the following estimates:

ML Estimates of t-DCC Model for Three Currency Futures over the Period 30 January 1985 to 30 April 2007

Currencies	ML Estimates		
	$\hat{\lambda}_1$	$\hat{\lambda}_2$	$1 - \hat{\lambda}_1 - \hat{\lambda}_2$
British pound	0.9576 (0.0048)	0.0374 (0.0039)	0.0050 (0.0013)[3.95]
Euro	0.9536 (0.0052)	0.0367 (0.0036)	0.0097 (0.0021)[4.62]
Yen	0.9396 (0.0077)	0.0439 (0.0048)	0.0164 (0.0038)[4.37]
$\hat{v} = 4.99$ (0.1472) , $\hat{\phi}_1 = 0.9702$ (0.0024), $\hat{\phi}_2 = 0.0268$ (0.0019)			
Note: Standard errors of the estimates are given in round brackets,			
t-statistics are given is square brackets.			

The estimates for the conditional volatilities of euro and British pound are also very close, and the very low estimate obtained for the degrees of freedom of the multivariate t distribution ($\hat{v} = 5$) once again high lights the importance of allowing for the fat tail properties of currency futures in volatility modelling. Estimates of conditional volatilities over the period January 2, 1986 to April 30, 2007 show a declining trend for euro and british pound but not for yen. The sample mean of conditional volatilities for yen has remained fairly constant at around 0.688 per cent per day over the two sub-samples: January 2, 1986 to December 30, 1996, and January 2, 1997 to April 30, 2007. But the mean estimates of the volatilities of euro and British pound have declined form 0.720 and 0.691 over the first sub-sample to 0.622 and 0.537, respectively, over the second sub-samples. This can be clearly seen in Figure 25.

7   A Concluding Remark

This paper proposes the use of t-DCC model for the analysis of asset returns as a way of dealing with the fat-tailed nature of their underling distributions. However, the multivariate t distribution used for this purpose implies marginal t distributions for the individual underlying returns with the *same* degrees of freedom. This is clearly not supported by the data. As can be seen from Table 2, the degrees of freedom parameter estimated separately for the different asset classes differ markedly across the asset classes (around 6 for currencies, 8 for bonds and 9.5 for equities). One possible way of dealing with this problem would be to combine the t-DCC models estimated separately by asset classes, filling the missing blocks of the full correlation matrix, $\mathbf{R}_{t-1}$, by means of exponential smoothers of the type used in Riskmetrics. However, in this case the distribution of returns on portfolios formed with assets from different asset classes will no longer follow a t distribution, and VaR calculations must be carried out by stochastic simulations as no closed form solution seems to exist for linear

combination of t-distributed variates with different degrees of freedom. Further research in this area is required.

In the case of some asset returns, particularly in the cash markets, it might also be important to consider using multivariate asymmetric distributions, such as the "multivariate skew-Student density" recently proposed by Bauwens and Laurent (2005). However, our preliminary analysis suggests that such asymmetries might not important in futures markets where there are little restrictions on long/short transactions.

Table 1: Summary Statistics for Futures Daily Returns and Devolatized Daily Returns - 02-Jan-95 to 30-Dec-06

Asset	Returns				Devolatilized returns			
	Mean	S.D.	Skewness	Ex. Kurtosis	Mean	S.D.	Skewness	Ex. Kurtosis
Currencies								
Australian dollar	0.008	0.674	-0.035	2.801	0.024	0.997	-0.056	0.307
British pound	0.013	0.516	0.012	1.549	0.030	0.998	0.039	0.396
Canadian dollar	0.006	0.408	0.021	1.460	0.005	1.000	0.048	-0.086
Swiss franc	-0.006	0.691	0.217	1.998	-0.016	1.000	0.135	0.267
Euro	-0.001	0.626	0.050	1.476	-0.002	0.996	0.065	0.221
Yen	-0.019	0.730	0.951	9.833	-0.039	1.000	0.271	0.701
Bonds								
Bunds	0.017	0.317	-0.407	1.548	0.075	0.994	-0.159	0.092
Gilt	0.011	0.359	-0.206	2.003	0.036	0.997	-0.057	0.081
JGB	0.018	0.288	-0.463	4.178	0.080	1.004	-0.206	0.384
TNote	0.015	0.372	-0.322	1.923	0.052	0.999	-0.088	0.258
Equities								
CAC	0.041	1.388	-0.027	2.790	0.042	1.001	-0.119	-0.160
DAX	0.037	1.510	-0.227	4.547	0.045	1.002	-0.169	-0.073
Nikkei	0.007	1.463	0.060	1.989	0.009	1.000	-0.058	0.039
S&P 500	0.033	1.118	-0.083	3.916	0.038	1.004	-0.192	0.105
FTSE	0.021	1.118	-0.066	2.819	0.027	1.005	-0.134	-0.172

Table 2: Maximized log-likelihood Values of DCC Models Estimatd with Daily
Returns over 02-Jan-95 to 31-Dec-04

Assets	Standardized Returns			Devolatized returns		
	Normal	t-distribution	D.F.	Normal	t-distribution.	D.F.
Currencies (6)	-9600.4	-8853.9	5.94 (0.2285)	-9602.1	-8848.4	5.91 (0.2263)
Bonds (4)	-1282.3	-1018.9	7.66 (0.4479)	-1284.2	-1019.6	7.62 (0.4438)
Equities (5)	-18189.4	-17894.2	9.37 (0.5578)	-18192.3	-17896.7	9.33 (0.5556)
All 15	-28604.8	-27460.8	10.16 (0.3889)	-28612.3	-27461.4	10.05 (0.3832)

Note: D.F. is the estimated degrees of the freedom of the multivariate t-distribution.

Standard errors of the estimates are given in round brackets.

Table 3: ML Estimates of t-DCC Model Estimatd with Daily Returns over 02-Jan-95 to 31-Dec-04

Asset	ML Estimates		
	$\hat{\lambda}_1$	$\hat{\lambda}_2$	$1 - \hat{\lambda}_1 - \hat{\lambda}_2$
Currencies			
Australian dollar	0.9631 (0.0069)	0.0246 (0.0038)	0.0124 (0.0044)[2.80]
British pound	0.9669 (0.0092)	0.0218 (0.0047)	0.0114 (0.0053)[2.13]
Canadian dollar	0.9687 (0.0050)	0.0293 (0.0043)	0.0021 (0.0017)[1.19]
Swiss franc	0.9647 (0.0057)	0.0260 (0.0036)	0.0094 (0.0028)[3.33]
Euro	0.9691 (0.0047)	0.0234 (0.0031)	0.0075 (0.0022)[3.42]
Yen	0.9505 (0.0085)	0.0409 (0.0062)	0.0086 (0.0029)[2.93]
Bonds			
Bunds	0.9525 (0.0075)	0.0361 (0.0049)	0.0114 (0.0036)[3.21]
Gilt	0.9675 (0.0058)	0.0280 (0.0044)	0.0045 (0.0020)[2.25]
JGB	0.9212 (0.0095)	0.0707 (0.0080)	0.0082 (0.0022)[3.69]
TNote	0.9571 (0.0061)	0.0389 (0.0046)	0.0040 (0.0028)[1.41]
Equities			
CAC	0.9524 (0.0056)	0.0392 (0.0041)	0.0084 (0.0023)[3.60]
DAX	0.9547 (0.0053)	0.0374 (0.0039)	0.0079 (0.0023)[3.44]
Nikkei	0.9097 (0.0143)	0.0584 (0.0078)	0.0319 (0.0087)[3.65]
S&P 500	0.9376 (0.0082)	0.0524 (0.0063)	0.0100 (0.0032)[3.12]
FTSE	0.9463 (0.0066)	0.0454 (0.0051)	0.0082 (0.0023)[3.54]

$\hat{v} = 10.05$ (0.3832) , $\hat{\phi}_1 = 0.9810$ (0.0012), $\hat{\phi}_2 = 0.0107$ (0.0005)

Note: Standard errors of the estimates are given in round brackets,

t-statistics are given is square brackets. λ_{i1} and λ_{i2} are the asset-specific

volatility parameters. ϕ_1 and ϕ_2 are the common conditional

correlation parameters.
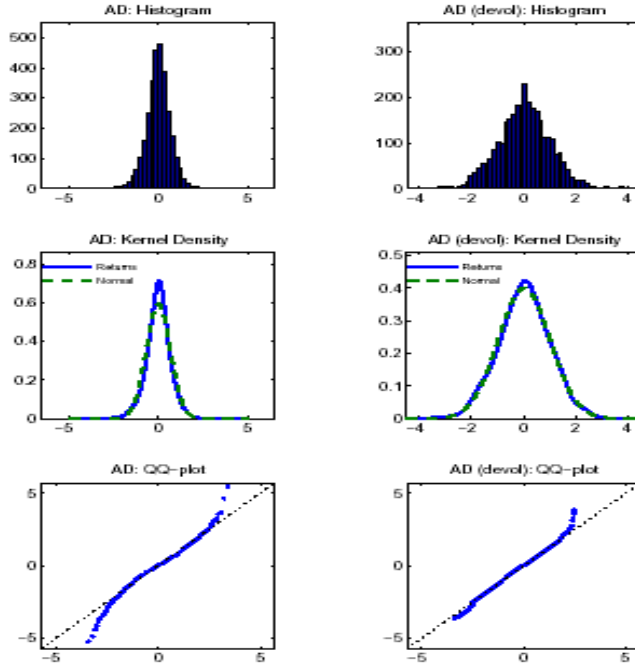
Figure 1: Australian dollar futures returns (simple and devolatized) 02-Jan-1995 to 31-Dec-2006
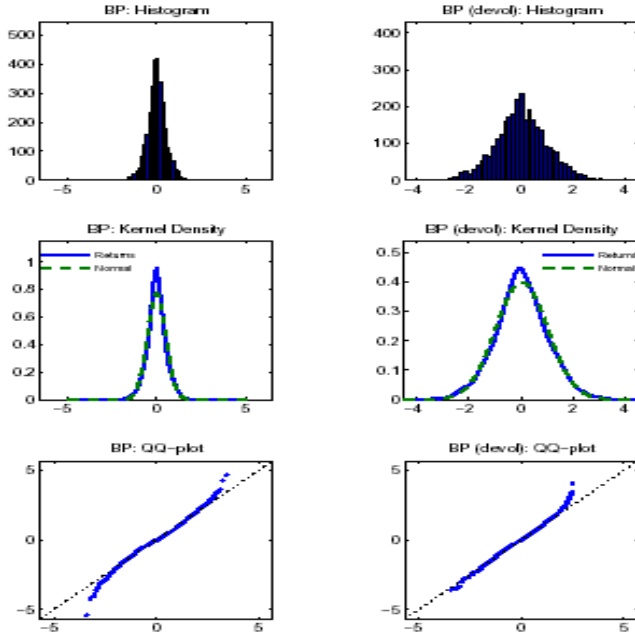
Figure 2: British pound futures returns (simple and devolatized) 02-Jan-1995 to 31-Dec-2006

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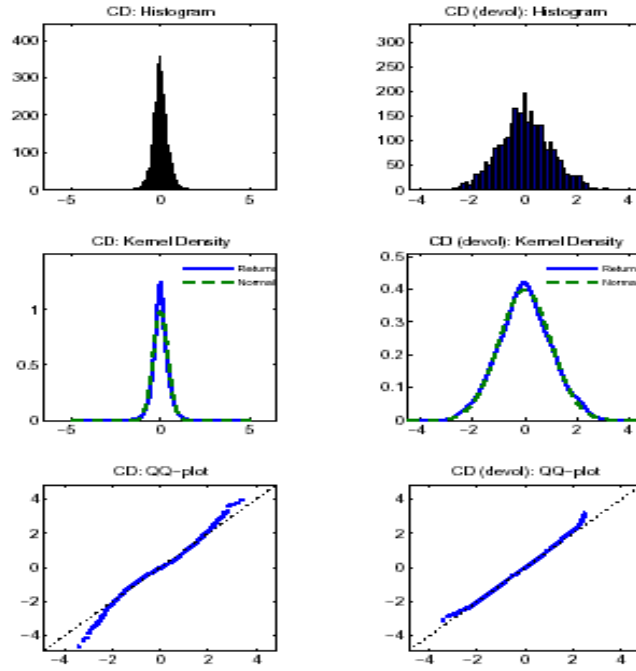
Figure 3: Canadian dollar futures returns (simple and de-volatized) 02-Jan-1995 to 31-Dec-2006
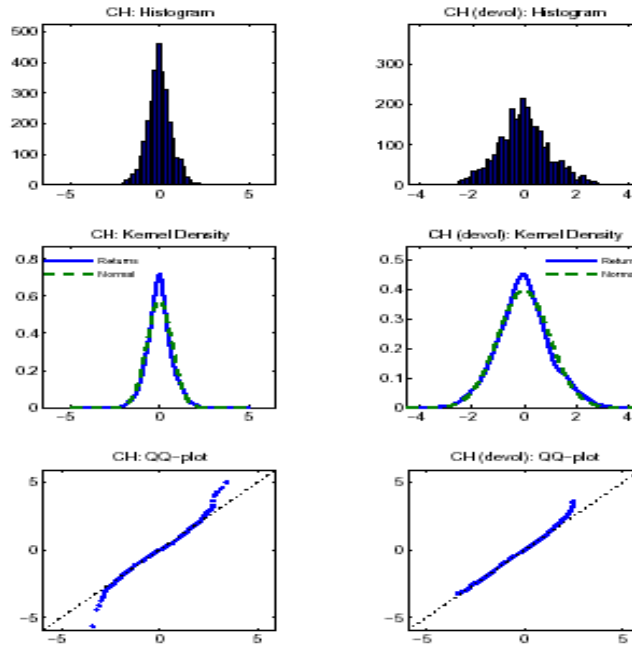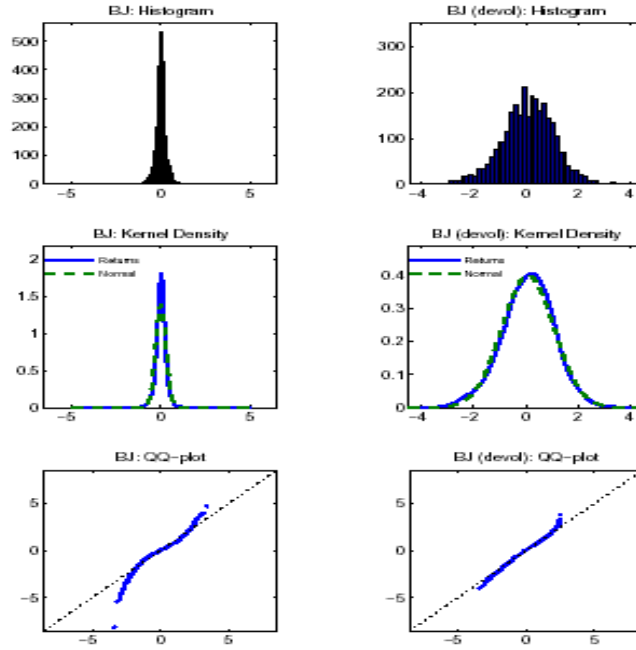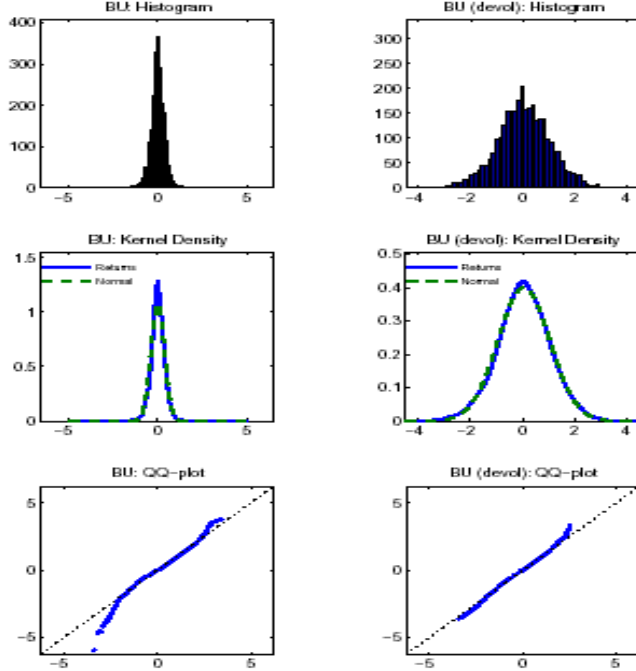

Figure 4: Swiss franc futures returns (simple and de-volatized) 02-Jan-1995 to 31-Dec-2006
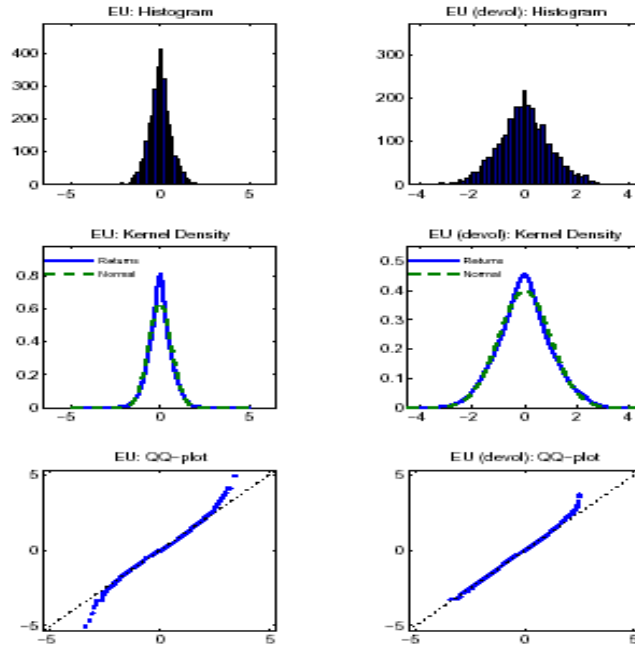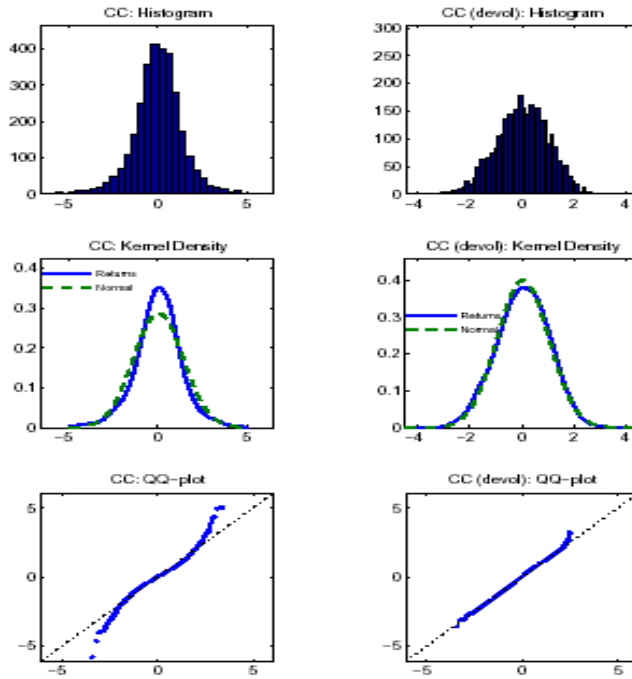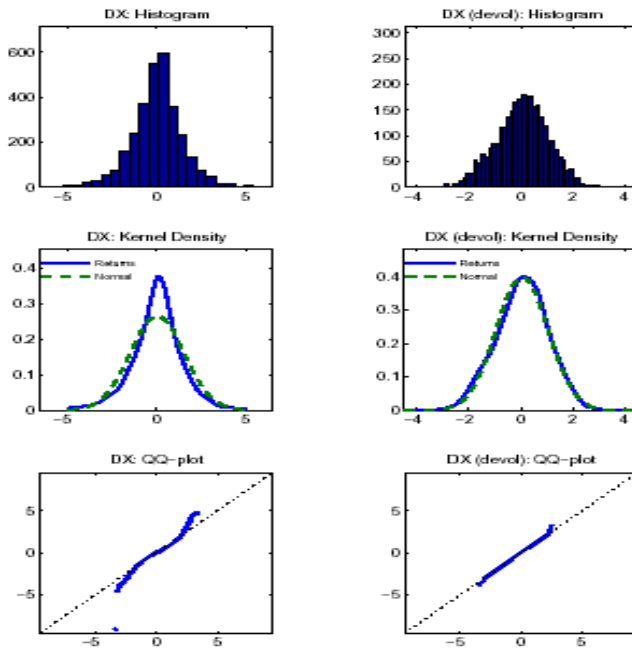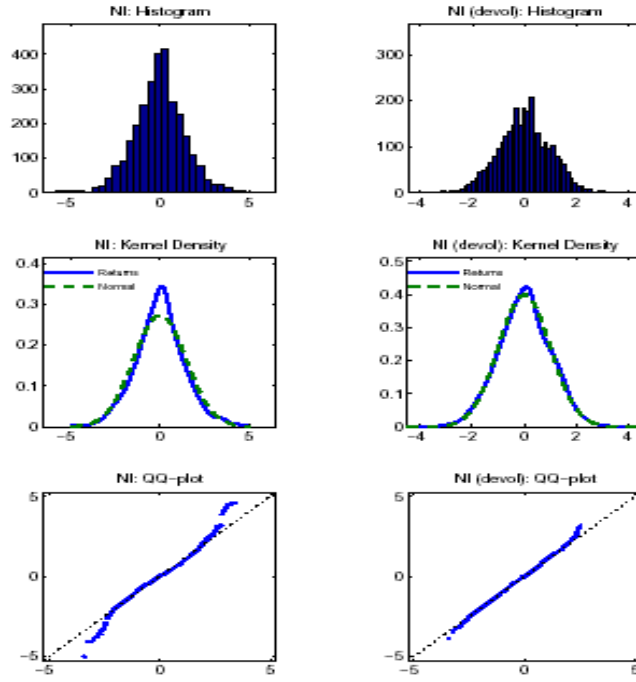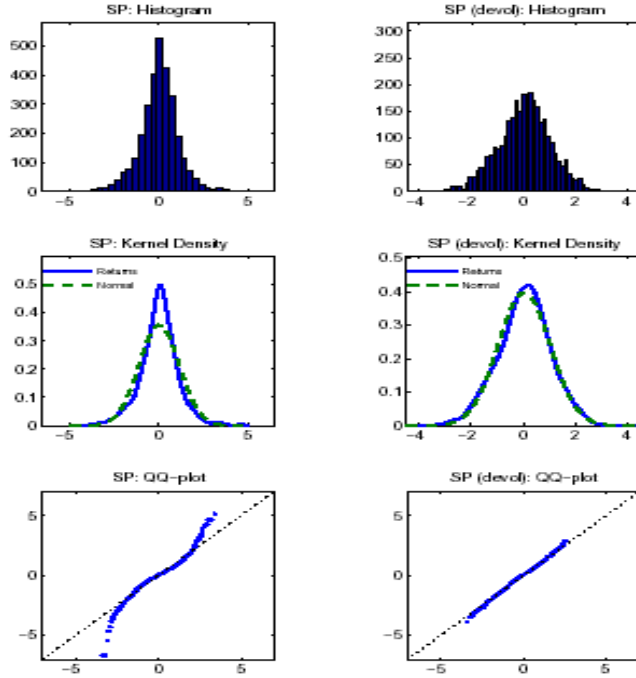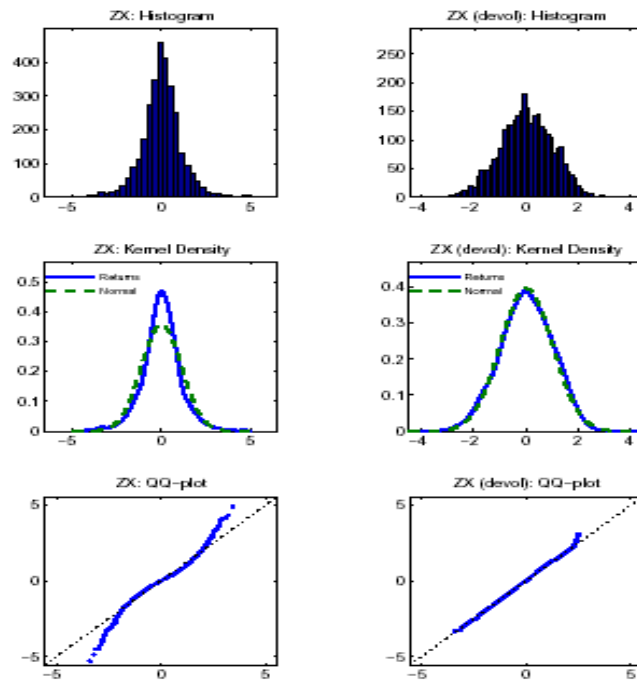
Figure 5: Euro futures returns (simple and de-volatized) 02-Jan-1995 to 31-Dec-2006
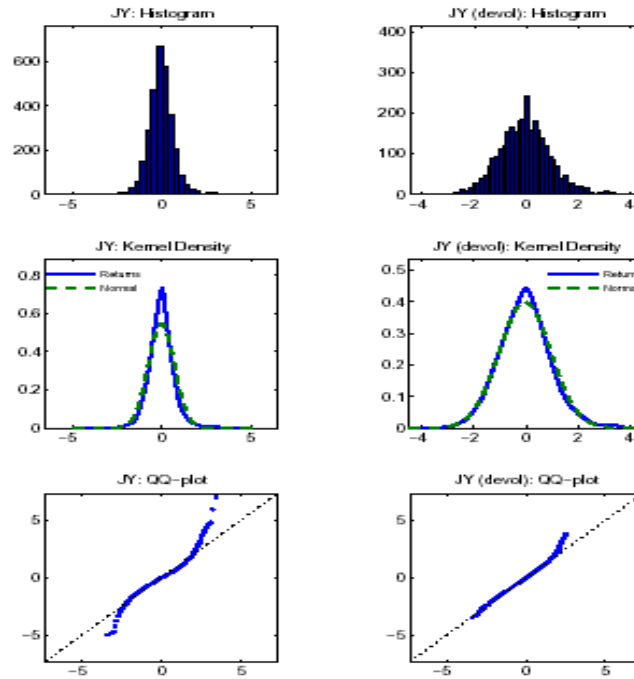
Figure 6: Japanese yen futures returns (simple and de-volatized) 02-Jan-1995 to 31-Dec-2006
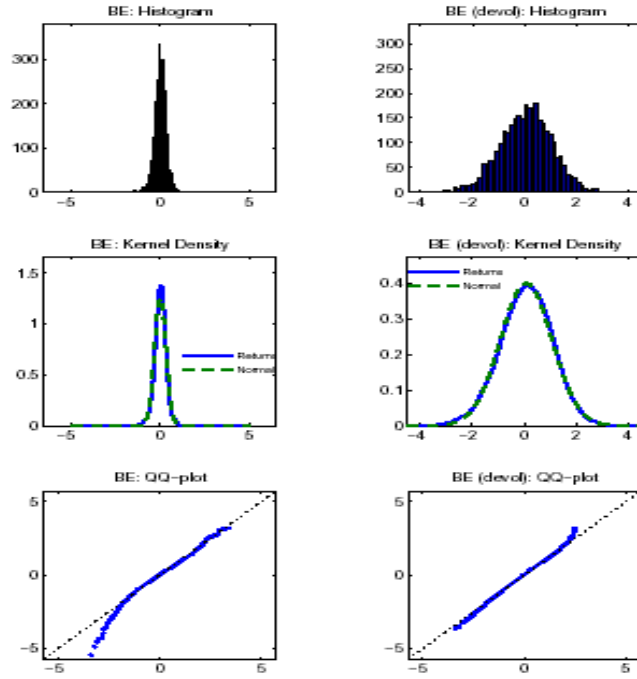
Figure 7: Bunds returns (simple and de-volatized) 02-Jan-1995 to 31-Dec-2006
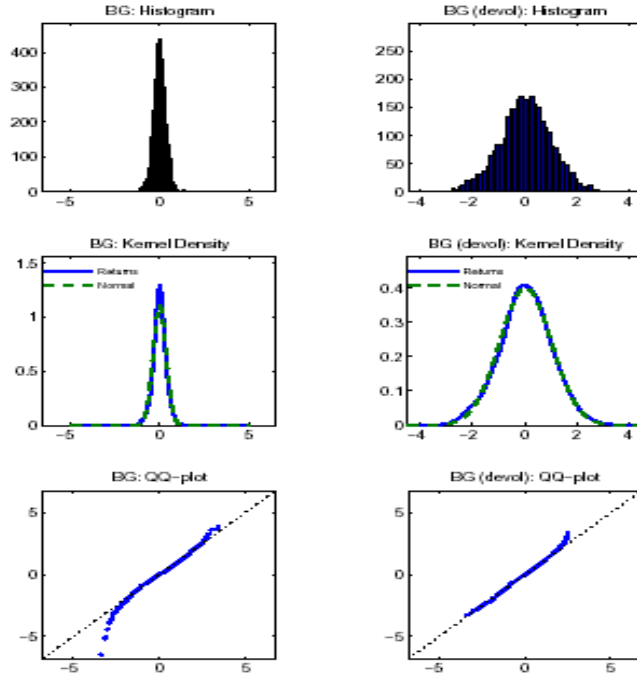
Figure 8: Gilt returns (simple and de-volatized) 02-Jan-1995 to 31-Dec-2006

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Figure 9: JGB returns (simple and de-volatized) 02-Jan-1995 to 31-Dec-2006

Figure 10: US Treasury note returns (simple and de-volatized) 02-Jan-1995 to 31-Dec-2006

Figure 11: CAC returns (simple and de-volatized) 02-Jan-1995 to 31-Dec-2006

Figure 12: DAX returns (simple and de-volatized) 02-Jan-1995 to 31-Dec-2006

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Figure 13: Nikkei returns (simple and de-volatized) 02-Jan-1995 to 31-Dec-2006

Figure 14: S&P 500 returns (simple and de-volatized) 02-Jan-1995 to 31-Dec-2006

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Figure 15: FTSE returns (simple and de-volatized) 02-Jan-1995 to 31-Dec-2006
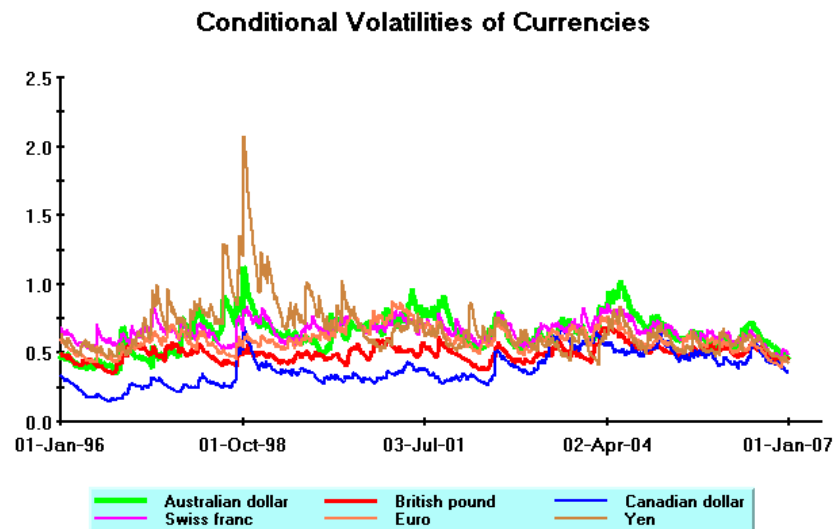
Figure 16: Conditional volatilities of currency futures returns

Conditional Volatilities of Equity Index Futures
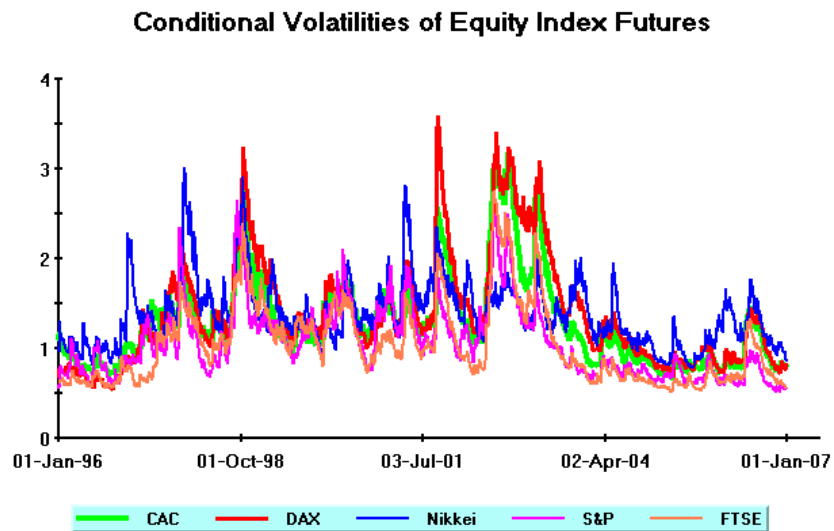
Figure 17: Conditional volatilities of equity futures returns

Conditional Volatilities of Bond Futures
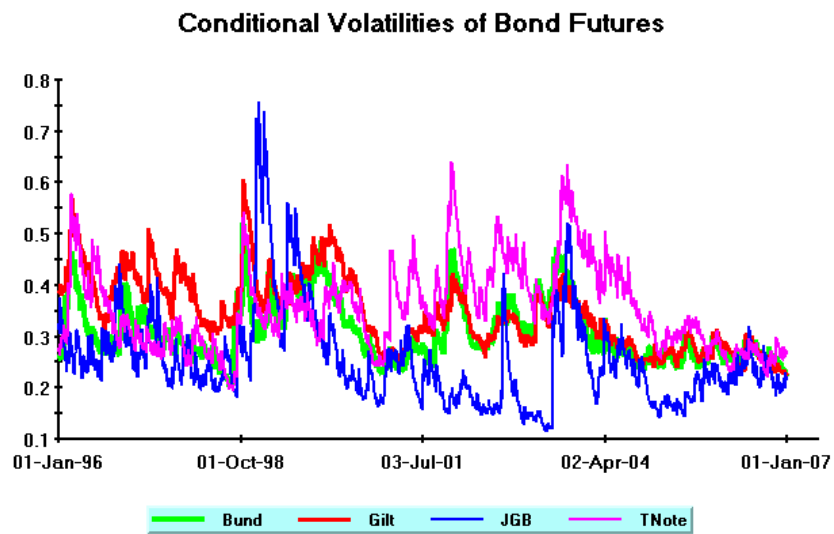
Figure 18: Conditional volatilities of bond futures returns

Conditional Correlations of Euro and other Currencies
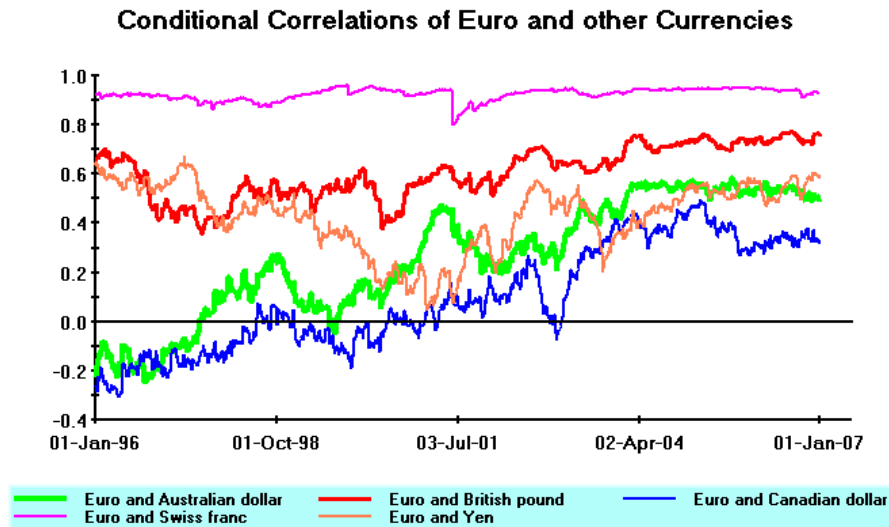
Figure 19: Conditional correlations of Euro with other currency futures returns

Conditional Correlations of S&P Futures with other Equity Index Futures
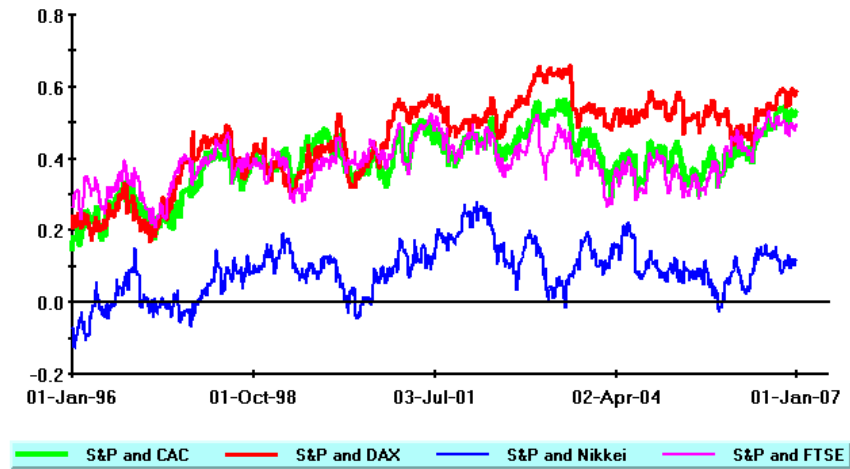
Figure 20: Conditional correlations of S&P 500 with other equity futures returns

Conditional Correlations of TNote Bond Futures with other Bond Futures
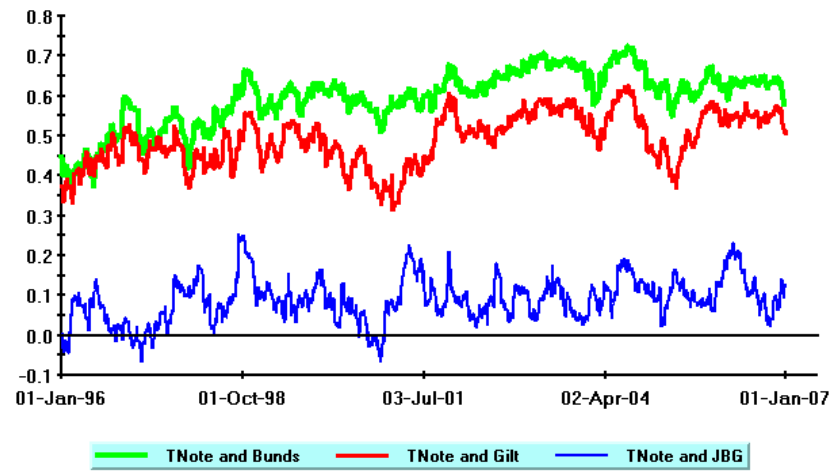
Figure 21: Conditional correlations of TNote with other bond futures returns

Conditional Correlations of Bonds and Equity Futures
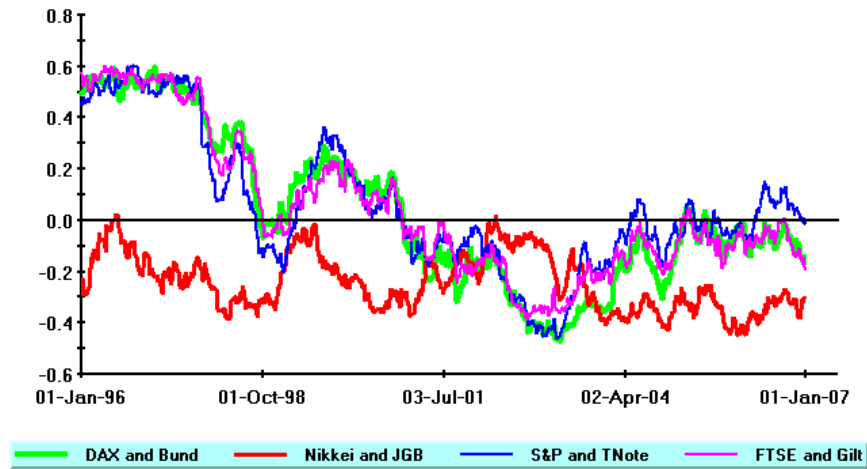
Figure 22: Conditional correlations of bond and equity futures returns

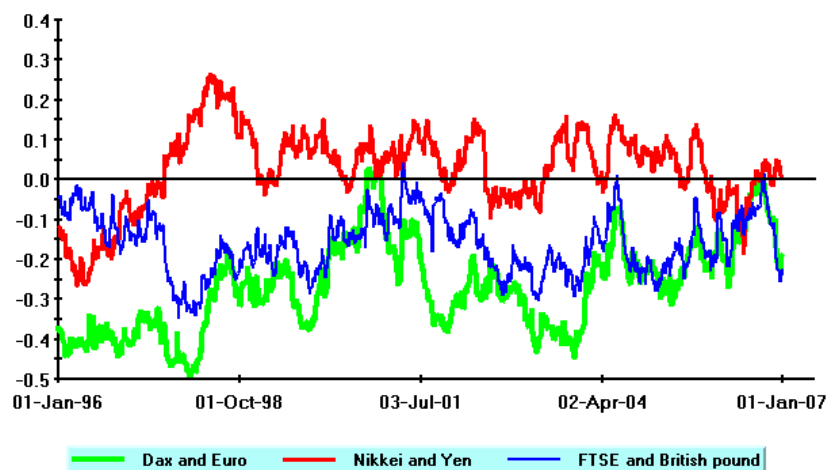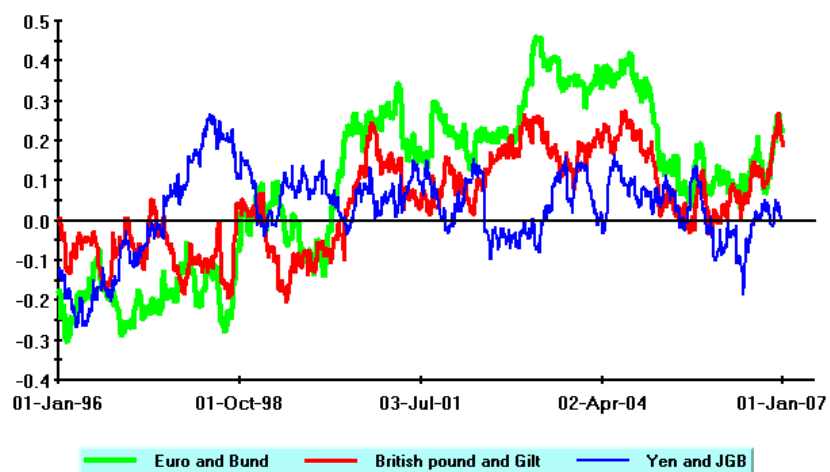Figure 23: Conditional correlations of equity and currency futures returns

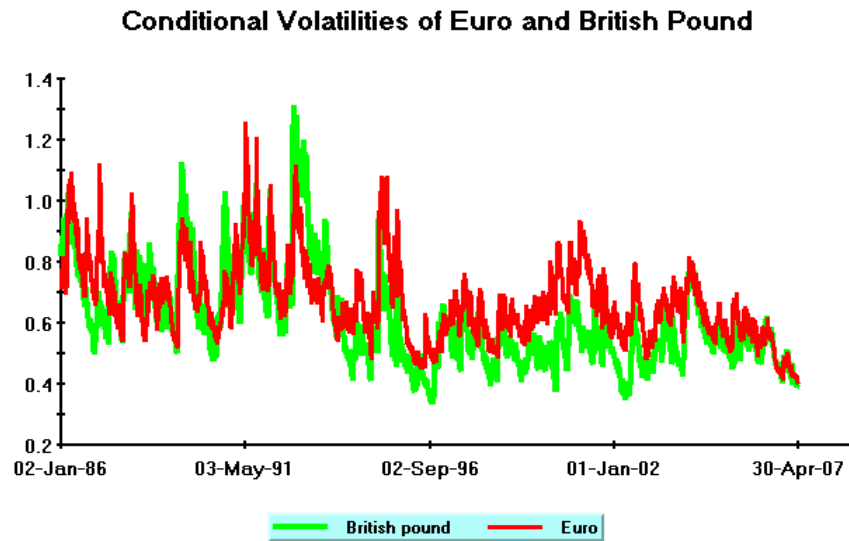Figure 24: Conditional correlations of bond and currency futures returns

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Figure 25: Conditional volatilities of euro and British pound over the 1986-2007 period

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