# Factor demand linkages and the business cycle: interpreting aggregate fluctuations as sectoral fluctuations.

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#### Abstract

This paper investigates the drivers of industry and aggregate fluctuations. We model the dynamics of a panel of highly disaggregated manufacturing sectors. This allows us to consider directly the linkages between sectors typical of any production system, in a framework where the sectors are fully heterogeneous. We establish that these features are fundamental for the propagation of the shocks in the aggregate economy. Aggregate fluctuations can be accounted for by small industry specific shocks. Moreover, a contemporaneous technology shock to all sectors in the economy, i.e. an aggregate technology shock, implies a positive response in both output and hours at the aggregate level. When this intersectoral channel is neglected we find a negative correlation as with much of the literature. This suggests that the standard technology driven Real Business Cycle paradigm is a reasonable approximation of a more complicated model featuring heterogenously interconnected sectors.

JEL Classification: E20, E32, C31, C51

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# 1 Introduction<sup>1</sup>

This paper investigates the drivers of industry and aggregate fluctuations. With few exceptions<sup>2</sup>, the literature has largely approached this question using aggregate time series. In this paper instead we take seriously the real business cycle paradigm (Long and Plosser, 1983, 1986) and consider an aggregate model featuring sectoral shocks and sectoral interactions. In a multisectoral model intermediate goods used in one sector are produced in other sectors, which themselves use intermediate goods produced in yet other sectors. In other words, the use of intermediate goods leads to a whole chain of intersectoral linkages that we have to take into account. The presence of an intermediate input channel is emphasized by Hornstein and Praschnick (1997) and recently analysed in detail in Kim and Kim (2006). Shea (2002) and Conley and Dupor (2003) provide evidence for the importance of factor demand linkages and sectoral complementarities in industry comovements and aggregate fluctuations. Here we consider explicitly the empirical relevance of this channel. Modelling aggregate time series directly implies that sectors are relatively homogeneous and most importantly that sectoral interactions among sectors is of second order importance for aggregate fluctuations<sup>3</sup>. Instead we model the dynamics of a panel of highly disaggregated manufacturing sectors. This allows us to consider directly the linkages between sectors typical of any production system, in a framework where the sectors are fully heterogeneous<sup>4</sup>. We show how these issues can be analyzed consistently, allowing for both aggregate and idiosyncratic sectoral shocks and sectoral interactions. We establish that these features are important for the propagation of shocks in the aggregate economy. Furthermore, we consider the implications of our results for the relative roles played by aggregate and sectoral shocks in explaining aggregate fluctuations.

We assume that industry dynamics are mainly driven by technology and non-technology shocks and use long run restrictions in a 'structural' VAR to identify the shocks. The main novelty is that all sectors in the economy are related by factor demand linkages captured by the input output matrix. According to the aggregation theorem in Blanchard and Quah (1989, p.670), the effect of the intermediate goods channel or the effect of aggregate shocks is correctly captured by

<sup>&</sup>lt;sup>1</sup>We would like to acknowledge the comments of Alex Al-Haschimi, Elisa Tosetti and Robert Vigfusson. Any remaining errors are the responsibility of the authors.

<sup>&</sup>lt;sup>2</sup>See Chang and Hong (2006) and to some extent Basu, Fernald and Kimball (2006).

<sup>&</sup>lt;sup>3</sup>See Dupor (1999) for a discussion of the theoretical conditions under which the latter hypothesis is verified, and Hovarth (1998) for a critique.

<sup>&</sup>lt;sup>4</sup>Swanson (2006) shows that the heterogeneity of agents in the economy might itself be a source of amplification of shocks hitting the economy. Indeed, different responses to a common signal result in a change in the relative competitiveness of sectors, therefore variation in "relative productivity" and "relative prices" can be a source of instability in the model.

the standard bivariate procedure applied to each sector separately, if and only if the response of a sector to other sectors' shocks is the same as the response of a sector to its own idiosyncratic sectoral shocks<sup>5</sup>. Therefore, for each sector we identify technology and non-technology shocks, where these shocks alone can explain the industry and aggregate fluctuations only if all sectors are analyzed contemporaneously, i.e. not in isolation. Nevertheless, we provide evidence that shocks appear to be idiosyncratic at the industry level, and the linkages between sectors is the key mechanism for explaining the observed comovement across sectors and aggregate fluctuations.

A common way of evaluating technology driven business cycles models is to look at the impulse response functions following a shock to technology. A widespread finding is the puzzling result that a technology shock is followed by a fall in the labour input<sup>6</sup>. This observation is inconsistent with the standard business cycle model, which instead implies positive comovement between output, productivity and the labor input. This has led many to conclude that the technology driven real business cycle hypothesis is "dead" (Francis and Ramey, 2005). Others consider various modifications of the original model. Gali (1999) suggests that the paradigm needs to be changed in favour of a business cycle model driven instead by preference shocks and featuring sticky prices. Christiano et al. (2003, 2006) argue that the negative response of the labour input use to a technology shock might be the result of a misspecification of the original model and specifically, the mistreatment of labour input in the empirical model. Indeed, they find that the effect of a technology shock on the labour input clearly depends on the treatment of the labour input; if this is included in levels the puzzling result disappears. Nevertheless, they find that technology shocks account only for a minimal part of aggregate fluctuations.

We make 3 contributions to the literature. First, we find that it is essential to model interactions between sectors that arise from the outputs of some sectors being inputs into other sectors<sup>7</sup>. A contemporaneous technology shock to all sec-

<sup>&</sup>lt;sup>5</sup>This view has been challanged empirically by Fisher (2006) who shows the qualitative difference between sector neutral and investment specific technology shocks. The sectoral specification we consider in this paper is fully consistent with the presence of investment specific shocks which would correspond to a contemporaneous shock to all the investment goods sectors in the economy. See Hansen and Prescott (1993) for a two sector model featuring investment specific shocks.

<sup>&</sup>lt;sup>6</sup>There is a fast growing literature regarding the dynamic effects of technology shocks. See, for example, Basu (1998), Basu et al. (2002, 2006), Chang and Hong (2006), Chari et al. (2005), Christiano et al. (2003, 2006), Fernald (2005), Francis and Ramey (2005), Gali (1999), Gali et al. (2003), Liu and Phaneuf (2006), Ramey (2004) and Vigfusson (2004), Wang and Wen (2007) among many others.

Gali and Rabalnal (2004) provide a comprehensive survey of the existing literature, while Erceg, Guerrieri and Gust (2005) and Chari, Kehoe and McGrattan (2004) present a critique.

<sup>&</sup>lt;sup>7</sup>Of course this is also the point of Horvath (2000). He describes a multisector dynamic

tors in the economy, equivalent to an aggregate technology shock, then implies a positive aggregate response in both output and hours. However, when the sectoral interactions are ignored we find a negative correlation as with much of the literature. This suggests that the standard technology driven Real Business Cycle paradigm is a reasonable approximation of a more complicated model featuring heterogeneously interconnected sectors.

Secondly, these additional channels for the transmission of shocks are not only qualitatively, but also quantitatively important. Sectoral interactions prove to be an important amplifier of sector specific shocks. Aggregate fluctuations can be generated by smaller idiosyncratic shocks at the industry level. Technology shocks appear to account for most sectoral fluctuations; but most significantly shocks to other sectors (transmitted though sectoral interactions) are fundamental for tracking individual sectoral cycles. Thirdly, our analysis suggests, once sectoral interactions are accounted for, that technology and non-technology shocks seems to be equally important in explaining aggregate economic fluctuations. Interestingly our results tend to show that the role of the technology shocks has gained in importance since the mid 1980s.

Our approach is to model each industry as a 'structural' VAR, identifying the technology shock as a permanent effect on productivity. We then construct an industry VAR (SecVAR) using the GVAR approach of Pesaran et al (2004)<sup>8</sup> and link sectors specifically through the input output matrix. This allows us to distinguish between the contribution made by technology shocks to particular industries and the overall effect amplified by sectoral interactions.

The remainder of the paper is organized as follows. In section 2, we briefly review the standard bivariate procedure for disentangling technology and non-technology shocks. Then, we show how to model heterogenous sectors consistently, when these are related by production linkages. We employ a structural VAR but applied to industrial sectors. Section 3 describes the data, and discusses some of the theoretical motivation for the specification of the model. Section 4 investigates the origins of shocks at the industry level. In section 5, we report estimates of the effects of technology on employment and disentangle the different contributions to the aggregate outcome. Section 6 reports our findings regarding the important role played by sectoral interactions captured by the input-output matrix in explaining both sectoral and aggregate fluctuations. Finally section 7 contains concluding remarks.

general equilibrium model which he calibrates to US industry data annul finds that aggregate fluctuations are driven by independent sectoral shocks. In this paper we provide a more direct empirical result.

<sup>&</sup>lt;sup>8</sup>See also Dees et al (2007a, 2007b).

# 2 Identifying the Effects of a Permanent Technology Shock

We follow Galì (1999), Galì, Lopez-Salido, and Valles (2002) and Francis and Ramey (2005) and adopt the identifying assumption that the only type of shock that affects the long-run level of labour productivity is a permanent shock to technology. This assumption is satisfied by a large class of standard business cycle models. See, for example, the real business cycle models in King, Plosser and Rebelo (1988), King, Plosser, Stock and Watson (1991) and Christiano and Eichenbaum (1992) which assume that technology shocks are a difference stationary process<sup>9</sup>. Reduced form time series methods, in conjunction with the long run identifying assumption are used to disentangle two fundamental (orthogonal) disturbances: technology and non-technology shocks.

The linear approximation to the equilibrium of any economic model has a moving average representation. Consider the following simple bivariate system<sup>10</sup>:

$$\varkappa_t = \mathbf{C}(L) \begin{bmatrix} \varepsilon_t^T \\ \varepsilon_t^{NT} \end{bmatrix}, \tag{1}$$

with  $E(\varepsilon_t'\varepsilon_t) = \Omega_{\varepsilon}$ , where  $\Omega_{\varepsilon}$  is a diagonal matrix, and  $E(\varepsilon_t'\varepsilon_s) = \mathbf{0} \ \forall t \neq s$ .  $\varkappa_t = [\Delta z_t, y_t]'$ , where  $z_t$  denotes the log of labour productivity<sup>11</sup>,  $y_t$  denotes the log of labour input or the change in log of labour input,  $\varepsilon_t^T$  denotes the technology shock and  $\varepsilon_t^{NT}$  denotes the non-technology shock.  $\mathbf{C}(L)$  is a polynomial in the lag operator, L. The treatment of the low frequency component of labour input, in the next section, has recently gained attention in the literature. The issue is intrinsically related to whether non-technology shocks are level or difference stationary. We discuss this issue later. The assumption identifying the technology shock implies that  $\mathbf{C}(1)$  is lower triangular  $(C^{12}(1) = \sum_{j=0}^{\infty} C_j^{12} = 0)^{12}$ .

The standard implication of real business cycle models is that the technology shock is the only variable to affect labour productivity,  $z_t$ , in the long run, i.e. is

<sup>&</sup>lt;sup>9</sup>Gali (1999) discuss the assumptions which are jointly sufficient to yield the identifying restrictions used. Notice that increasing returns, capital taxes, and some models of endogenous growth would all imply that non-technology shocks can change long-run labour productivity, thus invalidating the identifying assumption. Francis and Ramey (2005) investigates the distortionary effect that may derive from the exclusion of the permanent effect of capital taxes, they find that this does not affect the final inference of the simpler bivariate specification.

<sup>&</sup>lt;sup>10</sup>The constant terms are suppressed here for expositional convenience.

<sup>&</sup>lt;sup>11</sup>Chang and Hong (2006) suggests that total factor productivity should be used instead of labor productivity.

<sup>&</sup>lt;sup>12</sup>Here, we are implicitely assuming that the MA is fundamental; see Lippi and Reichlin (1993) for a discussion of this assumption.

the only variable that has a permanent effect, written as:

$$\lim_{h \to \infty} \left[ E_t(z_{t+h}) - E_{t-1}(z_{t+h}) \right] = f(\varepsilon_t^T)$$

This specification has been widely used in the literature using aggregate data, that is for business cycles models where the unit root in productivity originates solely in the technology shock.

The analysis of aggregate data highlights the properties of a business cycle model driven by a single aggregate shock, as in the original work of Kidland and Prescott (1983) and King et al. (1999). If the analysis is conducted at the sectoral level, more care is needed with the specification of the model. Specifically, the relations between the sectors in the economy need to be taken into consideration. In fact, another branch of the RBC literature has sought mechanisms at the sectoral level, whereby industry specific shocks can be amplified through sectoral linkages to the macroeconomic level. Following the pioneering work of Long and Plosser (1983) and 1987), RBC models have been generalized into a multi-sectoral environment where industry specific shocks are propagated through sectoral inter-dependencies which can generate business cycle fluctuations. The idea was revitalized by Horvath (1998, 2000) who shows how the input output structure of the economy is a good way of capturing the relations between sectors in the economy. Also, Conley and Dupor (2003) and Shea (2002) emphasize sectoral complementarity as the main mechanism of propagation of sectoral shocks at the aggregate level, the main idea being intrinsically related to the original result of Jovanovic (1987).

In the following section we discuss the implications of industry interdependence for the econometric analysis of the effect of technology shocks on hours.

# 2.1 The representation of the system with industry shocks and comovement

The simplest version of the RBC model assumes that the dynamics of the business cycle (and the sectoral comovements in business cycles) are driven by aggregate common shocks to the economy. At the same time, Long and Plosser (1986) showed that aggregate business cycles can be generated by industry specific shocks, where industries are inter-connected through input-output relations. Evidence in favour of the latter hypothesis is documented in Conley and Dupor (2003), where comovements are driven by strong complementarities between sectors. Horvath (1998) has explored the possibility that comovements originate directly from input-output relations<sup>13</sup>.

<sup>&</sup>lt;sup>13</sup>The two views are not mutually exclusive. It is possible that aggregate business cycles and sector comovements are the result of amplified sector shocks and aggregate shocks (e.g. monetary, fiscal and oil price shocks).

The discussion above suggests that when the analysis is conducted at the industry level, the identification of technology and non-technology shock needs to be modelled in a way which includes at the same time all the sectors in the economy. Specifically, the bivariate model of equation (1) can be specified as

$$\mathbf{x}_t = \mathbf{B}(L)\mathbf{u}_t,\tag{2}$$

with  $\varkappa_{it} = [\Delta z_{it}, y_{it}]'$  and  $\mathbf{x}_t = [\varkappa'_{1t}, ..., \varkappa'_{Nt}]'$ , similarly  $\varepsilon_{it} = [\varepsilon^T_{it}, \varepsilon^{NT}_{it}]$  and  $\mathbf{u}_t = [\varepsilon'_{1t}, ..., \varepsilon'_{Nt}]'$ . The specification in (2) does not impose any particular restriction on the nature of the identified shocks. Specifically, the shocks at the industry level can either be fully idiosyncratic or need to be decomposed into an aggregate component and an industry specific component. Whether the comovement between sectors and the aggregate business cycle is driven by aggregate shocks simultaneously affecting all the sectors in the economy is an empirical question that is addressed in section 4.

A similar specification has been used by Chang and Hong (2006) to recover (industry specific) technology shocks in order to study the effect of technology shocks on employment at different levels of disaggregation. However, Chang and Hong (2006) neglect the role of factor demand linkages in the production process, or any other interaction between sectors. Each sector is analyzed in isolation, i.e. each  $\mathbf{B}_j$  in  $\mathbf{B}(L)$  is composed of block diagonal matrices. This is a very strong restriction, as it neglects the widely documented comovement between the sectors in the economy. Nevertheless, estimating (2) without any restriction on the matrix polynomial  $\mathbf{B}(L)$ , is infeasible for any reasonably large number of industries.

A consistent way of analyzing (2), taking into account sector comovements ,is to estimate each specific sector model as:<sup>14</sup>

$$\mathbf{A}_{i0} \boldsymbol{\varkappa}_{it} = \mathbf{C}_{i0} \boldsymbol{\varkappa}_{it}^* + \mathbf{A}_{i1} \boldsymbol{\varkappa}_{it-1} + \mathbf{C}_{i1} \boldsymbol{\varkappa}_{it-1}^* + \boldsymbol{\varepsilon}_{it}, \tag{3}$$

with  $\varkappa_{it} = \left[ \Delta z_{it}, \ y_{it} \right]'$ ,  $\varkappa_{it}^* = \left[ \Delta z_{it}^*, \ y_{it}^* \right]'$  and  $\varkappa_{it}^*$  are appropriate industry specific cross sectional averages of the original variables in the system and reflect interactions between sectors. We construct the industry cross sectional average to capture factor demand linkages between manufacturing sectors in the economy<sup>15</sup>. Specifically, the averages  $\varkappa_{it}^* = \left[ \sum_{j=1}^N \omega_{ijt} \Delta z_{jt}, \ \sum_{j=1}^N \omega_{ijt} y_{jt} \right]'$ , where the (possibly time varying) weights  $\omega_{ijt}$  correspond to the share of commodities j

<sup>&</sup>lt;sup>14</sup>For ease of exposition we focus on the simple VARX(1,1) without any deterministic component. But the discussion applies equally to a more general formulation. In principle an appropriate number of lags of the endogenous and weakly exogenous variables are included such that the error term (i.e. the identified shocks) are serially uncorrelated.

Given the short annual time series we choose a single lag specification in the empirical section. This is also consistent with the Akaike and Schwarz information criteria for most of the sectors.

<sup>&</sup>lt;sup>15</sup>Notice that as long as the weights satisfy the usual granularity conditions, for  $N \to \infty$ , the

used as input material in sector i. Long and Plosser (1983, 1987), Dupor (1999) and Horvath (1998) derive similar specifications from microfoundations, where the weighting matrix is constructed directly from the input-output matrix<sup>16</sup>. Shea (2002) and Conley and Dupor (2003) find strong support for the importance of input-output linkages. One of the main criticisms in Chang and Hong (2006) is centered on the incorrect use of labour productivity in the system instead of a direct measure of total factor productivity. Nevertheless, if shocks to material and capital inputs reflect supply and demand shocks in the input/capital producing sectors, the inclusion of aggregate variables reflecting factor demand linkages should reduce the perverse effect of the wrong choice of variables emphasized in Chang and Hong (2006).

To estimate the dynamic effect of technology shocks we follow the procedure outlined in Shapiro and Watson (1988), and discussed in Christiano et al (2003) in the context of the dynamic effect of a technology shock. If we apply the same restriction as in (1) for each sector separately, we are able to identify sector specific shocks, such that  $E(\varepsilon_t'\varepsilon_t) = \Omega_{\varepsilon}$  in (3) is a diagonal matrix. However, in a multisectoral context we need to be more careful and consider the interrelation between sectors. Specifically, to identify the technology shocks, we restrict labour productivity in the long run to be affected by only the technology shocks. For each generic sector i we apply this restriction to shocks originating in the same sector i and shocks originating in other sectors that affect sector i through sectoral interactions. The contemporaneous relations between the sectoral specific variables and the aggregate variables can be estimated consistently as long as the artificial aggregate variables in the system are weakly exogenous, a condition satisfied for  $N \to \infty^{17}$ . Therefore, the restriction of the effect of other sectors shocks through the input channel is an overidentifying restriction<sup>18</sup>. The presence of the industry specific cross sectional averages enlarge the set of instrument that can be used to identify the technology shocks<sup>19</sup>, therefore addressing some of the concerns raised in Christiano et al. (2003) about possible biases arising from the use of weak instruments.

It is possible to recover the original specification in (2) by stacking the sector

cross section average can be thought of as aggregate shocks hitting the economy (see Pesaran, 2006, Forni and Reichlin, 1996). Later in the paper we carry out some robustness checks to establish whether our results are sensitive to the inclusion of some alternative 'aggregate' shocks.

<sup>&</sup>lt;sup>16</sup>Long and Plosser (1983) and Hovarth (1998) consider different time assumptions in production, such that the derived policy fuction will include the effect of the intermediate imput channel either contemporaneously or with a lag.

<sup>&</sup>lt;sup>17</sup>See Pesaran et al (2004) for a proof.

<sup>&</sup>lt;sup>18</sup>In principle, this additional restriction can be tested. Notice that the imposition of this additional restriction does not affect the qualititive results presented in the empirical section.

<sup>&</sup>lt;sup>19</sup>In appendix A we show that  $y_{it-1}^*$  can be used as an additional instrument when the long run restriction of other sectors non technology shocks has been imposed.

specific models outlined above in (3). For example, for a VAR(1), the model can be rewritten as

$$\mathbf{G}_0 \mathbf{x}_t + \mathbf{G}_1 \mathbf{x}_{t-1} = \mathbf{u}_t, \tag{4}$$

with  $E(\mathbf{u}_t'\mathbf{u}_t) = \mathbf{\Omega}_u$  and  $E(\mathbf{u}_t'\mathbf{u}_s) = \mathbf{0} \ \forall t \neq s$ . The identifying restriction to disentangle technology and non technology shocks is that  $\mathbf{\Omega}_{\varepsilon}$ , the *i*-th diagonal block of the covariance matrix  $\mathbf{\Omega}_u$ , in (3) is a diagonal matrix. However, the off diagonal elements of  $\mathbf{\Omega}_u$  are left unrestricted. In principle,  $\mathbf{\Omega}_u$  is a diagonal matrix only if the identified shocks are fully idiosyncratic also between sectors;  $\mathbf{\Omega}_u$  is not diagonal either in the presence of common shocks to the economy or as a result of local interactions. Conley and Dupor (2002) focus on this issue<sup>20</sup>. The matrix of coefficients are

$$egin{array}{lll} \mathbf{G}_{i0} & = & \left(egin{array}{c} \mathbf{A}_{ij}, & \mathbf{\overset{-C}{c}}_{ij} \ \end{array}
ight) & \mathbf{\overset{w}{\underbrace{w}}}_{it}, \ \mathbf{G}_{i1} & = & -\left(egin{array}{c} \mathbf{A}_{ij}, & \mathbf{C}_{ij} \end{array}
ight) \mathbf{W}_{it}, \end{array}$$

where n is the number of variables in the system, 2 in the bivariate case, and N is the number of industries analyzed. The weighting matrix is constructed, as outlined in Pesaran et al. (2004), such that for each sector this selects the sector specific variables and constructs the sector specific cross sectional averages in (3). The weights for the sectoral specific cross sectional averages reflect the factor demand linkages between sectors observable from the input-output matrix. Therefore, the polynomial matrix of the original model (2) can be recovered inverting  $\mathbf{G}(L)$ , where this is written as

$$\mathbf{G}(L) = \begin{bmatrix} \begin{pmatrix} \mathbf{A}_{10}, & -\mathbf{C}_{10} \end{pmatrix} \mathbf{W}_{1t} & -\begin{pmatrix} \mathbf{A}_{11}, & \mathbf{C}_{11} \end{pmatrix} \mathbf{W}_{1t}L \\ & \cdots & & \cdots \\ & \cdots & & \cdots \\ \begin{pmatrix} \mathbf{A}_{N0}, & -\mathbf{C}_{N0} \end{pmatrix} \mathbf{W}_{Nt} & -\begin{pmatrix} \mathbf{A}_{N1}, & \mathbf{C}_{N1} \end{pmatrix} \mathbf{W}_{Nt}L \end{bmatrix},$$

The SecVAR model analyzed in this section provides a further application of the GVAR model described in Pesaran et al. (2004) to the industry level. The difference is we consider a fully structural model, i.e. the contemporaneous relation is constrained not only between the endogenous and the weakly exogenous aggregate variables, but it also includes the contemporaneous relationships between the endogenous variables in the system<sup>21</sup>.

<sup>&</sup>lt;sup>20</sup>This issue is discussed in greater detail in section 4.

<sup>&</sup>lt;sup>21</sup>Specifically, the matrix of coefficients  $A_{i0}$ ,  $\forall i$ , are not constrained to be an identity matrix  $I_n$  as in the non structural formulation of the GVAR.

#### 3 Data and Estimation Results

The data used are collected from the NBER-CES Manufacturing Industry Database<sup>22</sup>. The database covers all 4-digit manufacturing industries from 1958 to 1996 (39 annual observations) ordered by 1987 SIC codes (459 industries)<sup>23</sup>. Labour input is measured as total hours worked<sup>24</sup>, while productivity is measured as real output divided by hours. Each variable is included as a log difference, where this choice is supported by panel unit root tests discussed below.

We match the dataset with the standard Input-Output matrix at the highest disaggregation, provided by the Bureau of Economic Activity<sup>25</sup>. Specifically, we make use of the "use" table, whose generic entry ij corresponds to the dollar value, in producers' prices, of commodity produced by industry j and used by industry i. This table is transformed into a weighting matrix by row standardization, such that each row sums to one. Note that before the transformation each row sum corresponds to total intermediate use, this information is likely to be recovered in the estimation of the coefficients  $C_{ij}$ , i = 1, ..., N and j = 0, 1, in  $(3)^{26}$ .

The input output "use" table clearly reflects demand factor linkages and therefore is a perfect measure of the intermediate input channel. Shea (2002) and Conley and Dupor (2003) use the same matrix to investigate demand factor linkages and sectoral complementarities. Ideally, we would need a time varying input-output matrix to take into consideration the change in the factor linkages between sectors in the economy. Nevertheless, we use the input-output matrix in 1987 as for this year only there exists an exact direct match between the classification of the NBER-CES database and the IO matrix from the BEA. Being in the middle of the estimation sample, 1987 should provide a reasonable approximation to the average

<sup>&</sup>lt;sup>22</sup>For a description of the data see Bartelsman, Becker, and Gray (1996). "NBER-CES Manufacturing Industry Database". Available at http://www.nber.org/nberces/nbprod96.htm.

<sup>&</sup>lt;sup>23</sup> As in other studies we exclude the "Asbestos Product" industry (SIC 3292) because the time series ends in 1993.

<sup>&</sup>lt;sup>24</sup>The results are robust to other measures of labor input, such as hours per worker and total employment. These additional results are available from the authors upon request.

<sup>&</sup>lt;sup>25</sup>The data are available at http://www.bea.gov/industry/io\_benchmark.htm. The original input output matrix when constrained to the manufacturing sector has 355 entries only. This means that the BEA original classification for the construction of the input output matrix aggregates more (4 digit SIC) sectors. As the entries in the original data correspond to the dollar value, in producers' prices, of each commodity used by each industry and by each final user, when more than one SIC sectors correspond to a single sector in the IO matrix we split the initial value equally between the SIC sectors.

The original IO matrix includes also within sectors trade. We exclude this from the calculation of the standardised weighting matrix.

<sup>&</sup>lt;sup>26</sup>The industry which has higher use of intermediate goods in production is likely to have higher values (in absolute value) of the coefficients associated with the "aggregate" components in (3).

input-output matrix throughout the sample.

Contemporaneous correlations of sectoral measures of activity are usually examined in order to quantify the comovement between sectors. Table 1 provides evidence of the cross sectional dependence between (the growth rate of) productivity and hours. The first row shows the average cross sectional correlation between sectors. In the second row is the cross-section dependence (CD) test of Pesaran (2004)<sup>27</sup>.

TABLE 1 ANALYSIS OF COMOVEMENT		
	Productivity	Hours
Average Cross Correlation	0.0554	0.1957
$^{\mathrm{CD}}$	109.28	386.01

Notes: The values in 'Average Cross Correlation' are the simple average of the pair-wise cross section correlation coefficients,  $\bar{\hat{\rho}} = [2/N(N-1)] \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \hat{\rho}_{ij}$  with  $\hat{\rho}_{ij}$  being the correlation coefficient of the variable of interest for the  $i^{th}$  and  $j^{th}$  cross section units.  $CD = \sqrt{2T/N(N-1)} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \hat{\rho}_{ij}$ , which tends to N(0,1) under the null hypothesis of no cross section dependence.

The results in Table 1 highlight the presence of substantial positive comovement of the variables, especially total hours worked. The CD test statistics clearly show that the cross correlations are statistically highly significant. Pesaran (2006) and Pesaran and Tosetti (2007), suggest that in this case the sectoral system should be estimated including cross sectional averages of the variables in the system, i.e. like (3), regardless of whether this is due to common shocks or to the links between sectors coming from factor demand linkages through the intermediate input channel.

## 3.1 Unit root versus stationary hours

Real business cycle theory interprets the bulk of macroeconomic fluctuations as the result of optimal responses to technology shocks. This in turn implies that there is a positive correlation between hours worked and labour productivity. The source of this correlation is a shift in the labour demand curve as a result of a technology shock combined with an upward sloping labour supply curve. However, there is a large literature suggesting that this is inconsistent with the data. Gali (1999), for example, uses the identifying assumption that innovations to technology are

 $<sup>^{27}</sup>$ The CD test is based on the average of pair-wise correlations, and tends to a standard normal distribution as  $N \to \infty$ . It does not require an *a priori* specification of a weighting matrix and is applicable to a variety of panel data models, including heterogeneous panels with structural breaks, with short time dimension, T, and large cross section, N (Pesaran 2004).

the only types of shock that have permanent effects on labour productivity, and finds that hours worked fall after a positive technology shock. There is an issue the literature concerning the representation of hours for extracting the technology shock. Galí (1999), using total rather than per capita hours, showed that his results did not hinge on the assumption of a unit root versus trend-stationary hours. However, Francis et al. (2005b) and Christiano, Eichenbaum and Vigfusson (2003) reach exactly opposite conclusions on the effect of a technology shocks when they consider different statistical properties of hours. Specifically, Francis et al. (2005b) fail to reject the unit root hypothesis in hours, but show that their conclusion (the negative permanent effect of technology shock on hours) is robust to other specification of hours, such as a quadratic trend. On the other hand, Christiano et al. (2004) argue that technology shocks lead to a positive response in hours when the technology shock is identified in a model in which hours per capita are assumed to be stationary. They reject the unit root hypothesis.

There may be a variety of reasons for a failure to reject the unit root hypothesis, including lack of power, shifts in mean, or misspecification of the low frequency deterministic components, or other forms of non-linearity<sup>28</sup>. In (3) we have not assumed any particular process for hours, indeed either specification can be accommodated (Pagan and Pesaran, 2007). Nevertheless, the presence of industry specific cross sectional averages as weakly exogenous variables in the system will help to avoid most of the problems related to the particular specification of the labour input. Indeed, the forcing variables will be acting to balance the distortionary effect of any low frequency components of the labour input, as well as possible breaks or nonlinearity in the variable.

When the analysis is conducted at the sectoral level the inclusion of the level or growth of the labour input might be dictated by economic theory. Chang and Hong (2006) suggest that at the sectoral level a reallocation effect could be at work such that (sectoral and aggregate) labour input should enter the system in first difference. Specifically, Campbell and Kuttner (1996) and Phelan et al. (2000) highlight the role of sectoral shifts in modelling employment at the industry level and their importance for a better understanding of the driving force of aggregate employment dynamics.

To determine the correct stationary transformation of the variables we apply the panel unit root test developed by Pesaran, Smith and Yagamata (2007)<sup>29</sup>.

 $<sup>^{28}</sup>$ Note that this problem would persist even in the difference specification. Fernald (2005) and Francis and Ramey (2005a) document trend breaks in productivity and hours.

<sup>&</sup>lt;sup>29</sup>This test extends the original test of Pesaran (2007) to the case with multiple common factors. The documented cross sectional dependence of the data can be related to the presence of intersectoral factor demand linkages and does not need to be directly related to the presence of common factors. Therefore in this case the specification of the factor structure of the data is not a trivial issue. The Pesaran et al. (2007) test can account for this uncertainty as it

The null hypothesis is that all the series have a unit root and are not cointegrated with the underlying factors. The results for the industry data are summarized in Table 2. Specifically, the null hypotheses cannot be rejected for the level of log labour productivity  $(z_{it})$  and hours  $(n_{it})$ , regardless of whether an intercept or an intercept and a linear trend are included whereas it is rejected for the growth rates.

	TABLE 2 UNIT ROOT TEST					
_	With intercept and linear trend					
	CADF(0)	CADF(1)	CADF(2)	CADF(3)		
$\overline{z_{it}}$	$-3.052^*$	-2.609	-2.181	-2.072		
$n_{it}$	-2.615	-2.476	-2.22	-2.081		
	With intercept					
	CADF(0)	CADF(1)	CADF(2)	CADF(3)		
$\overline{z_{it}}$	$-2.819^*$	-2.392*	-2.004	-1.904		
$n_{it}$	$-2.374^{*}$	-2.241	-2.031	-1.921		
$\Delta x_{it}$	-6.306*	$-4.547^{*}$	$-3.29^*$	$-2.671^*$		
$\Delta n_{it}$	$-7.409^*$	$-4.477^{*}$	$-3.523^*$	$-2.857^*$		

Notes: The reported values are CIPS(p) statistics, which are cross section averages of cross-sectionally Augmented Dickey-Fuller test statistics (Pesaran, Smith and Yagamata, 2007). The critical values for this test depends on the cross section, time dimension and number of lags included as well as the number of cross sectional average included. The values are tabulated in Pesaran, Smith and Yagamata (2007). When only the intercept is included the 5% critical value is -2.29 when no lag is included, -2.24 for 1 lag, -2.10 for 2 lags and -2.03 for three lags. When intercept and linear trend are included the critical value is -2.72 when no lag is included, -2.67 for 1 lag, -2.50 for 2 lags and -2.41 for three lags. The superscript "\*" signifies the test is significant at the five per cent level.

In the light of the results and the theoretical considerations outlined above we assume that there are a unit root in the labour input. Therefore we estimate and analyse (2)-(3) with both variables in log difference, i.e.  $y_{it} = \Delta n_{it}$ .

controls for the cross sectional dependence in the data without the need of prior specification of the factor structure of the data. Specifically, for each variable we augment the ADF regression with the weighted average of both productivity and hours. The weights are computed from the input output matrix as described above. As outlined in Pesaran et. al. (2004) for  $N \to \infty$  any set of weights that satisfy the usual granularity condition would capture the effect of aggregate common shocks. Indeed, we would obtain similar results in the test if a simple average is used to control for the cross sectional dependence in the data. Moon and Perron (2007) highlight that this type of test has a better performance than the other available panel unit root test with cross sectional dependent data for small panels where the estimation of factors is difficult.

## 4 Aggregate and Sectoral specific shocks

The positive comovement across sectors is a stylized fact that need to be addressed by any theory of the business cycle. Whether the comovement between sectors and the aggregate business cycle originates from aggregate shocks or sectoral shocks amplified by sectoral interactions, or a combination of the two is not clear a priori. This is a question that has attracted the interest of many researchers (see e.g. Cooper and Haltiwanger, 1996).

Following Long and Plosser (1983, 1987), Horvath (1998, 2000) shows that independent sectoral shocks can create aggregate persistence and comovements between sectors. On empirical grounds Long and Plosser (1987) first investigated whether the source of business cycle fluctuations is aggregate or sector specific. Their analysis is consistent with the existence of a single aggregate disturbance whose explanatory power is, however, limited. Similar results are reported by Cooper and Haltiwanger (1996), who highlight the fact that this particular methodology is bound to provide only an upper bound for a particular role for common shocks. Conley and Dupor (2003) from a completely different prospective propose an empirical strategy to identify the driving force of the business cycle, and conclude that the data support the sectoral origin of the business cycle.

The question whether it is common aggregate shocks or idiosyncratic industry shocks amplified by interactions between sectors that give rise to comovements, is closely related to the statistical property of weak and strong cross sectional dependence proposed by Pesaran and Tosetti (2007). Strong dependence between sectors is essential to replicate the aggregate cycle. Otherwise, by a standard diversification argument, as we disaggregate the economy into many sectors, independent sectoral disturbances will tend to average out, leaving aggregates unchanged (Dupor, 1999), which implies weak cross sectional dependence. Pesaran and Tosetti (2007) and Chudick and Pesaran (2007) show how strong dependence between sectors could arise if one or more sectors are dominant and/or if the shocks have a common factor structure, i.e. there are aggregate shocks to the economy. A similar argument is used by Horvath (1998), who relates the amplification effect of intersectoral linkages to a particular feature - the sparseness, of the input-output matrix.

Recent developments in factor analysis allow the determination of the number of common factors in a panel. Table 3 shows the results of the test proposed by Onatski (2007) and the information criteria introduced by Bai and Ng (2002)<sup>30</sup>. The table also shows the average pair-wise cross sectional correlations and the test

<sup>&</sup>lt;sup>30</sup>The information criteria of Bai and Ng (2002) and the test introduced by Onatski (2007) is used to determine the number of common static factors. As observed by Stock and Watson (2002), the number of static factors imposes a upper bound on the possible number of dynamic common factors.

for cross sectional dependence of Pesaran (2004). The test of Onatski (2007) starts from an a priori maximum number of factors,  $k_{max}$ , the null hypothesis of the test is  $H_0: r = k$  while the alternative is  $k < r = k + s \le k_{max}$ . The information criteria introduced by Bai and Ng (2002) select the number of common factors which minimises the penalised square sum of residuals<sup>31</sup>.

Table 2 provides some interesting results. When sectoral interactions are *ignored* the tests point to aggregate factors as the main explanation for comovement between sectors. However, these statistical procedures identify the presence of common factors by analysing the rate of the expansion of the eigenvalues of the covariance matrix of the original data (or residuals). However, the eigenvalues will be unbounded in both cases when there are complementarities between sectors and when aggregate shocks are necessary to reproduce aggregate fluctuations. The procedure cannot distinguish between these two alternatives<sup>32</sup>. When we take sectoral interactions into account the tests suggest that there are no common factors in the shocks of (2). Dependence between sectors in the economy would make it appear that there are aggregate shocks in the economy. This is exactly the argument made by Horvath (1998) to emphasize the sectoral origin of business cycle fluctuations. This is a potentially important result. By contrast, Acconcia and Simonelli (2008) in a similar exercise to this paper, but ignoring sectoral interactions, identify 2 aggregate common factors<sup>33</sup>.

<sup>&</sup>lt;sup>31</sup>The original information criteria might have substantial loss of power for pervasive weak cross sectional dependence in the factor residuals. Indeed, there is overestimation of the true number of factor because the scales of the Bai-Ng threshold functions happen to be too small when idiosyncratic terms are non-trivially correlated. This is recognized by Bai and Ng (2002) and proved in Onatski (2006). Bai and Ng (2002, p.207) observe that BIC3 has very good properties in the presence of cross sectional correlation can be explained by the fact that the multiplier on the average idiosyncratic eigenvalues in the threshold corresponding to BIC3 is relatively large. This last point is very important as it is to be expected that there will be non-trivial (weak) cross sectional correlation due to inter-sectoral linkages.

<sup>&</sup>lt;sup>32</sup>See Chudick and Pesaran (2007), Remark 5, p. 10.

<sup>&</sup>lt;sup>33</sup>Their analysis prefers the common component specification as they presuppose that it is aggregate shocks that drive the economy (see their discussion in footnote 3, p.6). Furthermore, the dataset they use is at a lower level of disaggregation, i.e. 2 digit SIC data.

TABLE 3 AN ANALYSIS OF COMOVEMENTS		
	$\Delta z_{it}$	$\Delta n_{it}$
Bai and Ng $(2002)^a$	1	2
Onatski $(H_0: r=0)$	33.089*	$7.105^{*}$
$(H_0: r = 1)$	23.66*	7.105*
$(H_0:r=2)$	1.252	$5.474^{*}$
No sectoral interactions <sup>b</sup>	$arepsilon_{it}^T$	$\frac{arepsilon_{it}^{NT}}{2}$
Bai and Ng $(2002)^c$	0	2
Onatski $(H_0: r=0)$	8.562*	3.647
$(H_0: r = 1)$	8.562*	3.647
$(H_0:r=2)$	1.402	3.647
Average Cross Correlation	0.048	0.212
CD	93.203	412.90
CD	93.203	412.90
	$\varepsilon_{it}^{T}$	$\varepsilon_{it}^{NT}$
Bai and Ng (2002)	$\frac{-}{0}$	$\frac{-it}{0}$
Onatski $(H_0: r=0)$	2.141	2.515
$\mathcal{O}_{\text{Hollow}}(H_0: r = 0)$	2.171	2.010
Average Cross Correlation	0.012	0.011
$\mathrm{CD}^d$	24.74	22.15

Notes: The first part of the table reports the Onatski (2007) test of the number of static factors. The second part of the table reports the choice of the number of static factor consistent with the information criteria of Bai and Ng (2002). Then, the average pair-wise cross section correlation is provided together with the CD test statistic. The superscript "\*" signifies the test is significant at the five per cent level. The critical values depend on  $\kappa = k_{\text{max}} - k$ , and these are tabulated in Onatski (2008). In the table we report the test for  $k_{\text{max}} = 5$ . The 5% values are 5.77 for  $\kappa = 5$ , 5.40 for  $\kappa = 4$  and 4.91 for  $\kappa = 3$ .  $k_{\text{max}} = 5$  is also the choice of the maximum possible aggregate factors allowed in the Bai and Ng (2002) procedure.

<sup>&</sup>lt;sup>a</sup> The number of factor selected is the same for all the selection criteria proposed (P1-3 and IP1-3). The BIC-3 criteria select one factor for total hours and 0 for labor productivity.

<sup>&</sup>lt;sup>b</sup> Specifically, this corresponds to setting the matrices  $\mathbf{C}_{ij}$  ( $\forall i$  and j=0,1) arbitrarily equal to the null matrix 0 in (3), i.e. the matrix of coefficients  $\mathbf{G}_j$ , for j=0,1, in (4) are block diagonal matrices.

<sup>&</sup>lt;sup>c</sup> The BIC-3 information criteria would select 1 common factor when the shocks are included all together, 0 among the technology shocks only, and 1 for non technology shocks alone. The P criteria (P1-P3) would all choose 2 factors in the technology shocks.

<sup>d</sup> Even though the value of the test statistics are highly reduced these are still not significant. Notice that failure to reject the null hypothesis might arise from the presence of weak cross sectional dependence of the identified shocks.

Recently, Franco and Philippon (2007) have argued that the source of aggregate fluctuations can be identified from pair-wise cross section correlations between shocks to firms. They conclude that the most important shock to explain aggregate fluctuations is the one with the highest average cross section correlation, as this shock is the one that is most likely to affect many firms/sectors at the same time. This reasoning is justified, implicitly, by appealing to the law of large numbers. However, from the result in Table 3 we note that if we do not take into account factor demand linkages and sectoral interactions this interpretation may be misleading. In this case, the natural conclusion would be to downsize the importance of technology shocks and emphasize non-technology shocks as the main drivers of aggregate dynamics. Interestingly, this is exactly one of the main messages of Franco and Philippon (2007). However, once the sectoral interactions are taken into account aggregate fluctuations can be reproduced even if the shocks are (almost) independent. Furthermore, using the same approach as Franco and Philippon (2007), our results identify technology shocks as at least as important as non-technology shocks for aggregate fluctuations.

Nevertheless, the results in the last panel of Table 3 - showing the average pairwise cross sectional correlations - suggest that the shocks that we have identified with our approach are not fully orthogonal. So the covariance matrix in (2) and (4),  $\Omega_u$ , is not diagonal. So, shocks to one sector are likely to be correlated with shocks to other sectors<sup>34</sup>. Conley and Dupor (2003) use a nonparametric technique to model the off diagonal elements of the covariance matrix  $\Omega_u$ . Here the issue is complicated as we identify not a single, but two types of shock. In principle the fact that the identified shocks are not fully orthogonal between sectors might cause some problems for the various exercises we perform. However, the magnitude of the average pair-wise cross section correlation suggests that we have weak cross sectional dependence, implying that dependence between sectoral shocks is not relevant for the aggregate dynamics of the economy<sup>35</sup>.

 $<sup>^{34}</sup>$ A similar finding is observed by Franco and Philippon (2007) with firm level data. However, in their case they do not control for possible sectoral interactions and/or for the presence of aggregate factors.

 $<sup>^{35}</sup>$ To understand how much information we lose by assuming that the shocks we have identified by capturing sectoral interactions through the input-output matrix are cross sectionally independent, the aggregate output and hours (growth) series were simulated assuming that  $\Omega_u$  is diagonal. The correlation between the aggregated series and the sum of sectors is 99.8% and 99.35% respectively for the growth rate of output and hours. We interpret this to suggest that it is safe to proceed as if  $\Omega_u$  is diagonal.

### 5 Technology shocks and hours

Real business cycle theory assigns a central role to technology shocks as the source of aggregate fluctuations. Moreover, positive technology shocks should lead to positive comovements of output, hours, and productivity. However, Gali (1999) finds that positive technology shocks appear to lead to a decline in labor input. Since then a number of studies have reported similar results (see Galì and Rabanal, 2004, for a review), which if confirmed would make a model of technology-driven business cycles unattractive (Ramey and Francis, 2005b).

Most of the empirical macroeconomic literature evaluating the effect of technology shocks focus on the analysis of aggregate data. So sectoral interactions through factor demand linkages do not matter. Chang and Hong (2006)<sup>36</sup> and Kiley (1997) examine the technology-hours question with sector level data, however, they consider each sector as a separate unit in the economy. In the previous section we found strong complementarities between sectors.

In our model an aggregate shock corresponds to a simultaneous shock to all sectors of the economy<sup>37</sup>. In Figure 1 we show the mean impulse response of labour productivity and hours to a 1-standard deviation shock, disregarding sectoral interactions<sup>38</sup>. The results are similar to Chang and Hong (2006) who use labour productivity and confirm the finding in the literature (see e.g. Gali, 1999, Francis and Ramey, 2005). The impact response for hours is negative, furthermore the effect persists in the long run<sup>39</sup>. The right side of the panel shows the impact response for each sector. Despite the magnitude, the sectoral impulse responses seem to be quite similar in shape, furthermore the negative response of hours is

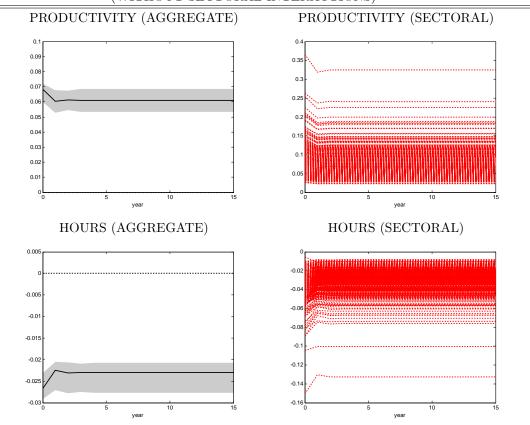
<sup>&</sup>lt;sup>36</sup>They focus on shocks to total factor productivity.

<sup>&</sup>lt;sup>37</sup>This corresponds to a weighted average of a 1-standard deviation shock to each sector in the economy. The weights are proportional to the average shipment value during the period. None of the sectors dominates the others by size. The average impulse response calculated in this way is very close to the actual impulse response to the aggregate economy, up to an approximation error.

<sup>&</sup>lt;sup>38</sup>This corresponds to setting the matrices  $\mathbf{C}_{ij}$  ( $\forall i$  and j=0,1) in (3) arbitrarily equal to  $\mathbf{0}$ , as in Chang and Hong (2006), but where the coefficients of the SVAR for each industry are estimated without bias. Pesaran and Tosetti (2007) and Chudick and Pesaran (2007) show that in the presence of cross sectional dependence the estimators would be biased.

<sup>&</sup>lt;sup>39</sup>Similar results are found in Franco and Philippon (2007), who use firm level data. They do not consider interdependencies (and their consequences) between firms. Basu, Kimball and Fernald (2006) reach the same conclusion identifying the shocks from a completely different prospective. They also identify the shocks at the sectoral level (2 digit SIC), but do not consider the potential effects of intersectoral relations.

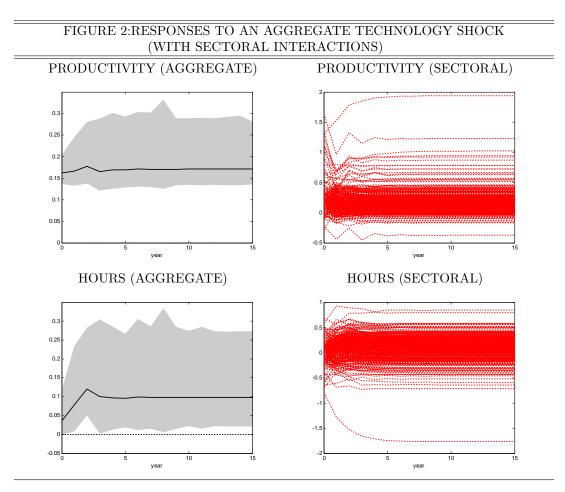
FIGURE 1. IMPULSE RESPONSES TO AN AGGREGATE TECHNOLOGY SHOCK (WITHOUT SECTORAL INTERACTIONS)



Notes: The figure shows estimated impulse responses of labor productivity and hours to a contemporaneous shock , where no interaction between sectors is allowed. The left hand panel provides the aggregate response, the shaded area represent the 90-percent confidence intervals based on bootstrapping 500 draws. The right hand panel shows the sectoral responses, whose weighted average corresponds to the figure on the left hand side.

In Figure 2, when we allow for sectoral interactions we have a very different outcome. An "aggregate" technology shock now has a positive (short and long run) impact on labour productivity and total hours. The impact of the shock is also generally much larger in magnitude, highlighting the importance of the sectoral interactions as an amplifier of sectoral shocks (Cooper and Haltiwanger, 1996). Even though the confidence intervals on the impulse responses are wider, the effect of technology on hours is always significant. The response of productivity at the sectoral level is negative for some sectors, something that is not observed when sectoral interactions are ignored. When sectors in the economy are all linked,

the sectoral technology shock also affects the relative competitiveness of each sector in the economy  $^{40}$ .



Notes: The figure shows impulse responses of labor productivity and hours to a contemporaneous change to the idiosyncratic sectoral technology shock when the interaction between sector reflects sector demand linkages. The left hand panel provides the aggregate response, the shaded area represent the 90-percent confidence intervals based on bootstrapping 500 draws. The right hand panel shows the sectoral responses, whose weighted average corresponds to the figure in the left hand side.

<sup>&</sup>lt;sup>40</sup>See Swanson (2006) for a different setting where a similar mechanism is at work. Fisher (2006) finds the same when the relative price of investment specific goods plays a role in the model.

#### 5.1 The role of sectoral interactions

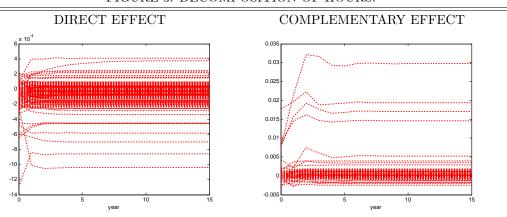
To get a better understanding of the results and the role that sectoral interactions play, it is convenient to return to the model we used to obtain the impulse responses, i.e. (2) or (4). In this VAR n variables for each of the N sectors are fully interacting, and idiosyncratic sectoral shocks can still be identified once the input-output linkages between sectors are imposed. The effect of a contemporaneous technology shock to the N sectors corresponds to an aggregate technology shock. In this case the aggregate impulse response is the weighted sum of each sectoral impulse response.

In the presence of sectoral interactions the effect of a shock in sector i is equal to the effect that the shock would have had in a system without interactions. In addition, this shock (through the input output system) affects all the sectors that are supplied by sector i; therefore directly or indirectly all sectors in the economy are affected by the shock to sector i, and this in turn echoes back to the original sector i, therefore amplifying of the original shock. So sectoral interactions induce a rich set of short-run dynamics. The first effect from sector i to all the other sectors in the economy is a downstream propagation from supplier to user, but at the same time we have the second effect, i.e. a reflex response, as the original sector is also a user of other sectors' supply<sup>41</sup>. The aggregate effect of any shock to sector i, is the industry weighted direct effect plus the weighted effect on the rest of the economy. When idiosyncratic shocks do not occur simultaneously it is possible to separate out the two components - the direct component, i.e. the effect of a shock to the ith sector and the complementary component, i.e. the effect of this shock on other sectors.

Figure 3 plots the estimated direct and complementary components at the sectoral level. The sectoral components are weighted such that their sum exactly matches the estimated aggregate impulse response of hours in Figure 2. The direct effect however does not correspond to the effect of the shocks when the sectoral complementarity between the sectors is considered. Indeed, this is shown in the right bottom panel of Figure 1 and it is clearly negative for every sector. Looking at the direct effect it is clear that the impulse response are still in line with the aggregate puzzle of a negative effect (the sum of the impulse response is negative). However, the indirect effect through the sectoral interactions provides the needed positive shift for a positive aggregate response of hours to occur

<sup>&</sup>lt;sup>41</sup>Following the notation in Shea (2002) the first effect is exactly a downstream propagation mechanism and the second effect can be thought as an upstream propagation from suppliers to user. This is because all the sectors in the economy are directly or indirectly (i.e. through a third sector) interconnected.





Notes: The figure shows the response of hours to an idiosyncratic technology shock at the sectoral level. The original impulse response are weighted according to industry size, measured by the real value of shipments, in this way the sum of the sectoral impulse responses exactly match the aggregate response reported in Figure 2.

Looking at the relative size of these effects, without the sectoral interactions, the aggregate effect of sectoral specific shocks tends to vanish due to the law of large numbers<sup>42</sup>. These results are in line with the view that factor demand linkages and other complementarities between sectors, can potentially be one of the main mechanisms for the propagation of shocks to the economy (Cooper et al. 1996).

From Figure 3 it is also evident how the aggregate results are driven by a few sectors in the economy. Sectors whose indirect effect is highest are those that are more connected as measured by the input output matrix<sup>43</sup>. This is in line with the argument put forward by Horvath (1998) and recently emphasized by Carvalho

The most important five sectors are all part of the "chemicals and allied products" (specifically SIC codes 2812-13-16 and 2865-69), and largely correspond to the sectors with the highest column sum of the weighting matrix.

<sup>&</sup>lt;sup>42</sup>This is in line with the Lucas (1981) result that disaggregation of the economy into finer industry details, tends to offset the impulse propagation mechanism due the sectoral interactions, though the law of large numbers.

 $<sup>^{43}</sup>$ The original input-output matrix is scaled such that the row sum is equal to one. Each element ij of this matrix corresponds to the share of commodity j in the production of sector i. The column sum is therefore a direct measure of the (relative) importance of each sector for aggregate fluctuations, as it reflects how each sector are connected to other sectors. In a different context Pesaran and Tosetti (2007) emphasise the role of the column sum as a direct measure of cross sectional dependence. A plot of the row sum of the input output matrix is provided in an Appendix available from the authors.

(2007). Shocks to sectors which are most connected in the economy are strongly amplified by factor demand linkages between sectors, and therefore are the sectors more likely to explain the aggregate business cycle.

#### 5.2 Sectoral heterogeneity

Even though the main element at work is the interactions between sectors, an important role is also played by heterogeneity between sectors. Swanson (2006) illustrates the importance of sectoral heterogeneity for the transmission of the shocks in the economy. Specifically, he shows that sectoral heterogeneity itself has important first order implications for the transmission of aggregate shocks to aggregate variables.

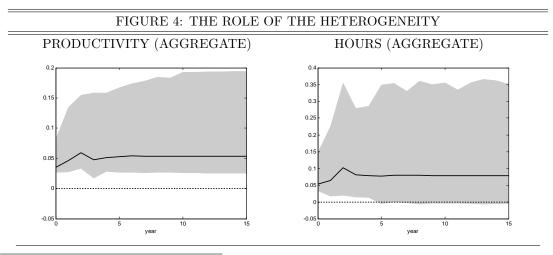
If we partition the structural SecVAR in equation (4) such that it identifies the sectoral specific components:

$$\mathbf{G}_{i0}\mathbf{x}_t + \mathbf{G}_{i1}\mathbf{x}_{t-1} = \mathbf{u}_{it},$$
  
 $\mathbf{G}_i(L)\mathbf{x}_t = \mathbf{u}_{it},$ 

where the matrices  $\mathbf{G}_{ij}$  in  $\mathbf{G}_i(L)$  are constructed as outlined in section 2. Furthermore, the model can be rewritten as

$$\left[\mathbf{G}_{i}(\mathbf{L}) - \overline{\mathbf{G}}(\mathbf{L})\right] \mathbf{x}_{t} + \overline{\mathbf{G}}(\mathbf{L}) \mathbf{x}_{t} = \mathbf{u}_{it}, \tag{5}$$

where  $\overline{\mathbf{G}}$  refers to the mean group estimator of the coefficients in  $(4)^{44}$ . Specifically, the coefficients in  $\overline{\mathbf{G}}$  are constructed such that the only source of heterogeneity between sectors appertains to the linkages with other sectors. This decomposition highlights the role of sectoral heterogeneity, which is exactly identified by the first expression on the right hand side of (5).



<sup>&</sup>lt;sup>44</sup>This is the same as saying that the mean group coefficients are calculated on the matrices  $\mathbf{A}_{ij}$  and  $\mathbf{C}_{ij}$  in (3), for j=0,1.

Notes: The figure shows the effect of sectoral heterogeneity on the responses of labour productivity and hours to a idiosyncratic contemporaneous shock when the interaction between sectors reflects sector demand linkages. The shaded area represents the 90-percent confidence intervals based on bootstrapping 500 draws.

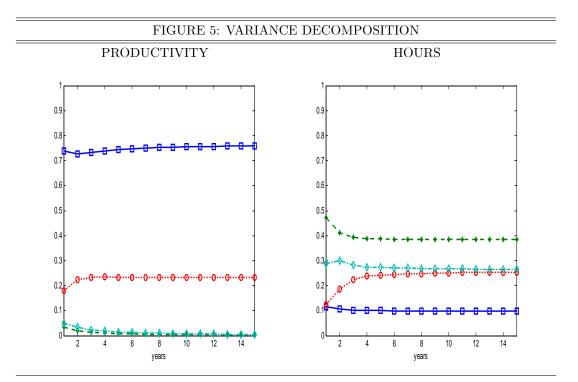
Notice that sectoral heterogeneity has important implications for the aggregate impulse response. Complementarities between sectors amplify the differences between sectors so that the effect of heterogeneity does not vanish with aggregation<sup>45</sup>.

#### 5.3 Variance decomposition.

In this section we decompose forecast variances at the sectoral level. This allows us to evaluate the importance of sectoral interactions for sector specific cycles and therefore for the aggregate business cycle. Also, we can evaluate the relative role of technology and non-technology shocks. Figure 5 shows the mean (weighted average) variance decomposition. Since each sector is related in turn to other sectors, productivity and hours in sector j are explained by shocks to the jth sector, and also by shocks (technology and non-technology) originated in other sectors. Labour productivity as expected is mostly explained by technology shocks, but with a quite sizable part (20 to 25%) originating in other sectors. Most interestingly, the variation in labour input is dominated by non-technology shocks, nevertheless, the role of technology shocks (in total) is not negligible. On impact technology shocks account for roughly 20% of the variation in hours, with its role rising steadily up to roughly 35\%, where this increase is due entirely to the increasing role of technology shocks in other sectors. Sectoral interactions in total account for roughly 20% of the variation in productivity and 50% of the variation in total hours worked<sup>46</sup>. Clearly we would get a very misleading picture if we ignore sectoral interactions. Technology shocks account for most of the variability in productivity, but its role in the explanation of total hours would be completely underestimated, as it accounts for only 15-20% of the variation when we ignore sectoral interactions.

<sup>&</sup>lt;sup>45</sup>Pesaran and Smith (1995) highlight the potential bias associated with the estimation of aggregate data, when the disaggregated relationship might be heterogeneous.

<sup>&</sup>lt;sup>46</sup>The results of this variance decomposition are similar to those of Christiano et al. (2004),



Notes: The figure shows the average forecast variance decomposition. The blue line with squares  $(-\boxdot - \boxdot -)$  represents the sector-specific technology component, the green line with stars (-\*-\*-) the sector-specific non-technology component. The red line with circles  $(-\bigcirc -\bigcirc -)$  is the component associated with technology shocks to other sectors , the light blue line with rhombus  $(-\bigcirc -\bigcirc -)$  is the non-technology shocks to other sectors.

In summary, sectoral interactions are a vital driver of sectoral fluctuations. Furthermore, once their role is correctly pinned down technology shocks as one of the main causes of aggregate fluctuations re-emerges.

# 5.4 Technology or non-technology shocks: The role of sectoral interactions.

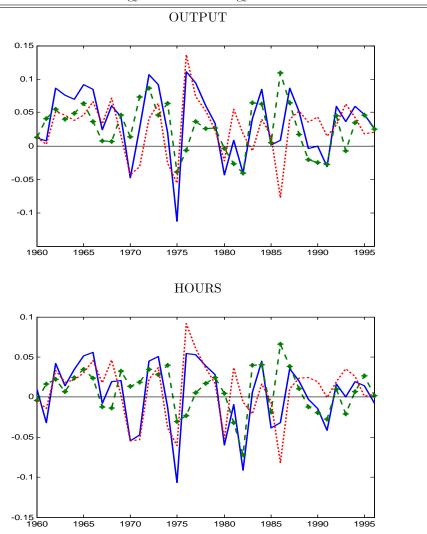
Another way to assess the role of technology shocks for aggregate fluctuations is to look at a simulated series when one type of shock at a time is shut down. Figure 6 shows simulated aggregate employment (hours) and output growth implied by the technology and non-technology shocks<sup>47</sup>. From the graphs it emerges that technology and non-technology shocks seems to be equally important for explaining

<sup>&</sup>lt;sup>47</sup>Note that computing an exact variance decomposition of aggregate data is impossible for the reason illustrated in section 4. The exact procedure for aggregation is discussed in the appendix.

aggregate fluctuations. Nevertheless, some difference are clear. Technology shocks appear to account for most of the cyclical volatility in the second part of the sample, from approximately 1980. By contrast, non-technology shocks appear to match the period from 1960 to 1980. Furthermore, the slow down at the beginning of the 90's seems to be largely as the result of technology shocks (Hansen and Prescott, 1993)<sup>48</sup>. These results are generally consistent with the view that demand shocks were the main driver of the business cycle before the 80s', whereas supply side shocks have gained importance since the 80s. Interestingly the latest period also corresponds to a steady decrease in aggregate volatility, the so called 'great moderation' (see e.g. Stock and Watson, 2002).

In the previous section we also emphasised the amplification due to factor demand linkages. Based on the impulse responses and the forecast variance decomposition of sectoral cycles, sectoral interactions appear to be the main driver of aggregate fluctuations, as emphasized for instance by Shea (2002). In Figure 7 we show the decomposition of the aggregate cycle that is directly attributable to sector specific shocks and that related to intersectoral amplification. The pattern that emerges from this figure is revealing. It clearly supports the idea that aggregate fluctuations are nothing more than the amplification of small shocks originating at the sectoral level, and transmitted to the whole economy by sectoral interactions.

<sup>&</sup>lt;sup>48</sup>Notice that Hansen and Prescott (1993) in their explanation of the downturn need to use a two sector RBC model, with an intermediate input channel, to make the technology shocks account for the recession at the beginning of the 90s.

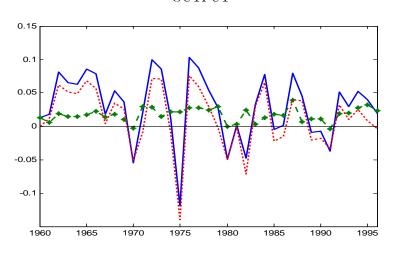


Notes: The figure shows a historical decomposition of the aggregate growth rate of output and hours into that attributable to technology and non-technology shocks. The blue continuous (—-) line represents actual data, the green dashed line with stars (-\*-) simulated data with only technology shocks, the red dotted line  $(\cdot \cdot \cdot)$  denotes the non technology shock.

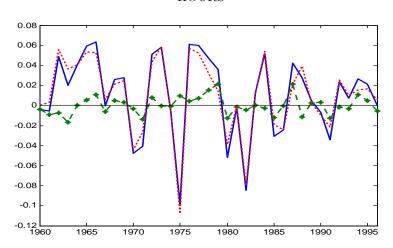
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FIGURE 7: BUSINESS CYCLE, HISTORICAL DECOMPOSITION
The role of factor demand linkages

#### OUTPUT



#### HOURS



Notes: The figure shows a historical decomposition of the aggregate growth rate of output and hours into sector specific shocks excluding the sectoral interactions, and including the sectoral interactions. The blue continuous (—) line represents the actual data, the green dashed line with stars (-\*-\*-) the simulated data with only sector specific shocks, and the red dotted ( $\cdots$ ) with the sectoral interactions.

#### 6 Some Robustness Checks

Our results suggest that aggregate technology shocks have little if any role in aggregate fluctuations. It is technology shocks at the industry/sectoral level that are the driving forces. In this section we turn to some robustness checks. First, we take the measure of the aggregate technology shock from Basu et al (2006). They construct the measure, controlling for aggregation effects, varying utilization of capital and labour, non-constant returns and imperfect competition. We then include this aggregate measure in the model so that the term appears as an additional conditioning variable for the estimation of each sectoral model.

The SecVAR analysis in previous sections is robust to the presence of aggregate shocks in the economy. As long as the weights used to construct the sector specific cross sectional averages in (3) satisfy the usual granularity conditions, for  $N \to \infty$ , cross sectional averages can be thought as a reflection of aggregate shocks hitting the economy<sup>49</sup>. Nevertheless complementarities between the sectors and specifically factor demand linkages, as reflected by the input-output matrix are an unequivocal feature of the production process.

In this section we discuss the robustness of our results in the presence of possible aggregate shocks to the economy. Specifically, we consider the robustness to the inclusion of an aggregate technology shock and monetary policy shock. For each sector we estimate

$$\mathbf{A}_{i0} \mathbf{\varkappa}_{it} = \mathbf{C}_{i0} \mathbf{\varkappa}_{it}^* + \mathbf{A}_{i1} \mathbf{\varkappa}_{it-1} + \mathbf{C}_{i1} \mathbf{\varkappa}_{it-1}^* + \mathbf{\Upsilon}_i \mathbf{d}_t + \boldsymbol{\varepsilon}_{it},$$

where  $\mathbf{d}_t$  is a k dimensional vector of aggregate shocks hitting the economy. We include aggregate total factor productivity (TFP) and the aggregate technology shock (ATS) constructed by Basu et al (2006). A monetary policy shock (MPS) is derived from an exactly identified VAR, estimated on quarterly variables averaged for each year<sup>50</sup>.

TABLE 4: THE IMPACT OF AGGREGATE SHOCKS				
	TFP	ATS	MPS	
	0.0552	0.2584	-0.1827	
$\Delta z_{it}$	(0.9415)	(4.9436)	(-1.9319)	
	53	44	54	
	0.0816	-0.2672	0 5005	
<b>A</b>			-0.5005	
$\Delta n_{it}$	(1.2044)	(-4.1964)	(-4.9461)	
	93	65	65	

<sup>&</sup>lt;sup>49</sup>See Pesaran (2006) Forni and Reichlin (1996).

<sup>&</sup>lt;sup>50</sup>The data are provided by Basu et al. (2006) and are available in the AER website (http://aea-web.org/aer/). Additional explanation of the data is available in their paper.

Notes: The first row of table 4 provides the mean group estimates of the coefficients on the 3 aggregate shocks. The second row are the t-test statistics on the null hypothesis that the mean group estimator is equal to 0. The third row shows the number of sectors where the null hypothesis is rejected (the total number of sectors is 458).

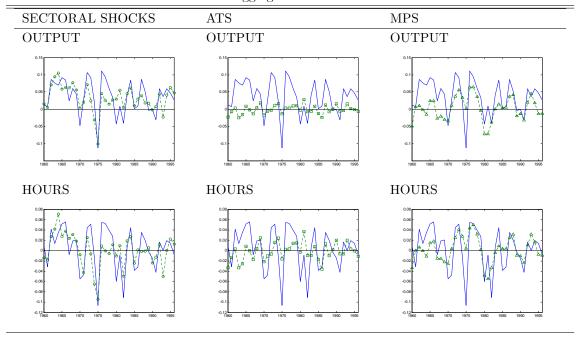
Table 4 presents the results of the inclusion of the aggregate shocks (one at a time)<sup>51</sup>. For ATS and MPS we are able to reject the null hypothesis on the mean group estimate, even though the null is rejected in only a small number of sectors. The signs on the monetary policy shock are consistent with the literature. Perhaps, very interestingly is the opposite sign associated to the TFP and the ATS, where the sign of the last one is consistent with the results in Basu et al. (2006)<sup>52</sup>. Similar tests of the importance of sectoral interactions (not reported, but available from the authors) show that the results are robust to the inclusion of aggregate measures of technology and monetary shocks. Moreover, the impulse responses at the sectoral level are consistent with the findings in the previous section (when we do not include the aggregate shocks).

In figure 8 we decompose the historical aggregate business cycle into that attributeable to the sector and aggregate shocks, where we differentiate between the aggregate technology shock (ATS) and the monetary policy shock (MPS). The figure clearly shows that the bulk of aggregate volatility is to be attributed to sectoral shocks. The aggregate technology shock has a very limited role. As expected a bigger role is played by monetary policy shocks. Interestingly, monetary policy seems to account for the recession in the early 1980s, corresponding to the Volcker disinflation.

<sup>&</sup>lt;sup>51</sup>If we include ATS and MPS together in the system the results are unchanged since by construction ATS and MPS are orthogonal.

<sup>&</sup>lt;sup>52</sup>We get very similar results if the shocks are included with a lag.

FIGURE 8: BUSINESS CYCLE, HISTORICAL DECOMPOSITION
Sectoral vs aggregate shocks



Notes: The figure shows a historical decomposition of the aggregate growth rate of output and hours into sector specific and aggregate shocks. The blue continuous (——-) line represents the actual data, the green dashed line with circles ( $-\bigcirc-\bigcirc$ ) the simulated data with only sector specific shocks, and the green dashed line with squares ( $-\bigcirc-\bigcirc$ ) with the aggregate technology shock and the green dashed line with triangles ( $-\triangle-\triangle-$ ) is the component associated with monetary policy shocks.

#### 7 Conclusions

This paper has investigated the role of factor demand linkages, i.e. the intermediate input channel for the propagation of the shocks using data on highly disaggregated manufacturing industries from 1958 to 1996. We construct a sectoral VAR, and estimate a series of bivariate models for productivity and hours at the sectoral level. Weighted averages of sectors, where the weights are derived from the input-output matrix, are used to control for cross sectional dependence. Once sectoral interactions are taken into account in this manner, we find that the aggregate business cycle is largely driven by sectoral shocks. This is in line with the real business cycle model of Long and Plosser (1983) and Horvath (1998, 2000). We show that taking into consideration sectoral interactions is important because they

prove to be an important amplifier for sector specific shocks, both technological and non-technological. But most importantly, we show that the perverse response of hours to a technology shock found in many other studies remains if sectoral interactions via the input-output matrix are ignored. When they are incorporated into the model we find a positive association.

Our results are potentially important as they show the problems that arise in empirical work when sectoral interactions and heterogeneity are ignored. As has been stressed by Cooper et al. (1996), Shea (2002) and Conley and Dupor (2003), results obtained from the analysis of aggregate data can be very misleading.

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## Appendix A: Estimation issues

To estimate the dynamic effect of a technology shock we follow the procedure outlined in Shapiro and Watson (1988), and discussed in Christiano et al (2004). As in Pesaran, Schuermann and S.M. Weiner (2004) the contemporaneous relations between sector specific variables and the aggregate variables can be estimated consistently as long as the weighted aggregate variables in the system are weakly exogenous, a condition satisfied for  $N \to \infty$ . To estimate the contemporaneous relation between the endogenous variables we need to rely on instrumental variables. Specifically, we make use of long run identification restrictions, in line with the literature. The analysis of disaggregated sectors as in (3)-(4) provides both a theoretically consistent estimate of an economy with sectoral interdependence and/or both sectoral and aggregate shocks to the economy and a new set of instruments. In this case, the weak instrument problem usually described in the literature might be avoided by using the industry specific cross sectional averages of the original variables in the system.

Specifically, for a specific sector i the system of simultaneous equations to be estimated is

$$(\mathbf{A}_{i0} - \mathbf{A}_{i1}L) \begin{bmatrix} \Delta z_{it} \\ \Delta n_{it} \end{bmatrix} = (\mathbf{C}_{i0} - \mathbf{C}_{i1}L) \begin{bmatrix} \Delta z_{it}^* \\ \Delta n_{it}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_{it}^1 \\ \varepsilon_{it}^2 \end{bmatrix}, \tag{A1}$$

where  $\mathbf{A}_{ij}$  and  $\mathbf{C}_{ij}$ ,  $\forall i$  and j=1,2, are  $2\times 2$  matrices, with the generic xj-element denoted with subscript. The restriction that only technological shocks have a permanent effect on productivity implies that  $A_{i0}^{12} = -A_{i1}^{12}$ . A similar restriction for other sectors' technology shocks is also imposed, i.e.  $C_{i0}^{12} = -C_{i1}^{12}$ . It follows that the technology shock for sector i,  $\varepsilon_{it}^{T}$ , can be recovered from

$$\Delta z_{it} = A_i^{12} \Delta^2 n_{it} + C_{i0}^{11} \Delta z_{it}^* + C_i^{12} \Delta^2 n_{it}^* + A_{i1}^{11} \Delta z_{it-1} + C_{i1}^{11} \Delta z_{it-1}^* + \varepsilon_{it}^T, \quad (A2)$$

with  $A_i^{12} = A_{i0}^{12} = -A_{i1}^{12}$  and  $C_i^{12} = C_{i0}^{12} = -C_{i1}^{12}$ . To estimate the equation above we need at least a single instrument to estimate the contemporaneous effect of productivity and labor input growth,  $A_i^{12}$ , the usual procedure of using  $\Delta n_{it-1}$  has been criticized as this practice may suffer from a weak instrument problem<sup>53</sup>. Specifically, consider the reduced form VARX representation of the system

$$\mathbf{\Phi}_{i}(L) \begin{pmatrix} \Delta z_{it} \\ \Delta n_{it} \end{pmatrix} = \mathbf{\Psi}_{i}(L) \begin{pmatrix} \Delta z_{it}^{*} \\ \Delta n_{it}^{*} \end{pmatrix} + \mathbf{e}_{it},$$

the first difference of the second variable  $(\Delta^2 n_{it})$ , considering the simple case of a VARX(1,1), i.e.  $\Phi_i(\mathbf{L}) = (I - \Phi_{i1}L)$  and  $\Psi_i(L) = (\Psi_{i0} - \Psi_{i1}L)$ , can written as

$$\Delta^2 n_{it} = \Phi_{i1}^{21} \Delta z_{it-1} + (\Phi_{i1}^{22} - 1) \Delta n_{it-1} + \Psi_{i0}^{21} \Delta z_{it}^* + \Psi_{i0}^{22} \Delta n_{it}^* + \Psi_{i1}^{21} \Delta z_{it-1}^* + \Psi_{i1}^{22} \Delta n_{it-1}^* + e_{it}^2,$$

 $<sup>^{53}</sup>$ See Staiger and Stock (1997) for a discussion of the weak instrument problem.

therefore the validity of  $\Delta n_{it-1}$  as an instrument clearly depends on the condition  $\Phi_{i1}^{22} \neq 1$ . So if  $\Phi_{i1}^{22}$  is close enough to 1 then the use of  $\Delta n_{it-1}$  as instrument for  $\Delta^2 n_{it}$  is subject to the weak instrument problem<sup>54</sup>. Rewriting the expression as a function of  $\Delta^2 n_{it-1}^*$  we obtain

$$\Delta^{2} n_{it} = \Phi_{i1}^{21} \Delta z_{it-1} + (\Phi_{i1}^{22} - 1) \Delta n_{it-1} + \Psi_{i0}^{21} \Delta z_{it}^{*} + \Psi_{i0}^{22} \Delta^{2} n_{it}^{*} + \Psi_{i1}^{21} \Delta z_{it-1}^{*} + (\Psi_{i1}^{22} + \Psi_{i0}^{22}) \Delta n_{it-1}^{*} + e_{it}^{2}.$$

The expression above makes clear that the aggregate labor input,  $\Delta n_{it-1}^*$ , constitutes an additional appropriate instrument for  $\Delta^2 n_{it}$  if  $(\Psi_{i0}^{22} + \Psi_{i1}^{22}) \neq 0$ , i.e. if the long run effect of an aggregate non-technology shock on the sector specific labour input is not zero. This condition corresponds to the long run neutrality of aggregate shocks to the labour input, as considered in Campbell et al. (1996). However, as they recognize this restriction is quite restrictive and not entirely innocuous<sup>55</sup>. In the light of this we include  $\Delta n_{it-1}^*$  as an additional instrument for the identification of  $A_i^{12}$  above.

Once (A2) has been estimated the residual (the technology shock,  $\varepsilon_{it}^T$ ) can be used to instrument the second relation for the labour input in (A1), which will deliver the non-technology shock to sector i,  $\varepsilon_{it}^{NT}$ , from

$$\Delta n_{it} = A_{i0}^{21} \Delta z_{it} + C_{i0}^{21} \Delta z_{it}^* + C_{i0}^{22} \Delta n_{it}^* + A_{i1}^{21} \Delta z_{it-1} + A_{i1}^{22} \Delta n_{it-1} + C_{i1}^{21} \Delta z_{it-1}^* + C_{i1}^{22} \Delta n_{it-1}^* + \varepsilon_{it}^{NT}.$$

The assumption of independence between the shocks insures that the shock is a good instrument to recover the contemporaneous effect of labour productivity on the labour input.

<sup>&</sup>lt;sup>54</sup>This is the well known condition  $A(1) \neq 0$  for a general VAR of order p, see Christiano et al. (2004) and Fry and Pagan (2005) for a discussion.

<sup>&</sup>lt;sup>55</sup>See Campbell el al. (1996), footnote 4 p. 96. For instance, theories of "reallocation timing" suggest that transitory aggregate shocks may be associated with permanent changes in industry size.

# Appendix B: Some details of the transmission mechanism of shocks

Here we discuss the interpretation of the impulse response function of a shock to a particular sector i. We focus on the impact effect, the generalization to any other horizon is straightforward. Recall that the SecVAR system<sup>56</sup> estimates a separate (n-dimensional) system for each sector i

$$\mathbf{A}_{i0}\varkappa_{it} = \mathbf{C}_{i0}\varkappa_{it}^* + \mathbf{A}_{i1}\varkappa_{it-1} + \mathbf{C}_{i1}\varkappa_{it-1}^* + \boldsymbol{\varepsilon}_{it},$$

stacking all the sectors in the economy a model for the full economy can be written

$$\mathbf{G}_0\mathbf{x}_t = \mathbf{G}_1\mathbf{x}_{t-1} - \mathbf{u}_t,$$

where  $\mathbf{x}_t$  is a  $nN \times 1$  vector containing all the n variables of the N sectors in the economy, therefore  $\mathbf{u}_t$  is a vector of the same size with the corresponding identified shocks. The matrix of coefficients  $\mathbf{G}_j$  for j = 1, 2 is an  $nN \times nN$  matrix composed such that

$$\mathbf{G}_j = \left[ egin{array}{c} \mathbf{B}_{j1} \mathbf{W}_1 \ & ... \ & ... \ \mathbf{B}_{jn} \mathbf{W}_n \end{array} 
ight],$$

with  $\mathbf{B}_{0i} = \begin{bmatrix} \mathbf{A}_{0i}, & -\mathbf{C}_{0i} \end{bmatrix}$  and  $\mathbf{B}_{1i} = \begin{bmatrix} \mathbf{A}_{1i}, & \mathbf{C}_{1i} \end{bmatrix}$ ,  $n \times 2n$  matrices. The sector specific weighting matrices  $\mathbf{W}_i$  are  $2n \times nN$  matrices, and (in this specific case) can be written as

$$egin{array}{lll} \mathbf{W}_i &=& \left[ egin{array}{ccc} \mathbf{0} & \mathbf{I}_n & \mathbf{0} \ & \mathbf{I}_n & \mathbf{0} & \mathbf{I}_n \ & \mathbf{IO}_i \otimes \mathbf{I}_k \end{array} 
ight] \ &=& \mathbf{\Xi}_i \otimes \mathbf{I}_k \end{array}$$

where  $\mathbf{I}_k$  is the k-dimension identity matrix,  $\mathbf{IO}$  is the input output matrix denoting the relation between the sectors in the economy, normalized such that the diagonal is all 0 and the row sum is equal to 1. Therefore,  $\mathbf{io}_i$  denotes the row i of the normalized matrix  $\mathbf{IO}$ .  $\mathbf{\Xi}_i$  for a particular sector i can be written as

$$\mathbf{\Xi}_i = \left[egin{array}{c} \mathbf{ind}_n(i) \ \mathbf{io}_i \end{array}
ight]$$

<sup>&</sup>lt;sup>56</sup>Notice that here we consider the simplest case with no deterministic component and no exogeneous variables. A system with number of weak exogenous aggregate variable equal to the number of endogenous variables in the system.

where  $\mathbf{ind}_n(i)$  is a  $1 \times n$  indicator vector, where the i-th element is equal to 1 and the rest equal to 0.

Note that the matrices  $\mathbf{G}_j$  can be rewritten such that the position in the matrix of the coefficients of the endogenous variables and the exogenous variables appears clearly in the matrix. This specification can be useful for disentangling the direct and complementarity (through the input output matrix) effect of a shock. Notice that the diagonal block of the matrix  $\mathbf{G}_j$  is composed of the matrices  $\mathbf{A}_{ji}$  for i = 1, ..., N.

As we focus on the impact effect the only relevant variable is  $\mathbf{G}_0$ , and we focus on this from now onwards. Let us introduce the  $nN \times n$  indicator matrix,  $\mathbf{IND}_{nk}^i$ , that extracts the i-th block of an  $nN \times nN$  matrix.

$$\mathbf{IND}_{nk}^i = \mathbf{ind}_n(i) \otimes \mathbf{I}_n,$$

where  $\mathbf{ind}_n(i)$  is the  $1 \times n$  indicator vector introduced above and  $\mathbf{I}_n$  the usual identify matrix. Then,  $\mathbf{G}_0$  can written such that the i-th  $n \times n$  block diagonal element is  $\mathbf{A}_{0i}$  and in general the i-th  $n \times nN$  block of the matrix can be written as

$$\left(\mathbf{IND}_{nk}^i\right)' imes \mathbf{G}_0 = \left[ \begin{array}{c} \mathbf{io}_i^{[1:(i-1)]} \otimes (-\mathbf{C}_{0i}), & \mathbf{A}_{0i}, & \mathbf{io}_i^{[i+1:N]} \otimes (-\mathbf{C}_{0i}) \end{array} 
ight],$$

where  $\mathbf{io}_i^{[j:k]}$  is the  $1 \times (k-j)$  vector corresponding to the j to k elements of  $\mathbf{io}_i$ . Let us focus on the impulse response to the first sector, the matrix of coefficients  $\mathbf{G}_0$  can therefore be easily partitioned as

$$\mathbf{G}_0 = \left[ egin{array}{cc} \mathbf{G}_{01} & \mathbf{G}_{0}^{12} \ \mathbf{G}_{0}^{21} & \mathbf{G}_{0}^{22} \end{array} 
ight],$$

with  $\mathbf{G}_0^{12} = \mathbf{io}_i^{[2:N]} \otimes (-\mathbf{C}_{0i})$   $(n \times (N-1)n \text{ matrix})$ , and the  $(N-1)n \times n \text{ matrix}$   $\mathbf{G}_0^{21}$ 

$$\mathbf{G}_0^{21} = \left[egin{array}{c} \mathbf{io}_2^{[1]} \otimes (-\mathbf{C}_{02}) \ & ... \ \mathbf{io}_2^{[N]} \otimes (-\mathbf{C}_{0N}) \end{array}
ight],$$

Understanding of the role of the matrices  $\mathbf{G}_0^{12}$  and  $\mathbf{G}_0^{21}$  is essential for the decomposition of the impulse response into all its components (direct and complementarity, and the amplification mechanism). Note that  $\mathbf{G}_0^{11} = \mathbf{A}_{01}$  and therefore it corresponds to the coefficients of the VAR for the sector under analysis (sector 1).  $\mathbf{G}_0^{21}$  summarizes the effect of a shock to sector 1 to all the other sectors. Specifically, for each sector different than 1 this is equal to the effect of the aggregate variables in those sectors scaled by the importance of sector 1 in those sectors, where this is measured by the factor share of intermediate input from sector 1. In addition,  $\mathbf{G}_0^{12}$  reflects the effect of the aggregate variables on sector 1, where the aggregate

variables are constructed by scaling the variables in the other sectors by size. The latter effect is the effect related to impact changes to the supplier sectors of sector 1.

The contemporaneous effect of an idiosyncratic shock in sector 1 to all the variables in the system can now be found as follows. The SecVAR above is inverted to give

$$\mathbf{G}_0 \mathbf{x}_t = \mathbf{G}_1 \mathbf{x}_{t-1} + \mathbf{u}_t,$$
  
 $\mathbf{x}_t = \mathbf{G}_0^{-1} \mathbf{G}_1 \mathbf{x}_{t-1} + \mathbf{G}_0^{-1} \mathbf{u}_t,$ 

Denote the matrix  $\mathbf{G}_0^{-1}\mathbf{G}_1 = \mathbf{F}$ . The impulse response at any horizon h from the shock j to sector i can be written as

$$\boldsymbol{\psi}(h) = \mathbf{F}^h \mathbf{G}_0^{-1} \mathbf{s}_{ii}$$

where  $\mathbf{s}_{ji}$  is a  $nk \times 1$  selection vector with the only non-null element, selecting the appropriate shock j in sector i. Here we consider the effect of a technology shock in the first sector, therefore ordering the variables as in the main text, such that

productivity comes first, 
$$\mathbf{s}_{11} = \begin{bmatrix} \boldsymbol{\varrho}_1' & \mathbf{0} \\ 1 \times [(N-1)n] \end{bmatrix}' = \begin{bmatrix} 1 & \mathbf{0} \\ 1 \times (2N-1) \end{bmatrix}'$$
, as in the

bivariate model  $\boldsymbol{\varrho}_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}'$ . The contemporaneous impulse response (i.e. the impact effect)<sup>57</sup> is

$$\boldsymbol{\psi}(0) = \mathbf{G}_0^{-1} \mathbf{s}_{11}$$

therefore, to understand the different effect we need to understand what happens when we invert  $\mathbf{G}_0$ . Applying the partition matrix inversion lemma<sup>58</sup>

$$\mathbf{G}_0 = \left[ egin{array}{ccc} \mathbf{A}_{01} & \mathbf{G}_0^{12} \ \mathbf{G}_0^{21} & \mathbf{G}_0^{22} \end{array} 
ight], \ \mathbf{G}_0^{-1} = \left[ egin{array}{cccc} \mathbf{A}_{01}^{-1} \left( \mathbf{I}_k + \mathbf{G}_0^{12} \mathbf{\Gamma}_0 \mathbf{G}_0^{21} \mathbf{A}_{01}^{-1} 
ight) & -\mathbf{A}_{01}^{-1} \mathbf{G}_0^{12} \mathbf{\Gamma}_0 \ -\mathbf{\Gamma}_0 \mathbf{G}_0^{21} \mathbf{A}_{01}^{-1} & \mathbf{\Gamma}_0 \end{array} 
ight],$$

<sup>58</sup> For the general  $2 \times 2$  partitioned matrix, one form of the partitioned inverse is

$$\left[ \begin{array}{cc} \boldsymbol{\Phi}_{11} & \boldsymbol{\Phi}_{12} \\ \boldsymbol{\Phi}_{21} & \boldsymbol{\Phi}_{22} \end{array} \right]^{-1} = \left[ \begin{array}{cc} \boldsymbol{\Phi}_{11}^{-1} \left( \mathbf{I} + \boldsymbol{\Phi}_{12} \mathbf{F}_2 \boldsymbol{\Phi}_{21} \boldsymbol{\Phi}_{11}^{-1} \right) & -\boldsymbol{\Phi}_{11}^{-1} \boldsymbol{\Phi}_{12} \mathbf{F}_2 \\ -\mathbf{F}_2 \boldsymbol{\Phi}_{21} \boldsymbol{\Phi}_{11}^{-1} & \mathbf{F}_2 \end{array} \right],$$

where

$$\mathbf{F}_2 = \left(\mathbf{\Phi}_{22} - \mathbf{\Phi}_{21}\mathbf{\Phi}_{11}^{-1}\mathbf{\Phi}_{12}\right)^{-1},$$

<sup>&</sup>lt;sup>57</sup>Starting from the impact effect, the impulse response for any horizon h can be calculated as  $\psi(h) = \mathbf{F}\psi(h-1)$ .

with  $\Gamma_0 = (\mathbf{G}_0^{22} - \mathbf{G}_0^{21} \mathbf{A}_{01}^{-1} \mathbf{G}_0^{12})^{-1}$ . Notice that for the impact effect the selection vector  $\mathbf{s}_{11}$  implicitly selects the first n column of  $\mathbf{G}_0^{-1}$ , specifically

$$\psi(0) = \mathbf{G}_{0}^{-1}\mathbf{s}_{11},$$

$$= \begin{bmatrix} \mathbf{A}_{01}^{-1} \left( \mathbf{I}_{k} + \mathbf{G}_{0}^{12}\boldsymbol{\Gamma}_{0}\mathbf{G}_{0}^{21}\mathbf{A}_{01}^{-1} \right) \boldsymbol{\varrho}_{1} \\ -\boldsymbol{\Gamma}_{0}\mathbf{G}_{0}^{21}\mathbf{A}_{01}^{-1}\boldsymbol{\varrho}_{1} \end{bmatrix},$$

$$= \begin{bmatrix} \mathbf{A}_{01}^{-1}\boldsymbol{\varrho}_{1} + \mathbf{A}_{01}^{-1}\mathbf{G}_{0}^{12}\boldsymbol{\Gamma}_{0}\mathbf{G}_{0}^{21}\mathbf{A}_{01}^{-1}\boldsymbol{\varrho}_{1} \\ -\boldsymbol{\Gamma}_{0}\mathbf{G}_{0}^{21}\mathbf{A}_{01}^{-1}\boldsymbol{\varrho}_{1} \end{bmatrix},$$

The  $((nN-2)\times 1)$  subvector  $\kappa_{comp} = (-\Gamma_0 \mathbf{G}_0^{21} \mathbf{A}_{01}^{-1} \boldsymbol{\varrho}_1)$  is what we have referred to as the complementary effect, i.e. this is the effect that a shock to sector 1 has on all the other sectors in the economy through sectoral complementarity. This is equal to the effect that the shock would have had to sector 1, if the sector was not connected to other sectors,  $\mathbf{A}_{01}^{-1}\boldsymbol{\varrho}_1$ , which is first transmitted to the other sectors through the downstream supplier user relations, exemplified by  $\mathbf{G}_0^{21}$ . These effects are further amplified by the interconnetitivity properties of the input output matrix, that directly or indirectly (i.e. through a third sector) links up all the sectors in the economy. This mechanism is embodied in  $\Gamma_0$ . Notice that the minus sign of  $\kappa_{comp}$  is going to balance the negative sign in  $\mathbf{G}_0^{21}$  that come by the fact that the matrix of coefficients associated with the intermediate input channel, the  $\mathbf{C}_{i0}, \forall i \neq 1$ , enters the system with a negative sign. Therefore, the sign of  $\kappa_{comp}$  reflects the sign of the estimated  $\mathbf{C}_{i0}, \forall i \neq 1$ .

What we label in the text as the *direct effect* is the effect of the shock to the same sector where the shock has originated. This corresponds to the first  $n \times 1$  subvector of  $\psi(0)$ . Rewriting this as

$$egin{array}{lll} oldsymbol{\kappa}_{dir} &=& \mathbf{A}_{01}^{-1} oldsymbol{arrho}_1 + \mathbf{A}_{01}^{-1} \mathbf{G}_0^{12} oldsymbol{\Gamma}_0 \mathbf{G}_0^{21} \mathbf{A}_{01}^{-1} oldsymbol{arrho}_1, \ &=& \mathbf{A}_{01}^{-1} oldsymbol{arrho}_1 - \mathbf{A}_{01}^{-1} \mathbf{G}_0^{12} oldsymbol{\kappa}_{comp}, \end{array}$$

makes clear that this is composed of the effect that the shock would have had if there were no interactions,  $\mathbf{A}_{01}^{-1}\boldsymbol{\varrho}_{1}$ , plus a component that comes as an echo from the complementary effect<sup>59</sup>.

To underline the fact that the effect of a shock in a system with no interactions corresponds only to the first part of the *direct effect*, notice that if each sector is considered in isolation, the matrix  $\mathbf{G}_0$  block diagonal and its i-th diagonal element is the generic matrix  $\mathbf{A}_{0i}$ . Therefore, the inverse matrix  $\mathbf{G}_0^{-1}$  is itself a block diagonal matrix whose i-th diagonal element is the generic  $\mathbf{A}_{0i}^{-1}$ . It follows

that in this case the impact effect is 
$$\psi(0) = \begin{bmatrix} (\mathbf{A}_{01}^{-1} \boldsymbol{\varrho}_1)', & \mathbf{0}' \\ \mathbf{1} \times [(N-1)n] \end{bmatrix}'$$
.

 $<sup>\</sup>overline{\mathbf{G}_{0}^{12}}$  Note that also in this case the negative sign is neutralized by the fact that the  $\mathbf{C}_{01}$  enters  $\mathbf{G}_{0}^{12}$  with a negative sign.

# **Appendix C: Aggregation**

Here we explain how to obtain the aggregate series and impulse response for output and hours<sup>60</sup>. Small capitals indicate the logarithms of the variables, aggregate variables are denoted with a tilda. By definition aggregate hours is

$$\widetilde{N}_t = \sum_i N_{it},$$

therefore the growth rate of (aggregate) total hours can be written as

$$\Delta \widetilde{n}_{t} = \log \left( \frac{\widetilde{N}_{t}}{\widetilde{N}_{t-1}} \right) = \log \left( \frac{\sum_{i} N_{it}}{\sum_{i} N_{it-1}} \right)$$
$$\simeq \log \left( \sum_{i} \omega_{i} \exp(\Delta n_{it}) \right),$$

where  $\omega_i$  is an appropriate aggregation weight that reflects industry size. In the application we use fixed weights and construct them from the average shipment value of sales during all the sample period.

Similarly, aggregate output growth is computed as

$$\Delta \widetilde{q}_t \simeq \log \left( \sum_i \omega_i \exp(\Delta x_{it} + \Delta n_{it}) \right),$$

<sup>&</sup>lt;sup>60</sup>Note that in the text we defined log of hours with  $n_{it}$ , and (labor) productivity  $x_{it}$ . Labor productivity is defined as output per hours worked, therefore we can define (the log of) output as  $q_{it} = x_{it} + n_{it}$ .