Preprint typeset in JHEP style - PAPER VERSION

Cavendish–HEP–06/15

Addendum to "Distinguishing Spins in Decay Chains at the Large Hadron Collider"*

Christiana Athanasiou[1], **Christopher G. Lester**[2], **Jennifer M. Smillie**[3] **and Bryan R. Webber**[4]

Cavendish Laboratory, University of Cambridge,
JJ Thomson Avenue, Cambridge CB3 0HE, U.K.
[1]*E-mail:* `ca274@cam.ac.uk`
[2]*E-mail:* `lester@hep.phy.cam.ac.uk`
[3]*E-mail:* `smillie@hep.phy.cam.ac.uk`
[4]*E-mail:* `webber@hep.phy.cam.ac.uk`

Abstract: We extend our earlier study of spin correlations in the decay chain $D \to Cq$, $C \to Bl^{\rm near}$, $B \to Al^{\rm far}$, where A, B, C, D are new particles with known masses but undetermined spins, $l^{\rm near}$ and $l^{\rm far}$ are opposite-sign same-flavour charged leptons and A is invisible. Instead of looking at the observable 2- and 3-particle invariant mass distributions separately, we compare the full three-dimensional phase space distributions for all possible spin assignments of the new particles, and show that this enhances their distinguishability using a quantitative measure known as the Kullback-Leibler distance.

Keywords: Hadronic Colliders, Beyond Standard Model, Supersymmetry Phenomenology, Large Extra Dimensions.

*Work supported in part by the UK Particle Physics and Astronomy Research Council.

Contents

1. Introduction

In the recent paper [1], to which we refer the reader for motivation, notation and relevant references, we examined the distinguishability of different spin assignments in the decay chain $D \to Cq$, $C \to Bl^{\text{near}}$, $B \to Al^{\text{far}}$, where A, B, C, D are new particles with known masses but undetermined spins, l^{near} and l^{far} are opposite-sign same-flavour charged leptons and A is invisible. This was done by comparing separately the invariant mass distributions of the three observable two-body combinations: dileptons (m_{ll}), quark- or antiquark-jet plus positive lepton (m_{jl^+}), and jet plus negative lepton (m_{jl^-}).[1]

If $P(m|S)$ represents the normalized probability distribution of any one of these three invariant masses predicted by spin assignment S, and T is the true spin configuration, then a measure of the improbability of S is provided by the *Kullback-Leibler distance*

$$\mathrm{KL}(T,S) = \int_m \log\left(\frac{P(m|T)}{P(m|S)}\right) P(m|T)dm \; . \tag{1.1}$$

In particular, the number N of events required to disfavour hypothesis S by a factor of $1/R$ under ideal conditions, assuming equal prior probabilities of S and T, would be

$$N \sim \frac{\log R}{\mathrm{KL}(T,S)} \; . \tag{1.2}$$

By ideal conditions we mean isolation of the decay chain with no background and perfect resolution. Therefore N sets a lower limit on the number of events that would be needed in real life. The results for $R = 1000$ are shown in tables 1-3, reproduced for convenience from [1], where a discussion of them can be found. Recall that the notation used is $DCBA$ with F for fermion, S for scalar, V for vector, so that squark decay in SUSY is SFSF and excited quark decay in UED is FVFV. Mass spectra I and II are SUSY- and UED-like respectively (see [1] for details).

[1]The three-body invariant mass m_{jll} was also studied but this is not independent of the two-body masses.

2. Three-dimensional analysis

To extract the most information from the data we should compare the predictions of different spin assigments with the full probability distribution in the three-dimensional space of m_{ll}, m_{jl^+} and m_{jl^-}. The ambiguity between near and far leptons means that this given by

$$P(m_{ll}, m_{jl^+}, m_{jl^-}) = \frac{1}{2} f_q \left[P_2(m_{ll}, m_{jl^+}, m_{jl^-}) + P_1(m_{ll}, m_{jl^-}, m_{jl^+}) \right] + \frac{1}{2} f_{\bar{q}} \left[P_1(m_{ll}, m_{jl^+}, m_{jl^-}) + P_2(m_{ll}, m_{jl^-}, m_{jl^+}) \right] , \tag{2.1}$$

where f_q and $f_{\bar{q}} = 1 - f_q$ are the fractions of quark- and antiquark-like objects D initiating the decay chain and we use $P_{1,2}(m_{ll}, m_{jl}^{\mathrm{near}}, m_{jl}^{\mathrm{far}})$ on the right-hand side, assuming both leptons are left-handed, otherwise f_q and $f_{\bar{q}}$ are interchanged. The subscripts 1 and 2 refer to processes 1 and 2 defined in [1] and the factors of one-half enter because $P_{1,2}$ are both normalized to unity.

Instead of trying to evaluate the three-dimensional generalization of the integral in eq. (1.1) analytically, it is convenient to perform a Monte Carlo integration. If we generate m_{ll}, m_{jl}^{near} and m_{jl}^{far} according to phase space, the weight to be assigned to the configuration $l^{\mathrm{near}} = l^+$, $l^{\mathrm{far}} = l^-$ is

$$P_{+-}(m_{ll}, m_{jl}^{\mathrm{near}}, m_{jl}^{\mathrm{far}}) = \frac{1}{2} \left[f_q P_2(m_{ll}, m_{jl}^{\mathrm{near}}, m_{jl}^{\mathrm{far}}) + f_{\bar{q}} P_1(m_{ll}, m_{jl}^{\mathrm{near}}, m_{jl}^{\mathrm{far}}) \right] \tag{2.2}$$

while that for $l^{\mathrm{near}} = l^-$, $l^{\mathrm{far}} = l^+$ is

$$P_{-+}(m_{ll}, m_{jl}^{\mathrm{near}}, m_{jl}^{\mathrm{far}}) = \frac{1}{2} \left[f_q P_1(m_{ll}, m_{jl}^{\mathrm{near}}, m_{jl}^{\mathrm{far}}) + f_{\bar{q}} P_2(m_{ll}, m_{jl}^{\mathrm{near}}, m_{jl}^{\mathrm{far}}) \right] . \tag{2.3}$$

In the former case, since the distinction between l^{near} and l^{far} is lost in the data (except when interchanging them gives a point outside phase space), we must use eq. (2.1) with $l^+ = l^{\mathrm{near}}$, $l^- = l^{\mathrm{far}}$ in the logarithmic factor of the KL-distance, i.e. the contribution is

$$\log\left(\frac{P_{+-}(m_{ll}, m_{jl}^{\mathrm{near}}, m_{jl}^{\mathrm{far}}|T) + P_{-+}(m_{ll}, m_{jl}^{\mathrm{far}}, m_{jl}^{\mathrm{near}}|T)}{P_{+-}(m_{ll}, m_{jl}^{\mathrm{near}}, m_{jl}^{\mathrm{far}}|S) + P_{-+}(m_{ll}, m_{jl}^{\mathrm{far}}, m_{jl}^{\mathrm{near}}|S)} \right) P_{+-}(m_{ll}, m_{jl}^{\mathrm{near}}, m_{jl}^{\mathrm{far}}|T) . \tag{2.4}$$

Similarly from the configuration $l^{\mathrm{near}} = l^-$, $l^{\mathrm{far}} = l^+$ we get the contribution

$$\log\left(\frac{P_{-+}(m_{ll}, m_{jl}^{\mathrm{near}}, m_{jl}^{\mathrm{far}}|T) + P_{+-}(m_{ll}, m_{jl}^{\mathrm{far}}, m_{jl}^{\mathrm{near}}|T)}{P_{-+}(m_{ll}, m_{jl}^{\mathrm{near}}, m_{jl}^{\mathrm{far}}|S) + P_{+-}(m_{ll}, m_{jl}^{\mathrm{far}}, m_{jl}^{\mathrm{near}}|S)} \right) P_{-+}(m_{ll}, m_{jl}^{\mathrm{near}}, m_{jl}^{\mathrm{far}}|T) . \tag{2.5}$$

Denoting the sum of these two contributions at the ith phase space point by $\mathrm{KL}_i(T, S)$, and summing over M such points, we have as $M \to \infty$

$$\frac{M \log R}{\sum_i \mathrm{KL}_i(T, S)} \to N , \tag{2.6}$$

which is the Monte Carlo equivalent of eq. (1.2). Results for $R = 1000$ and $M = 5 \times 10^7$ are shown in table 4. By comparing with tables 1-3, we see that, as might be expected, the three-dimensional analysis achieves a discrimination that is better than that of a one-dimensional analysis applied to any single invariant mass distribution. This could be particularly useful in difficult cases like that of distinguishing between SFSF (SUSY) and FVFV (UED).

Acknowledgements

We thank Sabine Kraml and members of the Cambridge Supersymmetry Working Group for helpful comments.

References

[1] C. Athanasiou, C. G. Lester, J. M. Smillie and B. R. Webber, arXiv:hep-ph/0605286.

(a)	SFSF	FVFV	FSFS	FVFS	FSFV	SFVF
SFSF	∞	60486	23	148	15608	66
FVFV	60622	∞	22	164	6866	62
FSFS	36	34	∞	16	39	266
FVFS	156	173	11	∞	130	24
FSFV	15600	6864	25	122	∞	76
SFVF	78	73	187	27	90	∞

(b)	SFSF	FVFV	FSFS	FVFS	FSFV	SFVF
SFSF	∞	3353	23	304	427	80
FVFV	3361	∞	27	179	232	113
FSFS	36	44	∞	20	22	208
FVFS	313	184	14	∞	13077	35
FSFV	436	236	15	12957	∞	39
SFVF	89	126	134	38	42	∞

Table 1: The number of events needed to disfavour the column model with respect to the row model by a factor of 0.001, assuming the data to come from the row model, for the $\widehat{m}^2_{ll}$ distribution: (a) mass spectrum I and (b) mass spectrum II.

(a)	SFSF	FVFV	FSFS	FVFS	FSFV	SFVF
SFSF	∞	1059	205	1524	758	727
FVFV	1090	∞	404	3256	4363	1746
FSFS	278	554	∞	418	741	870
FVFS	1605	3242	345	∞	1256	2365
FSFV	749	4207	507	1212	∞	1803
SFVF	813	1821	751	2415	1888	∞

(b)	SFSF	FVFV	FSFS	FVFS	FSFV	SFVF
SFSF	∞	3006	958	6874	761	1280
FVFV	2961	∞	4427	1685	2749	3761
FSFS	914	4201	∞	743	9874	4877
FVFS	6716	1699	752	∞	656	1306
FSFV	720	2666	10279	649	∞	4138
SFVF	1141	3517	5269	1276	4259	∞

Table 2: As in table 1, for the $\widehat{m}^2_{jl+}$ distribution.

(a)	SFSF	FVFV	FSFS	FVFS	FSFV	SFVF
SFSF	∞	1058	505	769	816	619
FVFV	1090	∞	541	5878	4821	445
FSFS	565	714	∞	1032	741	2183
FVFS	799	6435	882	∞	2742	510
FSFV	806	4641	507	2451	∞	413
SFVF	692	541	2272	576	521	∞

(b)	SFSF	FVFV	FSFS	FVFS	FSFV	SFVF
SFSF	∞	3037	689	8633	925	967
FVFV	2985	∞	2271	1431	4368	2527
FSFS	707	2297	∞	526	9874	5004
FVFS	8392	1450	525	∞	653	843
FSFV	924	4287	10279	640	∞	4036
SFVF	1047	2693	5213	870	4041	∞

Table 3: As in table 1, for the $\widehat{m}^2_{jl-}$ distribution.

(a)	SFSF	FVFV	FSFS	FVFS	FSFV	SFVF
SFSF	∞	455	21	47	348	55
FVFV	474	∞	21	54	1387	55
FSFS	33	34	∞	13	39	188
FVFS	55	67	10	∞	54	19
FSFV	341	1339	25	45	∞	66
SFVF	62	64	143	19	79	∞

(b)	SFSF	FVFV	FSFS	FVFS	FSFV	SFVF
SFSF	∞	1053	21	230	194	63
FVFV	1047	∞	27	135	190	90
FSFS	33	42	∞	19	22	175
FVFS	242	140	13	∞	332	33
FSFV	189	194	14	315	∞	37
SFVF	66	95	118	35	41	∞

Table 4: As in table 1, for the combined three-dimensional distribution.