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Mass bounds in a model with a Triplet Higgs

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Abstract

We perform an analysis of the Renormalization Group evolution of the couplings in an extension to the Standard Model which contains a real triplet in the Higgs sector. Insisting that the model remain valid up to 1 TeV allow us to map out the region of allowed mass for the Higgs bosons. We conclude that it is possible for there to be no light Higgs bosons without any otherwise dramatic deviation from the physics of the Standard Model.

1 Introduction

In a previous paper, we studied an extension of the Standard Model in which a real scalar $SU(2)$ triplet with zero hypercharge is added to the usual scalar $SU(2)$ doublet [1]. We showed that such an extension is allowed by the precision data and that the mass of the lightest Higgs boson can be as big as 500 GeV.

To recap, the Lagrangian of the model in terms of the usual Standard Model Higgs, Φ_1, and the new triplet, Φ_2, reads

$$\mathcal{L} \;=\; (D_\mu \Phi_1)^\dagger \; D^\mu \Phi_1 + \frac{1}{2}(D_\mu \Phi_2)^\dagger \; D^\mu \Phi_2 - V_0(\Phi_1, \Phi_2), \tag{1}$$

with a scalar potential

$$\begin{aligned} V_0(\Phi_1, \Phi_2) \;&=\; \mu_1^2 \, |\Phi_1|^2 + \frac{\mu_2^2}{2} \, |\Phi_2|^2 + \lambda_1 \, |\Phi_1|^4 + \frac{\lambda_2}{4} \, |\Phi_2|^4 + \frac{\lambda_3}{2} \, |\Phi_1|^2 \, |\Phi_2|^2 \\ &+\; \lambda_4 \, {\Phi_1}^\dagger \sigma^\alpha \Phi_1 \; \Phi_{2\alpha}. \end{aligned} \tag{2}$$

σ^α are the Pauli matrices. The expansion of the field components is

$$\Phi_1 = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}\,(h_c^0 + h^0 + i\phi^0) \end{pmatrix}_{Y=1}, \quad \Phi_2 = \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_c^0 + \eta^0 \end{pmatrix}_{Y=0} \tag{3}$$

where $\eta^\pm = (\eta_1 \mp i\eta_2)/\sqrt{2}$ and ϕ^0 is the Goldstone boson which is eaten by the Z^0.

The model violates custodial symmetry at tree level giving a prediction for the ρ-parameter of

$$\rho \;=\; 1 + 4\left(\frac{\eta_c^0}{h_c^0}\right)^2 . \tag{4}$$

As discussed in [1], it is precisely this violation of custodial symmetry which allows the lightest Higgs to be much heavier than in the Standard Model. By giving the triplet a non-zero vacuum expectation value, one is in effect making a positive tree-level contribution to the T-parameter, and this is enough to allow a heavier Higgs.

In the neutral Higgs sector we have two CP-even states which mix with angle γ. The mass eigenstates $\{H^0, N^0\}$ are defined by

$$\begin{pmatrix} H^0 \\ N^0 \end{pmatrix} \;=\; \begin{pmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \end{pmatrix} \begin{pmatrix} h^0 \\ \eta^0 \end{pmatrix} . \tag{5}$$

There is also mixing in the charged Higgs sector. We define the mass eigenstates $\{g^\pm, h^\pm\}$ by

$$\begin{pmatrix} g^\pm \\ h^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi^\pm \\ \eta^\pm \end{pmatrix}. \tag{6}$$

The $g^\pm$ are the Goldstone bosons corresponding to $W^\pm$ and, at tree level, the mixing angle is

$$\tan\beta = 2\frac{\eta_c^0}{h_c^0}. \tag{7}$$

The precision electroweak data constrain β to be smaller than about $4°$ [1].

In this paper we wish to examine the renormalization group flow of the couplings and hence establish bounds on the scalar masses under the assumption that the triplet model remain valid up to some scale Λ. We take $\Lambda = 1$ TeV and make no statements about physics at higher scales. For the Lagrangian of (1) to remain appropriate up to Λ, we demand that the scalar couplings λ_i remain perturbative and that the vacuum remain stable (i.e. is a local minimum) up to Λ. We begin in the next section with the calculation of the beta-functions. In Section 3 we present our results away from the decoupling limit of the model and in Section 4 we discuss decoupling.

2 The one-loop effective potential and the beta-functions

The effective potential [2, 3, 4, 5, 6, 7] has the following one-loop expansion in the $\overline{\text{MS}}$ renormalization scheme and 't Hooft-Landau gauge:

$$\begin{aligned}
V &= V_0 + V_{\text{CT}} + V_1 \\
&= \frac{1}{2}{\mu_1}^2\, h_c^{02} + \frac{1}{2}{\mu_2}^2\, \eta_c^{02} + \frac{1}{4}\lambda_1\, h_c^{04} + \frac{1}{4}\lambda_2\, \eta_c^{04} + \frac{1}{4}\lambda_3\, h_c^{02}\, \eta_c^{02} - \frac{1}{2}\lambda_4\, h_c^{02}\, \eta_c^0 \\
&+ \delta\Omega - \frac{1}{2}\delta{\mu_1}^2\, h_c^{02} - \frac{1}{2}\delta{\mu_2}^2\, \eta_c^{02} + \frac{1}{4}\delta\lambda_1\, h_c^{04} + \frac{1}{4}\delta\lambda_2\, \eta_c^{04} + \frac{1}{4}\delta\lambda_3\, h_c^{02}\, \eta_c^{02} - \frac{1}{2}\delta\lambda_4\, h_c^{02}\, \eta_c^0 \\
&+ \frac{1}{16\pi^2}\left\{\frac{3}{4}m_Z^4\left(\log\frac{m_Z^2}{\mu^2} - \frac{5}{6}\right) + \frac{3}{2}m_W^4\left(\log\frac{m_W^2}{\mu^2} - \frac{5}{6}\right) - 3\, m_t^4\left(\log\frac{m_t^2}{\mu^2} - \frac{3}{2}\right)\right. \\
&+ \frac{1}{4}m_{\phi^0}^4\left(\log\frac{m_{\phi^0}^2}{\mu^2} - \frac{3}{2}\right) + \frac{1}{2}m_{g^\pm}^4\left(\log\frac{m_{g^\pm}^2}{\mu^2} - \frac{3}{2}\right) + \frac{1}{2}m_{h^\pm}^4\left(\log\frac{m_{h^\pm}^2}{\mu^2} - \frac{3}{2}\right) \\
&+ \left.\frac{1}{4}m_{H^0}^4\left(\log\frac{m_{H^0}^2}{\mu^2} - \frac{3}{2}\right) + \frac{1}{4}m_{N^0}^4\left(\log\frac{m_{N^0}^2}{\mu^2} - \frac{3}{2}\right)\right\} \\
&- \frac{C_{\text{UV}}}{64\pi^2}\left\{3\, m_Z^4 + 6\, m_W^4 - 12\, m_t^4 + m_{\phi^0}^4 + 2\, m_{g^\pm}^4 + 2\, m_{h^\pm}^4 + m_{H^0}^4 + m_{N^0}^4\right\}.
\end{aligned} \tag{8}$$

μ is the renormalization scale and $C_{\text{UV}} = \frac{2}{4-D} - \gamma_E + \log 4\pi$. We have included the contributions from all the relevant physical states including the heaviest fermion, the top quark. The terms with δ correspond to the counterterms of the theory and the tree-level masses are

$$
\begin{aligned}
m_Z^2 &= \frac{1}{4} h_c^{02}\left(g^2+g'^2\right), && (9)\\
m_W^2 &= \frac{1}{4} g^2\, h_c^{02} + g^2\,\eta_c^{02}, && (10)\\
m_t^2 &= \frac{1}{2} {h_t}^2\, h_c^{02}, && (11)\\
m_{\phi^0}^2 &= {\mu_1}^2 + \lambda_1\, h_c^{02} + \frac{1}{2}\,\lambda_3\,\eta_c^{02} - \lambda_4\,\eta_c^0, && (12)\\
m_{g^\pm}^2 &= \mu_1^2 + \lambda_1\, h_c^{02} + \lambda_4\,\eta_c^0 + \frac{1}{2}\,\lambda_3\,\eta_c^{02} - \lambda_4\, h_c^0\,\tan\beta, && (13)\\
m_{h^\pm}^2 &= \mu_2^2 + \lambda_2\,\eta_c^{02} + \lambda_4\, h_c^0\,\tan\beta + \frac{1}{2}\,\lambda_3\, h_c^{02}, && (14)\\
m_{H^0}^2 &= {\mu_1}^2 + 3\,\lambda_1\, h_c^{02} + \frac{1}{2}\,\lambda_3\,\eta_c^{02} - \lambda_4\,\eta_c^0 + \lambda_4\, h_c^0\,\tan\gamma - \lambda_3\, h_c^0\,\eta_c^0\,\tan\gamma, && (15)\\
m_{N^0}^2 &= \mu_2^2 + 3\,\lambda_2\,\eta_c^{02} - \lambda_4\, h_c^0\,\tan\gamma + \frac{1}{2}\,\lambda_3\, h_c^0\left(h_c^0 + 2\,\eta_c^0\,\tan\gamma\right). && (16)
\end{aligned}
$$

It is understood that we should substitute explicitly for the mixing angles, which are solutions to the equations

$$
\lambda_4 h_c^0 + \tan\beta\left(\mu_1^2 - \mu_2^2 + \lambda_1 h_c^{02} - \frac{1}{2}\lambda_3 h_c^{02} + \lambda_4\eta_c^0 - \lambda_2\eta_c^{02} + \frac{1}{2}\lambda_3\eta_c^{02} - \lambda_4 h_c^0\tan\beta\right) = 0, \quad (17)
$$

$$
\begin{aligned}
&-\lambda_4 h_c^0 + \lambda_3 h_c^0\eta_c^0 + \tan\gamma\left(\mu_1^2 - \mu_2^2 + 3\lambda_1 h_c^{02} - \frac{1}{2}\lambda_3 h_c^{02} - \lambda_4\eta_c^0\right.\\
&\qquad \left. -3\lambda_2\eta_c^{02} + \frac{1}{2}\lambda_3\eta_c^{02} + \lambda_4 h_c^0\tan\gamma - \lambda_3 h_c^0\eta_c^0\tan\gamma\right) = 0. \quad (18)
\end{aligned}
$$

The expressions for the counterterms are thus

$$
\begin{aligned}
\delta\Omega &= \frac{C_{\rm UV}}{64\pi^2}\left(4\,\mu_1^4 + 3\,\mu_2^2\right), && (19)\\
\delta\mu_1^2 &= -\frac{C_{\rm UV}}{32\pi^2}\left(12\,\lambda_1\,\mu_1^2 + 3\lambda_3\,\mu_2^2 + 6\,\lambda_4^2\right), && (20)\\
\delta\mu_2^2 &= -\frac{C_{\rm UV}}{32\pi^2}\left(10\,\lambda_2\,\mu_2^2 + 4\,\lambda_3\mu_1^2 + 4\,\lambda_4^2\right), && (21)\\
\delta\lambda_1 &= \frac{C_{\rm UV}}{16\pi^2}\left(\frac{9}{16}\,g^4 - 3\,h_t^4 + 12\,\lambda_1^2 + \frac{3}{4}\,\lambda_3^2 + \frac{3}{8}\,g^2\,g'^2 + \frac{3}{16}\,g'^4\right), && (22)\\
\delta\lambda_2 &= \frac{C_{\rm UV}}{16\pi^2}\left(6\,g^4 + 11\,\lambda_2^2 + \lambda_3^2\right), && (23)\\
\delta\lambda_3 &= \frac{C_{\rm UV}}{16\pi^2}\left(3\,g^4 + 6\,\lambda_1\,\lambda_3 + 5\,\lambda_2\,\lambda_3 + 2\,\lambda_3^2\right), && (24)\\
\delta\lambda_4 &= \frac{C_{\rm UV}}{8\pi^2}\,\lambda_4\,\left(\lambda_1 + \lambda_3\right), && (25)
\end{aligned}
$$

where $\delta\Omega$ is the counterterm for the vacuum energy.

The fact that the theory should be independent of the unphysical mass μ implies that the couplings and masses acquire a μ dependence governed by the Renormalization Group (RG) equation for the one-loop effective potential, i.e.

$$\left(\beta_{\mu_1}\frac{\partial}{\partial\mu_1^2}+\beta_{\mu_2}\frac{\partial}{\partial\mu_2^2}+\beta_{\lambda_1}\frac{\partial}{\partial\lambda_1}+\beta_{\lambda_2}\frac{\partial}{\partial\lambda_2}+\beta_{\lambda_3}\frac{\partial}{\partial\lambda_3}+\beta_{\lambda_4}\frac{\partial}{\partial\lambda_4}\right.$$
$$\left.-\gamma_{h^0}\,h_c^0\,\frac{\partial}{\partial h_c^0}-\gamma_{\eta^0}\,\eta_c^0\,\frac{\partial}{\partial\eta_c^0}\right)V_0(h_c^0,\eta_c^0)=-2\frac{\partial}{\partial\log\mu^2}V_1(h_c^0,\eta_c^0). \qquad (26)$$

In terms of the tree level masses this equation is equivalent to

$$\begin{aligned}&\left(2\,\beta_{\mu_1}-4\,\gamma_{h^0}\,\mu_1^2\right)\,h_c^{0\,2}+\left(2\,\beta_{\mu_2}-4\,\gamma_{\eta^0}\,\mu_2^2\right)\,\eta_c^{0\,2}+\left(\beta_{\lambda_1}-4\,\gamma_{h^0}\,\lambda_1\right)\,h_c^{0\,4}+\left(\beta_{\lambda_2}-4\,\gamma_{\eta^0}\,\lambda_2\right)\,\eta_c^{0\,4}\\&\quad+\left(\beta_{\lambda_3}-2\,\left(\gamma_{h^0}+\gamma_{\eta^0}\right)\,\lambda_3\right)\,h_c^{0\,2}\,\eta_c^{0\,2}-2\,\left(\beta_{\lambda_4}-\left(2\,\gamma_{h^0}+\gamma_{\eta^0}\right)\,\lambda_4\right)\,h_c^{0\,2}\,\eta_c^0\\&\quad=\frac{1}{8\pi^2}\left(3\,m_Z^4+6\,m_W^4-12\,m_t^4+m_{\phi^0}^4+2\,m_{g^\pm}^4+2\,m_{h^\pm}^4+m_{H^0}^4+m_{N^0}^4\right), \qquad (27)\end{aligned}$$

and, matching powers of fields, we can derive the beta functions:

$$\begin{aligned}\beta_{\mu_1}&=-\frac{2}{C_{\rm UV}}\delta\mu_1^2+2\gamma_{h^0}\mu_1^2, &(28)\\ \beta_{\mu_2}&=-\frac{2}{C_{\rm UV}}\delta\mu_2^2+2\gamma_{\eta^0}\mu_2^2, &(29)\\ \beta_{\lambda_1}&=\frac{2}{C_{\rm UV}}\delta\lambda_1+4\gamma_{h^0}\lambda_1, &(30)\\ \beta_{\lambda_2}&=\frac{2}{C_{\rm UV}}\delta\lambda_2+4\gamma_{\eta^0}\lambda_2, &(31)\\ \beta_{\lambda_3}&=\frac{2}{C_{\rm UV}}\delta\lambda_3+2\left(\gamma_{h^0}+\gamma_{\eta^0}\right)\lambda_3, &(32)\\ \beta_{\lambda_4}&=\frac{2}{C_{\rm UV}}\delta\lambda_4+\left(2\,\gamma_{h^0}+\gamma_{\eta^0}\right)\lambda_4. &(33)\end{aligned}$$

We can now make use of the anomalous dimensions for the two neutral Higgs fields

$$\begin{aligned}\gamma_{h^0}&=\frac{1}{16\pi^2}\left(3\,h_t^2-\frac{9}{4}\,g^2-\frac{3}{4}\,g'^2\right), &(34)\\ \gamma_{\eta^0}&=-\frac{3}{8\pi^2}\,g^2, &(35)\end{aligned}$$

to write down our final expressions for the one-loop beta functions:

$$\begin{aligned}\beta_{\mu_1}&=\frac{1}{16\pi^2}\left(6\,\lambda_4^2+12\,\lambda_1\mu_1^2+3\,\lambda_3\,\mu_2^2\right)+\frac{1}{8\pi^2}\left(3\,h_t^2-\frac{9}{4}\,g^2-\frac{3}{4}\,g'^2\right)\,\mu_1^2, &(36)\\ \beta_{\mu_2}&=\frac{1}{16\pi^2}\left(4\,\lambda_4^2+4\,\lambda_3\,\mu_1^2+10\,\lambda_2\,\mu_2^2\right)-\frac{3}{4\pi^2}\,g^2\,\mu_2^2, &(37)\end{aligned}$$

$$
\begin{aligned}
\beta_{\lambda_1} &= \frac{1}{8\pi^2}\left(\frac{9}{16}\,g^4 - 3\,{h_t}^4 + 12\,\lambda_1^2 + \frac{3}{4}\,\lambda_3^2 + \frac{3}{8}\,g^2\,g'^2 + \frac{3}{16}\,g'^4\right) \\
&+ \frac{1}{4\pi^2}\left(3\,h_t^2 - \frac{9}{4}\,g^2 - \frac{3}{4}\,g'^2\right)\lambda_1, \qquad (38)\\
\beta_{\lambda_2} &= \frac{1}{8\pi^2}\left(6\,g^4 + 11\,\lambda_2^2 + \lambda_3^2\right) - \frac{3}{2\pi^2}\,g^2\,\lambda_2, \qquad (39)\\
\beta_{\lambda_3} &= \frac{1}{8\pi^2}\left(3\,g^4 + 6\,\lambda_1\,\lambda_3 + 5\,\lambda_2\,\lambda_3 + 2\,\lambda_3^2\right) + \frac{1}{8\pi^2}\left(3\,h_t^2 - \frac{33}{4}\,g^2 - \frac{3}{4}\,g'^2\right)\lambda_3, \qquad (40)\\
\beta_{\lambda_4} &= \frac{1}{4\pi^2}\,\lambda_4\,(\lambda_1 + \lambda_3) + \frac{3}{32\pi^2}\left(4\,h_t^2 - 7\,g^2 - \,g'^2\right)\lambda_4. \qquad (41)
\end{aligned}
$$

In the gauge and top quark sector the beta functions for the $U(1)$, $SU(3)$ and Yukawa couplings are the same as in the Standard Model, i.e.

$$
\begin{aligned}
\beta_{g'} &= \frac{41}{96\pi^2}\,{g'}^3, \qquad (42)\\
\beta_{g_S} &= -\frac{7}{16\pi^2}\,{g_S}^3, \qquad (43)\\
\beta_{h_t} &= \frac{1}{16\pi^2}\left\{\frac{9}{2}\,h_t^2 - 8\,{g_S}^2 - \frac{9}{4}\,g^2 - \frac{17}{12}\,g'^2\right\} h_t. \qquad (44)
\end{aligned}
$$

The $SU(2)$ coupling is modified due to the extra Higgs triplet in the adjoint representation, i.e.

$$
\beta_g = -\frac{5}{32\pi^2}\,g^3. \qquad (45)
$$

Working with the tree-level effective potential with couplings evolved using the one-loop β and γ functions we are able to resum the leading logarithms to all orders in the effective potential. It would be possible to include the next-to-leading logarithmic contributions by using the two-loop β and γ functions and including the one-loop part of the effective potential, see [5, 6, 8].

Let us now turn to the RG analysis. We first introduce the parameter t, related to the scale μ through $\mu(t) = m_Z \exp(t)$. We shall perform evolution starting at $t = 0$. The RG equations are coupled differential equations in the set

$$
\{g_s,\ g,\ g',\ h_t,\ \mu_1,\ \mu_2,\ \lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4\}. \qquad (46)
$$

We choose rather to use the following set to define the input to the RG equations:

$$
\left\{\alpha_s,\ m_Z,\ \sin^2\theta_W,\ m_t,\ m_{h^\pm},\ m_{H^0},\ m_{N^0},\ v,\ \tan\beta,\ \tan\gamma\right\}. \qquad (47)
$$

Within the accuracy to which we are working, the values of the couplings at $t = 0$ can be obtained from the input set using the appropriate tree-level expressions.

The vacuum conditions,

$$h_c^0\,\mu_1^2+\lambda_1\,h_c^{0^3}-\lambda_4\,h_c^0\,\eta_c^0+\frac{1}{2}\,\lambda_3\,h_c^0\,\eta_c^{0^2}=0, \tag{48}$$

$$\eta_c^0\,\mu_2^2+\frac{1}{2}\,\lambda_3\,h_c^{0^2}\,\eta_c^0-\frac{1}{2}\,\lambda_4\,h_c^{0^2}+\lambda_2\,\eta_c^{0^3}=0, \tag{49}$$

allow us to write (defining $h_c^0 \equiv v$ and $\eta_c^0 \ \equiv \ \frac{v}{2}\,\tan\beta$)

$$\begin{aligned}
m_Z^2 &= \frac{1}{4}\,v^2\,\left(g^2+g'^2\right), &(50)\\
m_W^2 &= \frac{1}{4}\,g^2\,v^2\,\left(1+\tan^2\beta\right), &(51)\\
m_t^2 &= \frac{1}{2}{h_t}^2\,v^2, &(52)\\
m_{\phi^0}^2 &= m_{g^\pm}^2 \ = \ 0, &(53)\\
m_{h^\pm}^2 &= v\,\lambda_4\,\left(\cot\beta+\tan\beta\right), &(54)\\
m_{H^0}^2 &= v\,\left\{2\,v\,\lambda_1+\left(\lambda_4-\frac{1}{2}\,v\,\lambda_3\,\tan\beta\right)\,\tan\gamma\right\}, &(55)\\
m_{N^0}^2 &= v\,\lambda_4\,\left(\cot\beta-\tan\gamma\right)+\frac{1}{2}\,v^2\,\tan\beta\left(\lambda_2\,\tan\beta+\lambda_3\,\tan\gamma\right), &(56)\\
\tan\left(2\,\gamma\right) &= \frac{2\,\tan\beta\,\left(-2\,\lambda_4+v\,\lambda_3\,\tan\beta\right)}{2\,\lambda_4-4\,v\,\lambda_1\,\tan\beta+v\,\lambda_2\,\tan^3\beta}. &(57)
\end{aligned}$$

Inverting these relations we can thus fix the $t = 0$ boundary conditions for the subsequent evolution:

$$\begin{aligned}
g_s &\equiv \sqrt{4\pi\alpha_s(m_Z)} \ \simeq \ 1.22, &(58)\\
v &\equiv \frac{1}{2^{1/4}\sqrt{G_{\rm Fermi}}} \ \simeq \ 246 \text{ GeV}, &(59)\\
g' &\equiv g\,\tan\theta_W \ \simeq \ 0.35, &(60)\\
g &\equiv 2\,\frac{m_Z}{v}\,\cos\theta_W \ \simeq \ 0.65, &(61)\\
h_t &\equiv \sqrt{2}\,\frac{m_t}{v} \ \simeq \ 1.01, &(62)\\
\lambda_1 &= \frac{1}{2\,v^2}\,\left(m_{H^0}^2\cos^2\gamma+m_{N^0}^2\sin^2\gamma\right), &(63)\\
\lambda_2 &= -\frac{1}{v^2}\,\left\{m_{h^\pm}^2-m_{H^0}^2-m_{N^0}^2+m_{h^\pm}^2\,\cos(2\,\beta)+\left(m_{H^0}^2-m_{N^0}^2\right)\,\cos(2\,\gamma)\right\}\,\cot^2\beta, &(64)\\
\lambda_3 &= \frac{1}{v^2}\,\cot\beta\,\left\{m_{h^\pm}^2\,\sin(2\,\beta)+\left(-m_{H^0}^2+m_{N^0}^2\right)\,\sin(2\,\gamma)\right\}, &(65)\\
\lambda_4 &= \frac{1}{v}\,m_{h^\pm}^2\,\cos\beta\,\sin\beta, &(66)\\
\mu_1^2 &= \frac{1}{8}\,\left\{-4\,m_{H^0}^2\cos^2\gamma+2\,m_{h^\pm}^2\sin^2\beta-4\,m_{N^0}^2\sin^2\gamma\right.
\end{aligned}$$

$$
\begin{aligned}
&+ \quad \left(m_{H^0}^2 - m_{N^0}^2\right) \sin(2\,\gamma)\,\tan\beta\Big\}\,, && (67)\\
\mu_2^2 \quad &= \quad \frac{1}{4}\left\{m_{h^\pm}^2 - m_{H^0}^2 - m_{N^0}^2 + m_{h^\pm}^2\,\cos(2\,\beta)\right.\\
&+ \quad \left.\left(m_{H^0}^2 - m_{N^0}^2\right)\left(\cos(2\,\gamma) + 2\,\cot\beta\,\sin(2\,\gamma)\right)\right\}. && (68)
\end{aligned}
$$

To ensure that the system remains in a local minimum we impose the condition that the squared masses should remain positive, i.e.

$$\lambda_4 > 0, \tag{69}$$

$$2\,v\,\lambda_1 + \left(\lambda_4 - \frac{1}{2}\,v\,\lambda_3\,\tan\beta\right)\,\tan\gamma > 0, \tag{70}$$

$$\lambda_4\,\left(\cot\beta - \tan\gamma\right) + \frac{1}{2}\,v\,\tan\beta\left(\lambda_2\,\tan\beta + \lambda_3\,\tan\gamma\right) > 0. \tag{71}$$

We impose the further requirement that the couplings remain perturbative. In particular we insist that $|\lambda_i(t)| < 4\pi$ for $i = 1,2,3$ and $|\lambda_4| < 4\pi v$. We run the evolution from $t = 0$ to $t_{\max} = \log\left(\Lambda/m_Z\right)$, with $\Lambda = 1$ TeV.

3 Results in the non-decoupling regime

In this section we present our results of the Higgs mass bounds in the regime where the triplet Higgs cannot be arbitrarily heavy. As we shall see in the next section, decoupling of the triplet occurs when both mixing angles and their sum $(\beta+\gamma)$ tend to zero and in this case, the triplet decouples from the doublet and can be arbitrarily heavy.

We are free to choose the 3 scalar masses and the 2 mixing angles at $t = 0$. In Figure 1 we show the range of Higgs masses allowed when there is no mixing in the neutral Higgs sector, $\gamma = 0$, for a value of $\beta = 0.04$. Such a value is towards the upper end of the range allowed by the precision data and is interesting because it allows a rather heavy lightest Higgs (e.g. for $\beta = 0.04$, $m_{H^0} > 150$ GeV and for $\beta = 0.05$, $m_{H^0} > 300$ GeV) [1]. The strong correlation between the $h^\pm$ and N^0 masses arises in order that λ_2 remain perturbative ($\Delta m \sim \beta^2 v$ for masses $\sim v$). The upper bound on the triplet Higgs masses (≈ 550 GeV) comes about from the perturbativity of λ_3 whilst that on H^0 (≈ 520 GeV) comes from the perturbativity of λ_1. These latter two bounds can be estimated crudely by ignoring the evolution of the couplings directly from equations (63) and (65). Evolution tightens the bounds due to the positivity of the beta functions, especially for the H^0 since $8\pi^2\beta_{\lambda_1} \approx 12\lambda_1^2$. The hole at low masses is due to vacuum stability.

In Figure 2 we show the allowed regions for $\gamma = 0.1$. The correlation of the mainly triplet Higgses is as in Figure 1. For large m_{H^0} (> 450 GeV), the upper limit on the triplet Higgs mass arises because λ_1 becomes too large (in this region $\lambda_1 \sim \lambda_3$). For smaller m_{H^0}, λ_1 is much smaller than λ_3 and the upper bound comes from the largeness of λ_3 with the tree-level estimate

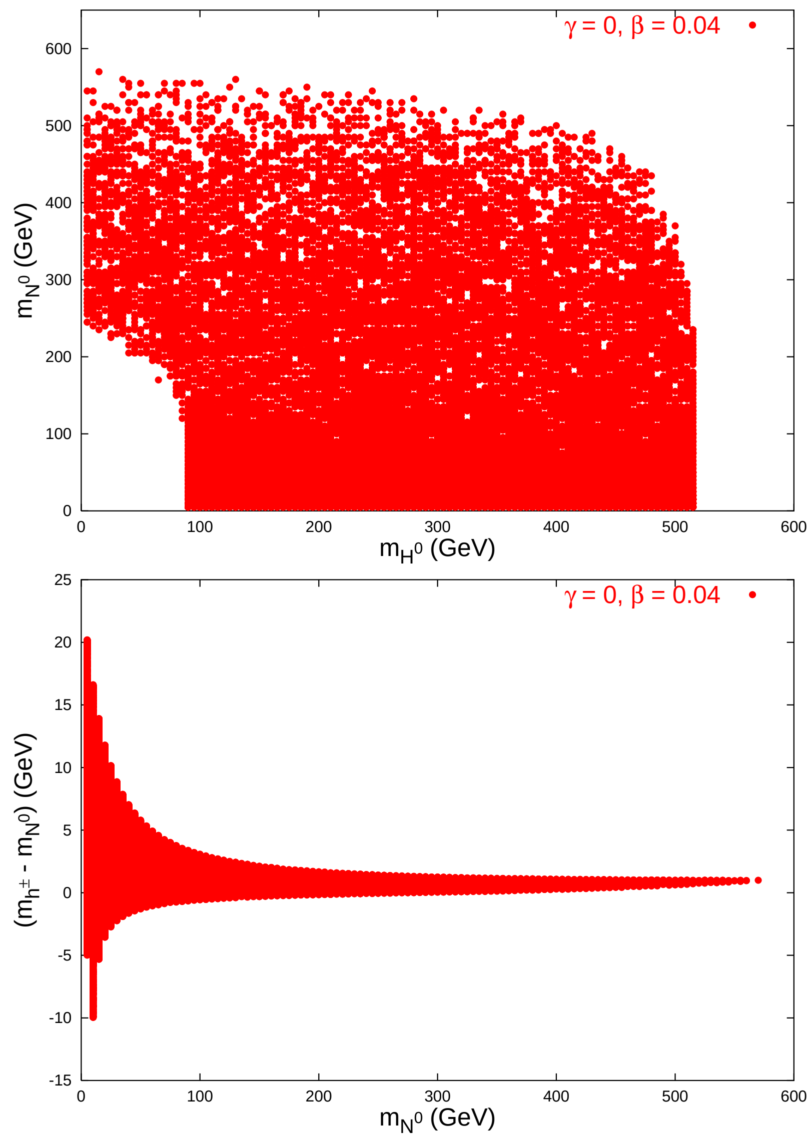

Figure 1: Allowed values of scalar masses for $\gamma = 0$

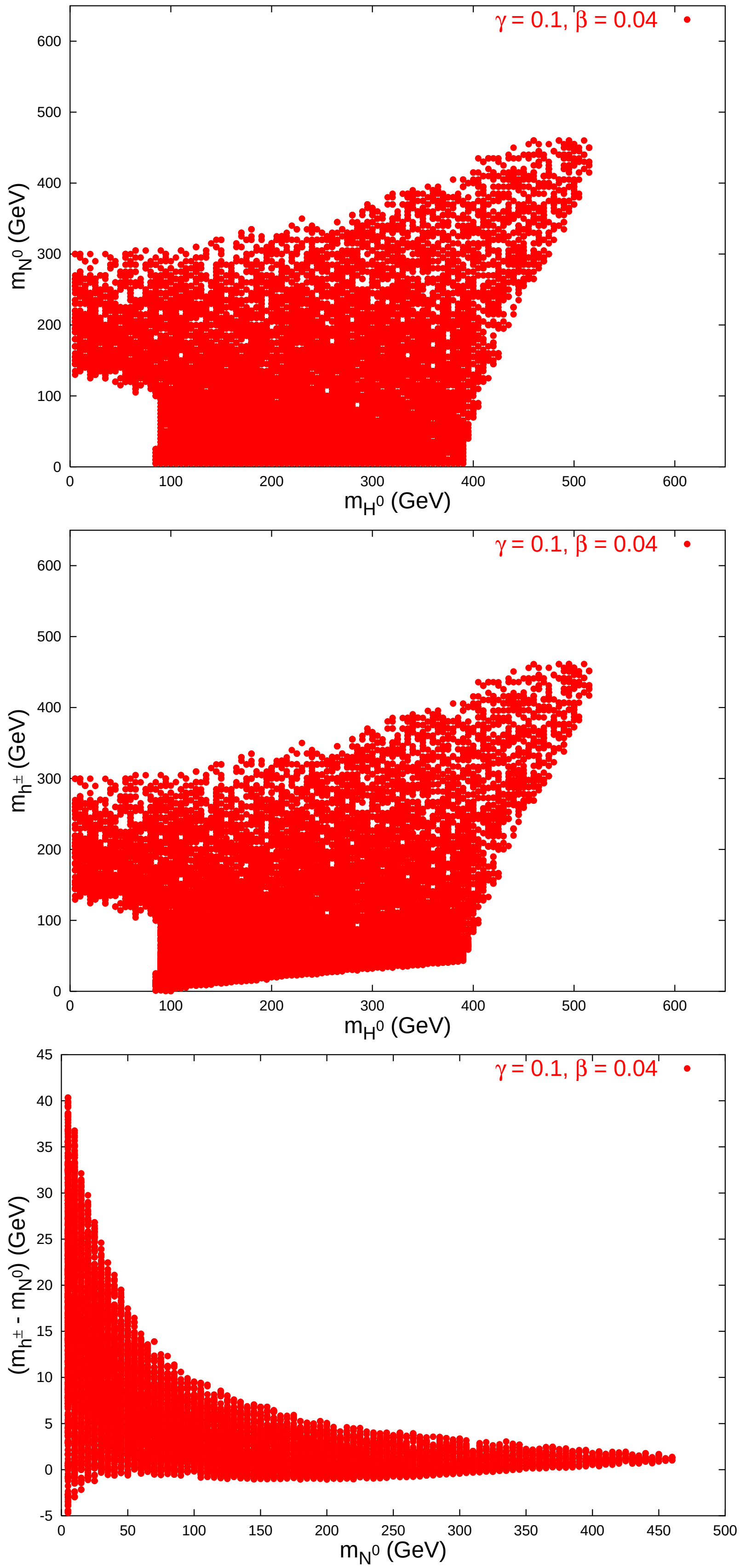

Figure 2: Allowed values of scalar masses for $\gamma = 0.1$

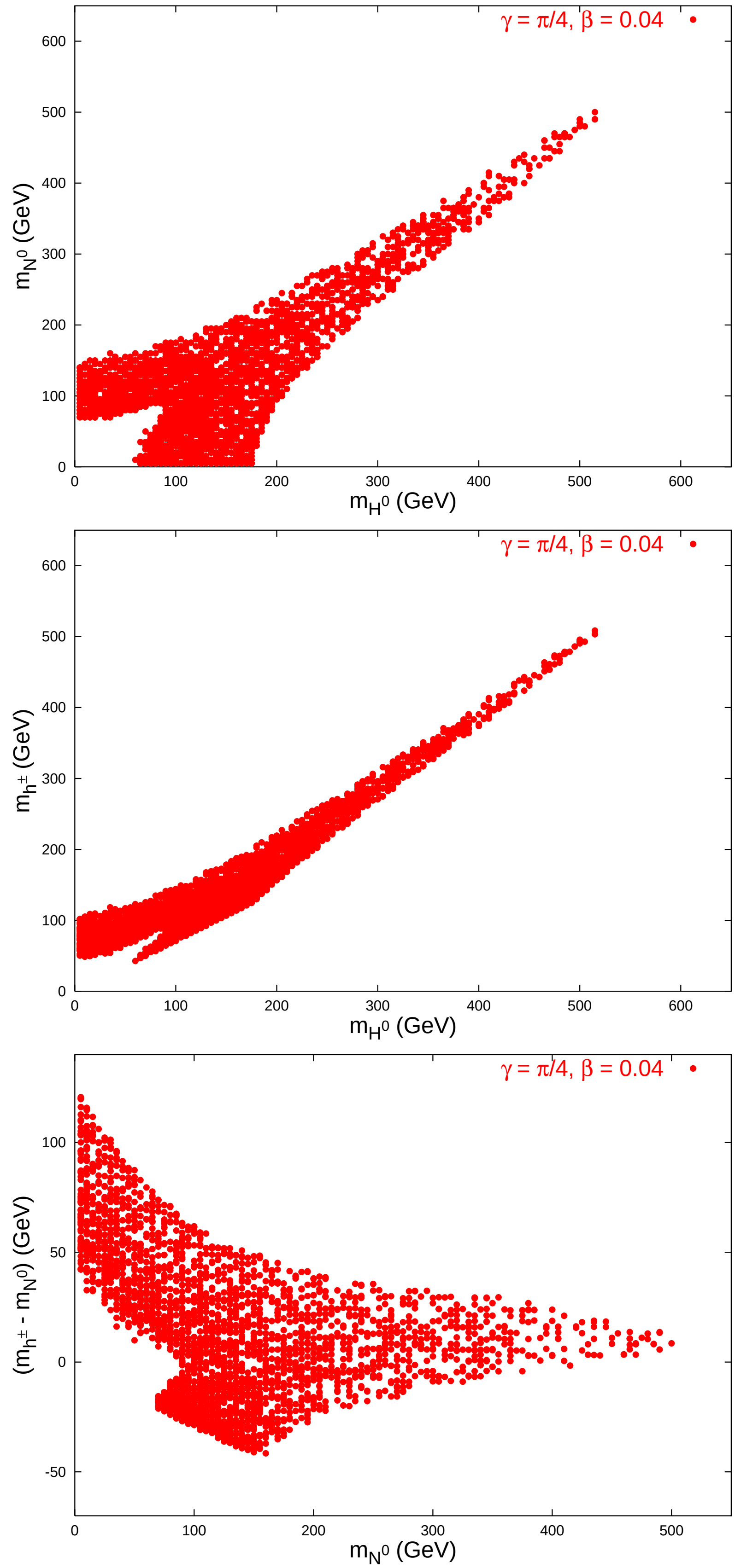

Figure 3: Allowed values of scalar masses for $\gamma = \pi/4$

being $m_{h^\pm}^2 < 2\pi v^2(\beta/\gamma)$. The upper limit on m_{H^0} is again a consequence of the perturbativity of λ_1, except at low $h^\pm$ masses, where it is due to the negativity of λ_3 driving the vacuum unstable. For very low $m_{h^\pm}$, λ_2 becoming too large is the problem.

In Figure 3 we show the allowed regions for $\gamma = \pi/4$. In this maximal mixing scenario one loses the distinction between doublet and triplet Higgses and the bounds are correspondingly more democratic. The largeness of $\tan(2\gamma)$ can be arranged either by tuning $2v\lambda_1 \approx \lambda_4/\beta$ or by having small enough λ_1 and λ_4. In the former case, all masses are approximately degenerate, as can be seen in the plot. In the latter case, which corresponds to light masses, the degeneracy is lifted. The bounds for $\gamma > \pi/4$ are very similar to those for $(\pi/2 - \gamma)$ on interchanging the neutral Higgses N^0 and H^0.

For $\beta < 0.04$ and small γ (but still away from the decoupling regime) the allowed regions are very similar to those for $\beta = 0.04$, i.e. as in Figure 1. For larger γ, the mass bounds are again as for larger β but with the correlation between the neutral and charged Higgs masses becoming even stronger than for larger β.

We should stress that all of the previous discussion is valid for strictly non-zero β. The situation is quite different for $\beta = 0$. If the neutral mixing is not zero (which is required if we are to avoid decoupling) then the vacuum conditions dictate that $\mu_1^2 = -\lambda_1 v^2$ and $\lambda_4 = 0$ and this renders equation (17) redundant. Equation (18) then yields $\mu_2^2 = 2\lambda_1 v^2 - \frac{1}{2}\lambda_3 v^2$ and we have complete degeneracy, i.e. $m_{H^0}^2 = m_{N^0}^2 = m_{h^\pm}^2 = 2\lambda_1 v^2$.

4 The decoupling limit

So far we have worked in a regime where the triplet does not decouple from the doublet. Clearly for $\beta = \gamma = 0$ there is no mixing between the doublet and triplet and there is no bound on the triplet mass. This is a special case of the more general decoupling scenario, which occurs when $|\beta + \gamma| \ll \beta$, which we now discuss.

For small mixing angles, the (mainly) triplet Higgs has mass squared $\sim \lambda_4 v/\beta$. One possible solution to the mixing angle equations (17) and (18) is that $\lambda_4 \sim \beta v$ and any γ. In this case the triplet Higgs has mass $\sim v$. This is the regime of the previous section. However, it is also possible to solve the mixing angle equations with $\lambda_4 \sim v$ by keeping μ_2^2 large, i.e. (17) gives $\lambda_4 v = \beta\mu_2^2 \sim v^2$. In this case, equation (18) forces $\beta + \gamma \approx 0$. This is the decoupling limit in which the triplet mass lies far above the mass of the doublet and the low energy model looks identical to the Standard Model.

Tree level arguments on the perturbativity of λ_3 allow us to quantify the approach to decoupling from the point of view of the triplet Higgs mass. In particular (65) dictates that, for small β and γ,

$$m_{h^\pm}^2 \approx m_{N^0}^2 < \frac{2\pi v^2 \beta + \gamma\, m_{H^0}^2}{\beta + \gamma}. \tag{72}$$

By virtue of the smallness of β_{λ_3} this relation picks up relatively small loop corrections. This bound clearly demonstrates decoupling. It also re-iterates the results of the previous section, i.e. for small $\beta \gg \gamma$ the limit is as in Figure 1 and for small $\beta \ll \gamma$ the limit is as in Figure 2.

We remark that the pseudo-decoupling regime, where β is not too small, is of particular interest in that it again allows one to relax the mass bound on the lightest Higgs coming from the precision data without otherwise changing the physics of the Standard Model [1].

5 Conclusions

We have computed the one-loop beta functions for the scalar couplings in an extension to the Standard Model which contains an additional real triplet Higgs. Through considerations of perturbativity of the couplings and vacuum stability we have been able to identify the allowed masses of the Higgs bosons in the non-decoupling regime. In the decoupling regime, the model tends to resemble the Standard Model.

We note that the theoretical mass bounds presented here will of course be tightened after considering the precision electroweak and direct search data. Such a study requires that the impact of the quantum corrections (to the T parameter) for non-zero γ be computed (they were not explored in [1]).

As a final remark, we wish to emphasise that the near degeneracy of the triplet Higgs masses (the mass splitting is naturally $\sim \beta^2 v$) ensures that, at least for small γ, the quantum corrections to the T parameter are negligible (the S parameter vanishing since the triplet has zero hypercharge) [1]. As shown in [1], this means that the lightest Higgs boson can be heavy as a result of the compensation arising from the explicit tree-level violation of custodial symmetry which the real triplet induces. Thus it is quite possible to be in a regime where all the Higgs bosons are heavy without any dramatic deviation from the physics of the Standard Model.

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