The POWHEG method applied to top pair production and decays at the ILC

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We study the effects of gluon radiation in top pair production and their decays for $e^+e^$ annihilation at the ILC. To achieve this we apply the POWHEG method and interface our results to the Monte Carlo event generator Herwig++. We consider a center-ofmass energy of $\sqrt{s} = 500$ GeV and compare decay correlations and bottom quark distributions before hadronization.

1 Introduction

In Table 1, we have highlighted some differences between matrix element (ME) and parton shower (PS) generators and have labelled as (M) or (D) those attributes we consider merits or drawbacks respectively. Note that most PS generators attempt to include NLO corrections

PS generators	ME generators
Resums leading logarithmic contributions	Can only go up to $N^{\sim 6}$ LO (D)
to all orders (M)	
High multiplicity hadrons	Low multiplicity partonic
in the final state (M)	final states (D)
Works well in regions	Works well in regions
of low relative p_T (M & D)	of high relative p_T (M & D)
Total rate is accurate to LO (D)	Total rate is accurate to $N^{(>0)}LO(M)$

Table 1: Differences between PS and ME generators

via a method called the matrix element correction which corrects the hardest shower emission so far to the exact matrix element and populates the high p_T regions according to the NLO cross-section. However, the total rate is still only accurate to LO and virtual corrections are not fully taken care of.

1.1 Getting the best of both worlds at NLO

The Positive Weighted Hardest Emission Generation (POWHEG) method [2, 3] aims to solve this problem. It

1. generates total rates accurate to NLO,

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- 2. treats hard emissions as in ME generators,
- 3. treats soft and collinear emissions as in PS generators,
- 4. and generates a set of fully exclusive events which can be interfaced with a hadronization model.

The POWHEG method achieves this by generating the hardest emission in the shower first to NLO accuracy using a modified Sudakov form factor. For angular ordered showers like Herwig++, it also includes a truncated shower of soft, wide angled emissions from the hard scale to the scale of the hardest emission. This maintains the correct soft emission pattern. This is illustrated in Figure 1.1. It then showers the resulting partons subject to a $p_{\rm T}$ veto



Figure 1: POWHEG emissions

to ensure that no harder emissions are generated. Unlike MC@NLO [4], it is independent of the PS generator used and all events have positive weight. In this talk [1], we will focus on the description and applications of the method in conjunction with the PS generator, Herwig++ [5].

1.2 The parton shower hardest emission cross-section

For a single parton, the cross-section for the hardest emission with transverse momentum $p_{\rm T}$ is given by

$$d\sigma = d\sigma_{\rm B} \left[\Delta_{\rm V}(0) + \Delta_{\rm V}(p_{\rm T}) \frac{\alpha_{\rm S}}{2\pi} P dz \frac{dq^2}{q^2} \right] \,, \tag{1}$$

where P is the splitting function for the hardest emission and $\Delta_{\rm V}(p_{\rm T})$ is the Sudakov form factor for no emissions with $k_{\rm T} > p_{\rm T}$ which is given by

$$\Delta_{\rm V}(p_{\rm T}) = \exp\left[-\int dz \frac{dq^2}{q^2} \frac{\alpha_{\rm S}}{2\pi} P\Theta(k_{\rm T} - p_{\rm T})\right] \,. \tag{2}$$

The cross-section (1) expanded to order $\alpha_{\rm S}$ gives

$$d\sigma = d\sigma_{\rm B} \left[\left\{ 1 - \int \frac{\alpha_S}{2\pi} P dz \frac{dq^2}{q^2} \right\} + \frac{\alpha_S}{2\pi} P dz \frac{dq^2}{q^2} \right] \,. \tag{3}$$

The POWHEG method aims to substitute (3) with the exact NLO result within the parton shower.

1.3 Correcting to the exact NLO cross-section

The exact NLO cross-section can be written as

$$d\sigma_{\rm NLO} = d\sigma_{\rm B} + d\sigma_{\rm V} + d\sigma_{\rm R} \equiv d\sigma_{\rm B} + d\sigma_{\rm V} + d\sigma_{\rm B} dr \mathcal{M} . \tag{4}$$

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Adding and subtracting $d\sigma_{\rm B} \int_{\delta} dr (\mathcal{M} - \mathcal{C})$ we get

$$d\sigma_{\rm NLO} = d\sigma_{\rm B} + d\sigma_{\rm V} + d\sigma_{\rm B} \int_{\delta} dr (\mathcal{M} - \mathcal{C}) + d\sigma_{\rm B} dr \mathcal{M} - d\sigma_{\rm B} \int_{\delta} dr (\mathcal{M} - \mathcal{C}) .$$
(5)

where C is a counter-term and δ is the subtraction region. This can be rearranged to give

$$d\sigma_{\rm NLO} = d\sigma_{\rm V'} + d\sigma_{\rm B} \int_{\delta} dr (\mathcal{M} - \mathcal{C}) + d\sigma_{\rm B} \left[\left\{ 1 - \int_{\delta} dr \mathcal{M} \right\} + \mathcal{M} dr \right]$$
(6)

with $d\sigma_{V'} = d\sigma_V + d\sigma_B \int_{\delta} dr C$ now finite. Comparing (6) with (3) above, we can write down an analog of (1) as

$$d\sigma_{\rm NLO} = d\sigma_{\bar{\rm B}} \left[\Delta_{\rm NLO}(0) + \Delta_{\rm NLO}(p_{\rm T}) \mathcal{M} dr \right]$$
(7)

where

$$d\sigma_{\bar{B}} = d\sigma_{B} + d\sigma_{V'} + d\sigma_{B} \int_{\delta} dr (\mathcal{M} - \mathcal{C})$$

$$\Delta_{\rm NLO}(p_{\rm T}) = \exp\left[-\int \mathcal{M}\Theta(k_{\rm T} - p_{\rm T})\right] .$$
(8)

Note that in defining $d\sigma_{\bar{B}}$, we have neglected terms of higher order than α_{S} and if it is negative, perturbation theory has broken down.

1.4 POWHEG formalism and applications

With

$$d\sigma_{\rm NLO} = d\sigma_{\bar{\rm B}} \left[\Delta_{\rm NLO}(0) + \Delta_{\rm NLO}(p_{\rm T}) \mathcal{M} dr \right] \,, \tag{9}$$

the POWHEG method can be applied by

- 1. generating the $p_{\rm T}$ of the hardest emission and its emission variables r, according to the term in square brackets using well known Monte Carlo techniques,
- 2. distributing the underlying Born variables according to $d\sigma_{\bar{B}}$, (This defines the event weight and since it is always positive definite, all event weights are positive)
- 3. for angular ordered showers, implementing a truncated shower of soft emissions between the *hard* scale and the scale of the hardest emission,
- 4. and finally showering the resulting partons as in a PS generator subject to a $p_{\rm T}$ veto.

The POWHEG method has been applied successfully to the following processes Z pair hadroproduction [6], heavy flavour production [7], e^+e^- annihilation to hadrons [8], Drell-Yan vector boson production [9, 10] and Higgs boson production via gluon fusion [11].

2 Top-pair production and decay at the ILC

The application to top pair production and decay at the ILC takes the following into account.

1. Spin correlations are taken into account in the matrix elements, \mathcal{M} for the production and decays of the top pairs.

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Figure 2: Top-pair production and decay

- 2. The narrow width approximation is applied so that production and decay interference can be neglected. This independence enables us to apply the method in separate frames: the lab frame for production and the top rest frame for its decay.
- 3. In the lab frame, the transverse momentum k_T is defined relative to the original $t \bar{t}$ axis whilst in the top rest frame it is relative to the original b W axis.
- 4. The scale range available for production emissions ($\approx \log(\sqrt{s}/m_t)$) and is much less than the range available for decay emissions ($\approx \log(m_t/m_b)$).
- 5. There are two different sources of the decay emissions: one from the top quark before it decays and the other from the b quark after the decay. Hence, there are three different regions for truncated emissions labelled g(tr) in Figure 2. These are before the hardest emission in the production, from the top quark before it decays and before the hardest emission from the b quark.

Further discussion of the method and its application can be found in [12]. Setting $\sqrt{s} = 500$ GeV and $m_{\rm t} = 175$ GeV, we considered the following four cases with POWHEG interfaced with Herwig++:

- 1. Leading order (LO) with no POWHEG emissions,
- 2. Only POWHEG emissions in the production process (Prod),
- 3. Only POWHEG emissions in the decays of the tops (Dec),
- 4. Both production and decay emissions allowed (Prod + Dec).

For the two different e^+e^- initial polarizations, we investigated the correlations between the decay products (we consider leptonic decays only for the W bosons) and the momentum distributions of the *b* quarks before hadronization. A selection of the plots obtained are presented in Figure 3.



Figure 3: Correlations and b quark momentum distributions.

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3 Summary

NLO improvements of parton showers are essential for near accurate predictions of angular correlations and momentum distributions at future colliders. The POWHEG method achieves this by distributing the *hardest* emission according to the NLO matrix element and yields events with positive weights. For angular ordered showers, the addition of a truncated shower is required. The method, though not very straightforward to apply, has demonstrated success in comparison with existing collider data.

In this talk, we have extended this to top-pair production and decay at the ILC and made predictions for some distributions in comparison to leading order predictions.

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References

- [1] Presentation:
- http://ilcagenda.linearcollider.org/contributionDisplay.py?contribId=93&sessionId=18&confId=2628
- [2] P. Nason, JHEP **11** 040 (2004) hep-ph/0409146;
- [3] S. Frixione, P. Nason, and C. Oleari JHEP 11 070 (2007);
- [4] S. Frixione and B. Webber JHEP 06 029 (2002) hep-ph/0204244;
- [5] M. Bahr et. al Eur. Phys. J. C58 639-707 (2008) arXiv:0803.0883 (2008); arXiv:0806.4560 (2008);
- [6] P. Nason, and G. Ridolfi JHEP 08 077 (2006) hep-ph/ 0606275 arXiv:0709.2092;
- [7] S. Frixione, P. Nason, and G. Ridolfi JHEP 09 126 (2007) arXiv:0707.3088;
- [8] O. Latunde-Dada JHEP 02 051 (2007) hep-ph/ 0612281;
- [9] S. Alioli, P. Nason, C. Oleari and E. Re JHEP 07 060 (2007) arXiv:0805.4802;
- [10] K. Hamilton, P. Richardson and J. Tully JHEP 10 015 (2008) arXiv:0806.0290 (2008);
- [11] S. Alioli, P. Nason, C. Oleari and E. Re arXiv:0812.0578 (2007);
- [12] O. Latunde-Dada Eur. Phys. J. C58 553-554 (2008)