

Review



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Multiple solutions in
supersymmetry and the Higgs

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Weak-scale supersymmetry is a well-motivated, if speculative, theory beyond the Standard Model of particle physics. It solves the thorny issue of the Higgs mass, namely: how can it be stable to quantum corrections, when they are expected to be 10^{15} times bigger than its mass? The experimental signal of the theory is the production and measurement of supersymmetric particles in the Large Hadron Collider (LHC) experiments. No such particles have been seen to date, but hopes are high for the impending run in 2015. Searches for supersymmetric particles can be difficult to interpret. Here, we shall discuss the fact that, even given a well-defined model of supersymmetry breaking with few parameters, there can be multiple solutions. These multiple solutions are physically different and could potentially mean that points in parameter space have been ruled out by interpretations of LHC data when they should not have been. We shall review the multiple solutions and illustrate their existence in a universal model of supersymmetry breaking.

1. Introduction to supersymmetry

The recent discovery of the Higgs boson of mass 125–126 GeV at the Large Hadron Collider (LHC) experiments [1,2] introduces a new problem: the hierarchy problem. This problem concerns quantum corrections to the Higgs boson mass. As other particles (which couple to the Higgs boson) fluctuate in and out of the vacuum, they give a contribution to the Higgs boson mass squared of order

$$\delta m_h^2 = \frac{1}{16\pi^2} M^2, \quad (1.1)$$

where M is the mass of the particles that are in the vacuum fluctuations.

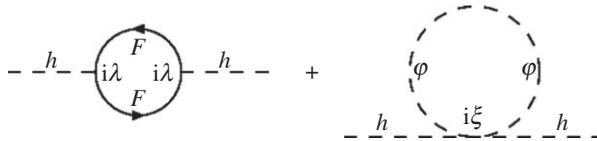


Figure 1. Examples of Feynman diagrams that give a large quantum correction to the Higgs boson mass. The masses of the particles F or ϕ may be much larger than the measured Higgs boson mass. λ and ξ denote coupling strengths between the various fields. The first Feynman diagram shows a Higgs particle splitting up into a particle and an anti-particle, which recombine into a Higgs particle. The second diagram shows a Higgs particle interacting with ϕ particles that are fluctuating out of the vacuum.

Thus, the dominant Higgs boson mass correction is expected to be from the heaviest such mass scales. The hierarchy problem originates from the observation that there are expected to be values that are many orders of magnitude heavier than the Higgs boson mass. For example, we know of the existence of the Planck scale, whose associated mass scale is $M_{\text{Pl}} \sim 10^{19}$ GeV. If this mass scale is (as we might expect) derived from microscopic propagating degrees of freedom, they are then expected to contribute to the Higgs boson mass with quantum corrections that are some 10^{15} times higher than the measured mass. One could hope that several such corrections cancel each other to 1 part in 10^{15} , but many of us feel that this is unrealistic unless there is some underlying reason for the cancellation.

It must be said that these Planck scale masses are associated with gravity, and a full quantum gravitational theory remains unverified by experiment. So one possibility is that using the standard quantum field theory arguments about vacuum fluctuations is simply wrong for some unknown reason (for example, perhaps M_{Pl} is a coupling constant that is not generated by some microscopic degrees of freedom). However, there are also other well-motivated extensions to the Standard Model of particle physics where the forces are unified (grand unified theories). These still have a huge mass scale associated with them of order 10^{16} GeV and would generate a correction to the Higgs boson mass that is much larger (10^{12} times) than its measured value. In the Standard Model, it is only the Higgs boson that has this problem of being sensitive to large quantum corrections. All of the particles other than the Higgs boson are protected by various symmetries: matter fields by the chiral symmetry of the model, and the force-carrying gauge bosons are protected by the gauge symmetry upon which the model is built. We emphasize, however, that there is technically no fine-tuning in the pure Standard Model Higgs boson mass computation: there is no higher mass scale within the Standard Model, because it does not include gravity or higher scales such as those derived from grand unified theories. However, we shall proceed on the basis that the hierarchy problem is pointing us in an interesting direction if we take it seriously as a problem, expecting that gravitational degrees of freedom will induce a huge quantum correction to the Higgs boson mass.

If we examine the correction to the Higgs boson mass shown in figure 1, we note an interesting fact: the large corrections have an different sign for the first contribution compared to the second. In fact, this is a property of quantum field theory: fermions give a *negative* sign, whereas bosons give a positive sign. Supersymmetry provides a mathematical reason for a large cancellation between the two diagrams, by imposing a symmetry on the quantum field theory between bosons and fermions. For every fermion degree of freedom, supersymmetry imposes that there must be a corresponding bosonic one, with identical mass and couplings. Thus, for instance in figure 1, supersymmetry imposes $m_F = m_\phi$ and $\lambda^2 = \xi$ [3].

In fact, fermions in the Standard Model (the quarks and leptons), each have two degrees of freedom (left and right handed, meaning that their spins are in the same direction or in opposite direction to the motion of the particle). When we supersymmetrize the model, we end up with two scalar bosons for each Standard Model fundamental fermion. The supersymmetric scalar

boson copies are prepended with an 's' to denote their different spin. Thus, we talk of right- and left-handed squarks and sleptons to be the spin 0 copies of the fermions. Spin $\frac{1}{2}$ copies of the Standard Model bosons have 'ino' appended to their name. In the minimal supersymmetric extension of the Standard Model, we therefore have the following particles in addition to the Standard Model ones:

- three families of up- and down-type squarks;
- three families of sleptons: left- and right-handed selectrons, and left-handed sneutrinos;
- two extra electrically charged Higgsinos, plus two extra electrically neutral ones. These come with two extra charged and two extra neutral Higgs bosons.

Thus, the minimal supersymmetric standard model (MSSM) is a *two Higgs doublet model*. The extra Higgs doublet is needed for mathematical consistency of the fully quantum theory, and also to give masses to down-type quarks and leptons.

The spin 1 force-carrying gauge bosons of the Standard Model (prior to the Higgs mechanism, these are gluons, W and Z bosons and the photon, respectively) attain a spin $\frac{1}{2}$ supersymmetric partner, collectively known as *gauginos*. Specifically, we have the following spin $\frac{1}{2}$ particles additional to the Standard Model:

- gluinos,
- winos,
- zinos, and
- photinos.

The supersymmetric prediction that, for instance, the slepton masses are identical to the lepton masses leaves us with a phenomenological problem. To date, no supersymmetric particles have been directly observed. But if they were of identical mass to their Standard Model counterparts, they would have been observed in past experiments. The resolution to this problem is the introduction of *supersymmetry breaking* in a way that does not reintroduce the hierarchy problem. Supersymmetry breaking which does not reintroduce the hierarchy problem is called soft. If we introduce a mass splitting between F and φ , we induce a correction to the Higgs boson mass squared of order

$$\frac{1}{16\pi^2}(m_F^2 - m_\varphi^2). \quad (1.2)$$

Thus, no matter the size of m_F , as long as the *splitting* with its partner is small, there is no large quantum correction induced to the Higgs mass. The mass of the Higgs then gives us a rough order of magnitude estimate for what the splitting should be: it should not be too much greater than the Higgs boson mass times 4π (i.e. approx. 1000 GeV). Thus, the masses of the supersymmetric partners of the Standard Model particles (which only have a comparatively negligible mass) should not be too much greater than about 1000 GeV. Many different models have been suggested that successfully exhibit such soft supersymmetry breaking, conveniently making supersymmetric partners heavier while not significantly affecting the masses of the Standard Model particles. Although there are many different models, with different predictions for the patterns of masses of supersymmetric particles and slightly different advantages or disadvantages, there is no outstanding candidate. In any case, we are faced with the hope that supersymmetric particles will be produced at the LHC, and measurements of their masses and couplings will subsequently be made, allowing for an empirical determination of the pattern of supersymmetry breaking.

In order to deduce what signatures are expected in LHC collisions which produce supersymmetric particles, we must examine the model *after* supersymmetry breaking and electroweak symmetry breaking via the Higgs mechanism [4,5]. Various particles mix after the effects of breaking these two symmetries are taken into account. For example, the electrically neutral spin $\frac{1}{2}$ particles with $L=0$ (i.e. the Higgsinos, the zino and the photino) mix: their

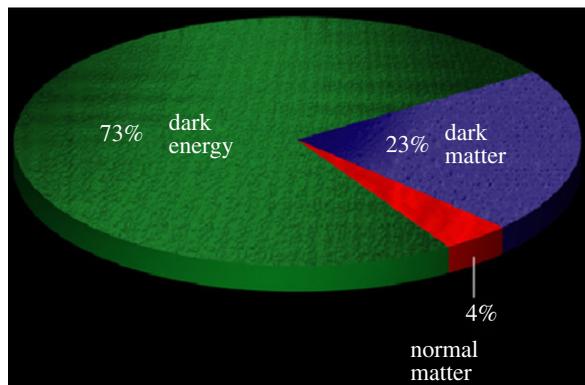


Figure 2. Energy budget of the universe: there is approximately six times the amount of mass in dark matter compared to normal matter. The rest of the energy is in the mysterious ‘dark energy’ of the universe, about which very little is known. Figure taken from M. Weiss, NASA/CXC (http://cosmictimes.gsfc.nasa.gov/universemashup/archive/complete_archive.html). (Online version in colour.)

mass eigenstates are called neutralinos, and denoted $\chi_{1,2,3,4}^0$. Neutralinos can help solve another problem associated with cosmological and astrophysical observations of our universe: namely, the dark matter problem.

Dark matter is a hypothesized new form of particle which is transparent to light, and which only interacts very weakly with matter. However, it is heavy and can affect the gravitational field in the universe. It was initially postulated to be present in a halo around spiral galaxies, because the speeds of the stars rotating around their centres did not match up with the predictions coming from standard gravitational theory (the stars more towards the edges of the galaxies were going far too fast). By postulating some heavy invisible matter present in the galaxy though, the predictions changed and the speed of the outer stars in such galaxies could be understood. Other corroborating inferences from observations soon were made: clusters of galaxies were seen to be moving with peculiar velocities that could be explained if dark matter were present. Also, measurements of the bending of light (weak gravitational lensing) from distant galaxies indicate that the light has passed through dark matter. Recently, observations including those of the afterglow of the big bang (effectively the angular correlations in the temperature spectrum of the cosmic microwave background) allow a fit in order to determine the amount of dark matter in the universe, compared to the amount of visible matter. While ordinary (baryonic) matter only makes up around 4% of the energy budget of our universe, dark matter makes up some 23% or so [6] (figure 2).

The lightest neutralino, χ_1^0 can have the right properties to make up the dark matter of the universe, since it is massive and does not interact electromagnetically. For this to be the case, it must be stable, which in practice means that it must be the lightest supersymmetric particle (supersymmetric particles can only decay to another lighter supersymmetric particle and ordinary Standard Model particles). This then leads to a rewriting of the early universe’s history: in the first instants after the big bang, the universe is very hot and energetic. There are all sorts of particles, including various supersymmetric ones. The supersymmetric particles all then very quickly decay away into ordinary (Standard Model) particles and dark matter, which remains in the universe to this day. The ‘thermal dark matter relic density’ prediction in supersymmetry comes from a quantification of this history. In the early universe, there are many interactions between the particles, which form a hot plasma. As the universe expands and cools down, eventually a given particle species is too far apart from the others to have a high chance of interacting with them, and so they ‘freeze out’. At this stage, if the particles are unstable they are decaying into other particle species, or otherwise they simply remain in the universe in their current numbers if they are stable. The idea at the LHC is that, although

the supersymmetric particles (aside from one that is the dark matter candidate) have all long since decayed, we can convert the energy E in the proton beams of the LHC into mass m of supersymmetric particles through the famous relation $E = mc^2$, so that we can measure some of their decay products and confirm their existence. If we know the strength of the particle physics interactions between the various particle species, we can calculate how many of them are in each species as a function of time after the big bang, assuming some cosmological history. The interaction cross section of dark matter annihilation in the early universe (where two particles of dark matter coalesce to make two particles of ordinary matter) is of crucial importance in this calculation. Since recent measurements by the Planck satellite, among others, constrain the temperature fluctuations of the cosmic microwave background, a highly constrained picture of cosmological history has emerged. Thus, if we fix the masses and interactions of the supersymmetric and ordinary particles, we can calculate their relative abundances today. Of particular importance is the predicted thermal relic abundance of the supersymmetric dark matter candidate.

2. Universality and Large Hadron Collider searches

The preceding decades have seen many different colliders with successively higher and higher energies. Since the 1960s, every decade the most energetic man-made particle collisions on the Earth have had their energies increased by roughly a factor 10. During this period, as the energy was increasing, various particles were discovered. E was not high enough in previous experiments in order to produce the heavier states (which have a larger m). Today, the LHC has the highest energy of any artificial particle collider that has ever existed on the Earth. Thus, if m is significantly higher than the E of any of the previous collisions, but still somewhat less than the LHC design energy of 14 000 GeV, we would expect to be able to first produce them at the LHC, provided that nature follows the supersymmetric model. We argued above that the supersymmetric particles should have masses less than 1000 GeV or so. This in turn implies that the LHC, with its centre of mass energies, at 7000 GeV = $-14\,000$ GeV, should be energetic enough to produce the supersymmetric partners. We must note that though that, in fact, it is the point-like constituents of the protons (the quarks and gluons) that may collide to produce supersymmetric partners, and they come with some *a priori* unknown fraction of the proton's momentum described phenomenologically by probability distribution functions in each collision (figure 3). A combination of increasing the energy and recording more collisions allows for the greatest chance of directly producing the supersymmetric partners. The hope is that, after production, they can be detected and measured in the ATLAS and CMS general purpose LHC detectors. These large machines act like three-dimensional digital cameras, measuring the properties of the final state objects (momenta, charges, etc.) resulting from the proton collisions. Some detective work is required to work backwards from the tracks of the final state objects and tell what is happening directly after the moment of collision. We must note that the collisions are quantum processes and their outcomes are inherently random. One cannot predict, for a given collision, what will result. However, if one uses the correct quantum theory to describe the collisions, one can predict the relative probabilities of various possible final states of a collision. Thus, after performing many collisions, one can check the predicted frequencies of various final states against the measured frequencies. These should match within uncertainties, if we have the correct theory, provided there have been no mistakes in the analysis.

The classic signature for supersymmetry in the LHC is that of missing transverse momentum. School-level physics tells us that momentum is conserved in any LHC collision. Initially, we have protons of equal and opposite momentum, and so the total momentum of the initial state is 0. The law of conservation of momentum implies that the vector sum of the final state momenta should therefore also be zero. However, if we add up the components of the momenta transverse to the beam (there are additional difficulties with measuring the total momentum parallel to the beam which renders it impractical) and there is a significant amount 'missing', we can infer one of two

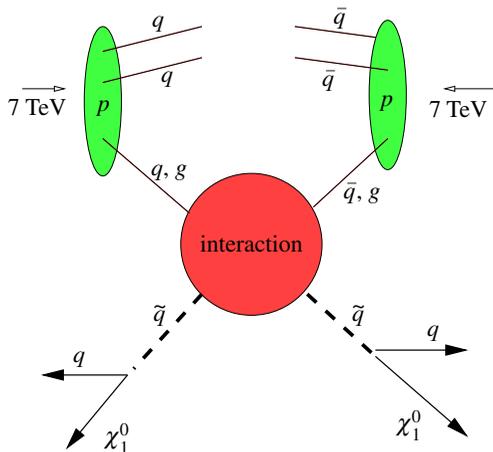


Figure 3. Sketch of a collision producing supersymmetric particles at the LHC. The initial protons of the beams (denoted 'p') are made of quarks ('q') and gluons ('g'). It is these which may collide to produce supersymmetric particles (these are pair produced, and shown as \tilde{q}). The supersymmetric particles subsequently decay to ordinary particles and dark matter particles χ_1^0 . (Online version in colour.)

things: either some particles fell in cracks in the detector and were unmeasured, or a particle went right through the detector without leaving a trace. One can account (by careful modelling and calibration) for the former effect. Each supersymmetric particle would decay to some ordinary particles and a single dark matter particle. The dark matter particle interacts so weakly with matter that it would just go straight through the detector without leaving a trace. It thus acts like a thief, stealing momentum away from the collision, undetected. If we measure too many collisions which have a large amount of 'missing transverse momentum', we can infer the production of such particles. By measuring some of the details of the visible particles that are produced, we can hope to check aspects of the supersymmetric model.

Unfortunately, the different ways in which supersymmetry can be broken leads to many different possible patterns of the details of the final fiery fragments of the collisions. For a particular pattern of supersymmetry breaking, we can make predictions for the various relative frequencies of the possible final states. In order to provide a realistic example, we often resort to *universal* models of supersymmetry breaking. These assume that all of the supersymmetry breaking spin 0 particle masses are equal (m_0), all of the supersymmetry breaking gaugino masses are equal ($M_{1/2}$), and that each of the supersymmetry breaking trilinear interactions between spin 0 particles are equal (A_0). There is another input parameter in the theory, $\tan \beta$, which measures how different the two Higgs doublet in the model are. Once these four parameters are set (and a sign in the Higgs potential, the sign of the so-called μ parameter), the model can be matched to current data on Standard Model particle masses, and the masses of all supersymmetric particles and the Higgs bosons can be predicted. In fact, the initial equal masses for the supersymmetric particles split apart because of differing quantum corrections (figure 4). We shall assume the universal pattern of supersymmetry breaking throughout this article.

So far, no direct evidence for supersymmetric particle production at the LHC has been found (in other words, not enough collisions have been seen which predict high amounts of missing transverse momentum). In order to display this fact, experiments interpret their data in terms of exclusion bounds on supersymmetric models. For example, in figure 5, we see exclusion bounds on the universal model described above. As can be seen from the figure, there are many ways of sieving the data and looking for supersymmetric particles, with much associated activity by the experimental collaborations in doing so.

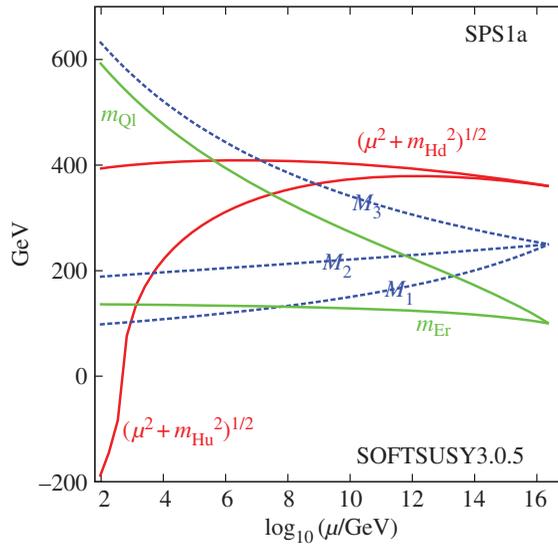


Figure 4. Example of quantum corrections splitting the various supersymmetric particle masses. On the right-hand side of the plot, the spin 0 particle masses m_{Er} and m_{QI} are equal. As the model is evolved down to lower energies μ relevant for experiments, the masses split apart due to differing quantum corrections. The same can be said of the gaugino masses M_1, M_2, M_3 . Obtained by the publicly available SOFTSUSY [7] program. The parameter point (SPS1a) is defined in [8]. (Online version in colour.)

3. Multiple solutions

In order to interpret the various supersymmetric particle searches, one must solve the differential equations that dictate the behaviour of the supersymmetric particle masses. One such solution is shown in figure 4. Such differential equations are shown by the Cauchy–Lipschitz theorem to have a unique solution, provided certain conditions hold (Lipschitz continuity, and that the boundary condition is fixed at one point; in this case at $M_{GUT} \sim 10^{16}$ GeV). In practice though, one of these conditions is violated, since, actually, the boundary conditions are set at radically different points: $\mu = M_{GUT}$, $\mu = M_Z = 91$ GeV and $\mu = M_S \sim 1000$ GeV. The problem is then a *boundary value problem* rather than an *initial value problem*. A cartoon of the situation is shown in figure 6. This means that the conditions of the uniqueness theorem do not hold.

In a recent publication [9], it was shown how the multiple boundary conditions allow several solutions to the system of boundary conditions and differential equations. These multiple solutions have some parameters being different, and therefore have different associated spectra. In principle, the different spectra can lead to different predictions for the outcome of collisions at the LHC. This leads to a potential loophole in interpretations of data, such as those in figure 5: *if one does not know of the existence of the additional solutions, one could be ruling out a point in parameter space from interpreting the data where one should not*. It was decided to investigate the properties of the additional solutions, and determine whether they could cause such loop-holes. New techniques had to be used to find the multiple solutions, because they are unstable to the usual algorithm for solving the system (fixed point iteration). In [10], we showed that the shooting method (with plenty of ‘shots’) can solve the problem.

Here, we shall illustrate some of the properties of the multiple solutions. For instance, in figure 7, we show the predictions for the Standard Model-like Higgs boson mass for a particular parameter choice, allowing the universal scalar mass m_0 to vary. $m_0 > 2000$ GeV is consistent with recent LHC measurements of a Higgs boson [1,2]. For $m_0 = 7572\text{--}7595$ GeV¹ there are three

¹As is well known, getting $m_h \sim 125$ GeV in the MSSM in general requires unnaturally large stop quark masses, and hence large m_0 in the universal model.

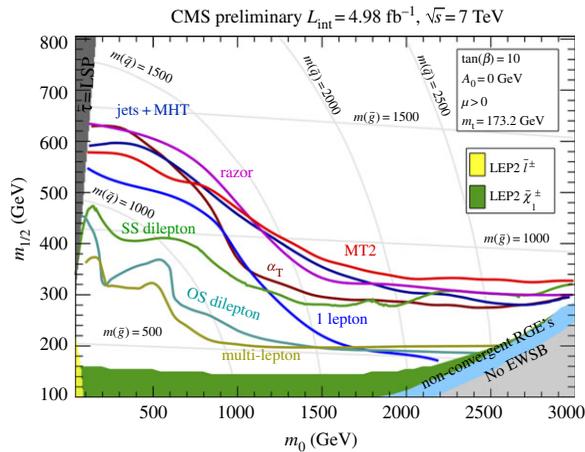


Figure 5. Exclusion limits from different experimental searches carried out at a centre of mass energy of 7 TeV at the LHC. Each curve shows exclusion limits derived from a different search. For each point in this parameter plane, the properties (such as masses) of the supersymmetric particles are set, although they vary from point to point. Each of the searches yielded a null result and so the area under each curve is ruled out by the corresponding search. The number of curves illustrates the many different ways in which supersymmetry is searched for at the LHC. (Online version in colour.)

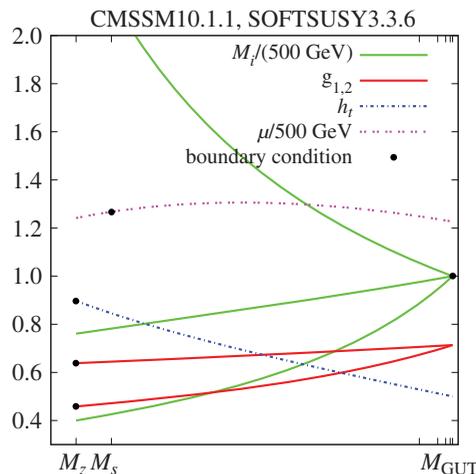


Figure 6. Boundary conditions in universal supersymmetry breaking models. We have shown the evolution of some of the mass parameters as the curves. The points show where boundary conditions are imposed. At M_z , we impose those coming from experimental measurements of Standard Model particle masses and couplings, at M_S boundary conditions coming from minimizing the Higgs potential are imposed, and at M_{GUT} , theoretical boundary conditions on the supersymmetry breaking terms are imposed (adapted from [9]). (Online version in colour.)

solutions, two with $\mu(M_S) < 0$, which have m_{h^0} in the range 125.4–125.7 GeV. We show another parameter point in table 1. The spectra show some notable differences between the different solutions, illustrating the fact that the solutions are physically different, leading to the possibility of their discrimination by collider measurements. Masses whose tree-level values depend upon the value of μ , such as the heavier neutralino and chargino masses, show the largest differences. Other sparticle and Higgs boson masses do have small per-mille level fractional differences for this parameter point.

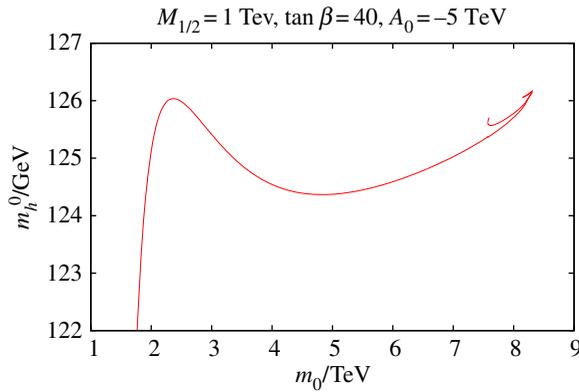


Figure 7. Multiple branches of solutions in the universal model at $M_{1/2} = 1$ TeV, $A_0 = -5$ TeV and $\tan \beta = 40$. The value of m_0 ranges as per the horizontal axis. We plot here the predicted values of the Higgs boson mass, m_{h^0} . There are three solutions in the range $7572 \text{ GeV} \leq m_0 \leq 7595 \text{ GeV}$ which are consistent with recent LHC measurements of a Higgs boson mass (adapted from [9]). (Online version in colour.)

Table 1. Differences in universal parameters and spectra for the multiple solutions of the parameter point $m_0 = 2.8$ TeV, $M_{1/2} = 660$ GeV, $\tan \beta = 40$ and $A_0 = 0$. We display here some masses (above the central horizontal line) and parameters (below the central horizontal line) of interest for the three solutions that predict the correct value of M_Z . Standard techniques only find solutions B and C, which are discriminated from each other by the input parameter which is the sign of the μ parameter. The first quantities listed are selected supersymmetric particle masses, whereas those under the horizontal lines are some underlying parameters of the model that are fixed by the boundary conditions (adapted from [9]).

quantity	solution A	solution B	solution C
$M_{\tilde{\chi}_1^0}/\text{GeV}$	282	282	281
$M_{\tilde{\chi}_2^0}/\text{GeV}$	502	497	471
$M_{\tilde{\chi}_3^0}/\text{GeV}$	558	548	510
$M_{\tilde{\chi}_4^0}/\text{GeV}$	610	605	593
$M_{\tilde{\chi}_1^\pm}/\text{GeV}$	503	497	470
$M_{\tilde{\chi}_2^\pm}/\text{GeV}$	609	604	592
$m_{\tilde{g}}/\text{GeV}$	1612	1612	1612
$\mu(M_S)/\text{GeV}$	-545	-535	497
$m_S^2(M_S)/10^5 \text{ GeV}^2$	0.800	0.809	1.07
$m_{H_u}^2(M_S)/10^5 \text{ GeV}^2$	-1.94	-1.83	-1.42
$h_t(M_S)$	0.840	0.839	0.836
$A_t(M_S)/\text{GeV}$	-1056	-1057	-1064
$M_X/10^{16} \text{ GeV}$	1.94	1.93	1.89
$g_1(M_Z)$	0.460	0.470	0.456
$g_2(M_Z)$	0.634	0.640	0.633

We show the number of solutions found along a particular parameter plane in figure 8a. We have chosen $A_0 = 0$, $\tan \beta = 10$ and $\mu > 0$ in particular because the LHC experiments ATLAS and CMS have interpreted their most sensitive searches in terms of exclusion regions with those values of the parameters (e.g. figure 5). We see that their excluded regions include points where up to

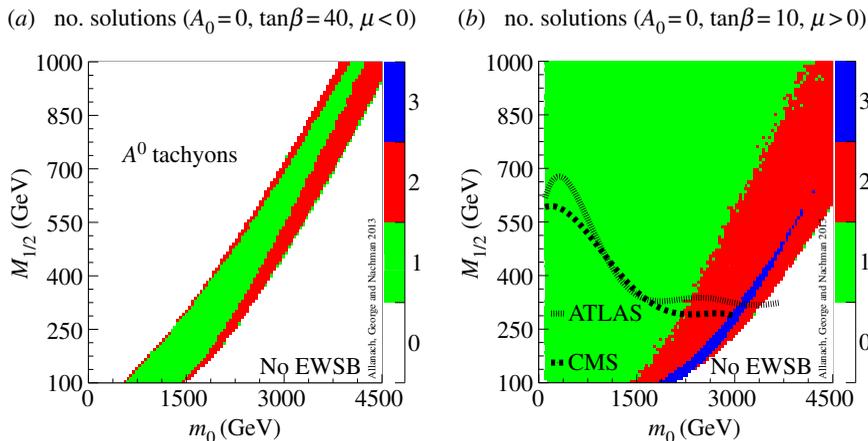


Figure 8. Number of solutions in the universal model as shown as the background colour and labelled in the key on the right-hand side of each plot. White regions have no solutions for the reasons labelled: ‘No EWSB’ denotes a region where there is no acceptable electroweak minimum of the Higgs potential. The lines in (a) display 95% exclusion contours from ATLAS [11] and CMS [12] jets plus missing transverse momentum searches. The region below each contour is excluded (adapted from [10]). (Online version in colour.)

three solutions are predicted. In fact, it turns out that in this plane, all of the multiple solutions have already been ruled out by previous experimental searches for charginos. Thus they do not present a problem.

At higher values of $\tan\beta$ and $\mu < 0$, there is a strip where this is not a problem, and is shown in figure 8b: the uppermost strip with two solutions (the lower strip is ruled out by having light charginos that would have been seen at the LEP2 collider). In fact, the multiple solutions in table 1 are taken from this ‘phenomenologically plausible strip’. Squarks from the first two generations and gluino masses display a negligible difference between the different solutions in this strip. The neutralino mass can be different by about 1–2%, depending upon the position in the plane. This means that, for the most stringent searches involving multiple jets of hadrons and the missing transverse momentum referred to above, if one of the solutions is ruled out by the analysis, the other solution will also be ruled out. On the other hand, chargino masses can be affected by 10% or so, and so interpretations of analyses which depend upon charginos in decay chains may be sensitive to the multiple solutions, and should be checked on a case-by-case basis.

Here, we exemplify the most notable difference between the solutions: that of a different predicted thermal relic density of dark matter in the universe. Recently, data from the Planck satellite have been used to derive the constraint [6] on the thermal dark matter relic density (see §1 for a heuristic explanation)

$$\Omega_{\text{CDM}}h^2 = 0.1198 \pm 0.0026. \quad (3.1)$$

We place a dominant theoretical uncertainty on our prediction of 0.01 coming from loops (the thermal relic density is only calculated by the publicly available micrOMEGAS [13,14] program to tree-level order), and therefore require the predicted thermal relic density of neutralinos to be $\Omega_{\text{CDM}}h^2 \in [0.0998, 0.1398]$. After a brief scan, we found a parameter point in the phenomenologically plausible strip ($m_0 = 760$ GeV, $M_{1/2} = 141.72$ GeV, $A_0 = 0$, $\tan\beta = 40$, $\mu < 0$) where the standard solution predicts $\Omega_{\text{CDM}}h^2 = 0.34$, i.e. it predicts far too much dark matter, but where the *additional* solution prediction of $\Omega_{\text{CDM}}h^2 = 0.118$ agrees with observations, being near the observationally constrained central value. It turns out that this point has the χ_1^0 mass being approximately half of the Higgs boson mass. The dark matter candidate χ_1^0 mass, which changes slightly between the solutions, is closer to half the Higgs boson mass for the additional solution, which leads to very efficient annihilation through a Higgs boson into quark or lepton

pairs, significantly reducing the relic density from 0.34 to 0.118. Here, we see the importance of the multiple solution: if we were not aware of it, we would certainly rule the model point out because it predicts far too much dark matter compared to observations. But then we see that the extra solution has just the right amount of dark matter and is allowed.

4. Summary

Supersymmetry is a well-motivated theory that explains how the Higgs mass is insensitive to potentially huge quantum corrections. It predicts a gamut of new particles with specific properties, which are being actively searched for at the LHC. In the absence of a signal of the production of supersymmetric particles (given simply by an excess of collisions in which there is a large apparent missing transverse momentum), the data are interpreted in terms of exclusion limits on models of supersymmetry breaking. Such exclusion limits have a potential loop-hole, due to the existence of multiple solutions, which have only just been found recently in the literature. Each limit, which was interpreted only as a single solution, should be checked to see whether it changes when the multiple solutions are taken into account. We have checked in universal supersymmetry breaking models that the most stringent searches—involving jets of hadrons and missing transverse momentum—are insensitive to the multiple solutions, for several reasons. Either the multiple solutions are ruled out by previous experiments because they predict very light charginos, or the masses of lightest neutralinos and squarks and gluinos are similar enough between the solutions such that if one solution is ruled out by an analysis, to a good approximation the other solution will be as well. However, more particular searches such as those involving searches for charginos are likely to display significant differences between solutions, and the limits must be interpreted carefully for each one in turn. Also, we have demonstrated with a parameter point example that the predicted density of dark matter left today in the universe can be very sensitive. The parameter point had far too much dark matter in the standard solution compared with the amount derived from observations, whereas the additional solution had just the right amount. Analyses employing the relic density of dark matter can therefore be particularly vulnerable to changes coming from the existence of additional solutions. We expect multiple solutions as a possibility whenever a high-scale supersymmetry breaking mechanism is active, such as in superstring-inspired models.

Data accessibility. All electronic supplementary material data are stored on the arxiv versions of refs [9,10].

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